Time Dep. S. Eq: \(- \frac{\hbar^2}{2m} \frac{d^2\Psi(x,t)}{dx^2} + U(x)\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}\)

Time Indep. S. Eq: \(- \frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)\)

Particle in box of length L: \(\psi_n(0 < x < L) = \sqrt{\frac{2}{L}} \sin \left(\frac{n\pi x}{L}\right)\) & \(E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}\)

Planck's constant \(\hbar = 6.626 \times 10^{-34}\text{J.s} = 4.136 \times 10^{-15}\text{eV.s}\)

1 eV = 1.60 \times 10^{-19} \text{J}

Speed of Light \(c = 3 \times 10^8 \text{m/s}\)

\(\bar{L} = \text{I\bar{\omega}}\)

Electron mass \(= 9.1 \times 10^{-31} \text{Kg} = 0.511 \text{MeV/c}^2\)

Energy in Hydrogen atom \(E_n = -\frac{ke^2}{2a_0} \left(\frac{1}{n^2}\right) = \left(-13.6 \text{eV}\right)\)

\(\sin \alpha \sin \beta = \frac{1}{2} (\cos (\alpha - \beta) - \cos (\alpha + \beta))\)

\(\int \sin^2 x \, dx = -\frac{1}{2} \cos x \sin x + \frac{1}{2} x = \frac{1}{2} x - \frac{1}{4} \sin 2x\)

Please consult the proctor if you don’t understand any part of the questions
Problem 1: Déjà vu All Over Again! [12 pts]

Consider a particle of mass \( m \) moving in a two-dimensional box defined by potential \( U=0 \) for \( 0 < x < L \) and \( -L/4 < y < L/4 \) (a) Draw this potential form in two dimensions (b) Find the normalization constant, the complete wavefunctions and probability densities for \( n = 1 \), \( n = 2 \) and \( n=3 \) states (c) Find the expression for energies \( E_n \) for these states. (d) Calculate the expectation value \( \langle y \rangle \) for the particle in \( n=2 \) state.

Problem 2: A “Spinning” Electron Is Such a Bogus Concept? [8 pts]

When the idea of electron spin was introduced, the electron was thought to be a tiny charged sphere of radius of about \( 3.0 \times 10^{-6} \) nm. The moment of inertia for a sphere is \( I = \frac{2}{5} mR^2 \). (a) What is the magnitude of the total angular momentum of an electron in the 1s state of the Hydrogen atom? (b) calculate the value of \( \omega \), the angular velocity of the electron’s spin rotation (c) Find the equatorial speed of the electron and compare it with the speed of light. (d) Based on your finding in (c) is the concept of a spinning electron Bogus or not Bogus? Pl. explain why?