



*Department of Physics  
University of California San Diego*

*Quantum Universe (4E)  
Prof. V. Sharma  
Quiz # 4 (May 27, 2005)*

$$\lambda = \frac{h}{p} \quad ; \quad \Delta x. \Delta p \geq \frac{h}{4\pi} \quad ; \quad \Delta E. \Delta t \geq \frac{h}{4\pi}$$

$$\text{Time Dep. S. Eq: } -\frac{\hbar^2}{2m} \frac{d^2\Psi(x,t)}{dx^2} + U(x)\Psi(x,t) = i\hbar \frac{\partial\Psi(x,t)}{\partial t}$$

$$\text{Time Indep. S. Eq: } -\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E \psi(x)$$

$$\text{Particle in box of length L: } \psi_n(0 < x < L) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad \& \quad E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

$$\text{Planck's constant } h = 6.626 \times 10^{-34} \text{ J.s} = 4.136 \times 10^{-15} \text{ eV.s}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$\text{Electron mass} = 9.1 \times 10^{-31} \text{ Kg} = 0.511 \text{ MeV}/c^2$$

$$\text{Energy in Hydrogen atom } E_n = \frac{-ke^2}{2a_0} \left( \frac{1}{n^2} \right) = \left( \frac{-13.6 \text{ eV}}{n^2} \right)$$

$$\sin \alpha \sin \beta = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\int \sin^2 x \, dx = -\frac{1}{2} \cos x \sin x + \frac{1}{2} x = \frac{1}{2} x - \frac{1}{4} \sin 2x$$

Please consult the proctor if you don't understand any part of the questions



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**Problem 1: Lazy “R” Us ! [8 pts]**

Consider a particle of mass  $m$  moving in a one-dimensional box with rigid walls (infinite potential) at  $x = -L/4$  and  $x = L/4$ . (a) Draw this potential form (b) Find the normalization constant, the complete wavefunctions and probability densities for state  $n = 1$ ,  $n = 2$  &  $n = 3$ . (c) Neatly sketch the wavefunctions and probability densities in each case. Indicate the locations of the walls.

**Problem 2: Episode II: Attack Of The Clones [12 pts]**

A free particle of mass  $m$  with wave number  $k_1$  is traveling in the right direction. Its energy  $E = \frac{\hbar^2 k_1^2}{2m} = 2V_0$ . At  $x=0$ , the potential step jumps from zero to  $-V_0$  and remains at this value for all positive  $x$ . (a) draw the potential (b) does the particle slow down or speed up in region  $x>0$ ? (c) What is the wave number  $k_2$  of the particle in  $x>0$ ? (d) Calculate the reflection coefficient  $R$  at the potential step. (e) What is the transmission coefficient  $T$ ? (f) If one million such particles with wave number  $k_1$  are incident on the potential step, how many particles are expected to continue along the positive  $x$  direction? (g) How does this answer compare with the classical prediction?