\[ \lambda = \frac{h}{p} ; \quad \Delta x. \Delta p \geq \frac{h}{4\pi} ; \quad \Delta E. \Delta t \geq \frac{h}{4\pi} \]

Time Dep. S. Eq: \[ -\frac{\hbar^2}{2m} \frac{d^2\Psi(x,t)}{dx^2} + U(x)\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t} \]

Time Indep. S. Eq: \[ -\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E \psi(x) \]

Particle in box of length L: \[ \psi_n(0 < x < L) = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) \] & \[ E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \]

Planck's constant \( \hbar = 6.626 \times 10^{-34} \text{ J.s} = 4.136 \times 10^{-15} \text{ eV.s} \)

1 eV = \( 1.60 \times 10^{-19} \) J

Electron mass \( = 9.1 \times 10^{-31} \) Kg = 0.511 MeV/c^2

Energy in Hydrogen atom \( E_n = -\frac{ke^2}{2a_0} \left( \frac{1}{n^2} \right) = \left( -13.6 \text{ eV} \right) \)

\[ \sin \alpha \sin \beta = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta)) \]

\[ \int \sin^2 x \, dx = -\frac{1}{2} \cos x \sin x + \frac{1}{2} x = \frac{1}{2} x - \frac{1}{4} \sin 2x \]

Please consult the proctor if you don’t understand any part of the questions.
Problem 1: Lazy “R” Us! [8 pts]

Consider a particle of mass m moving in a one-dimensional box with rigid walls (infinite potential) at x = - L/4 and x = L/4. (a) Draw this potential form (b) Find the normalization constant, the complete wavefunctions and probability densities for state n = 1, n = 2 & n = 3. (c) Neatly sketch the wavefunctions and probability densities in each case. Indicate the locations of the walls.

Problem 2: Episode II: Attack Of The Clones [12 pts]

A free particle of mass m with wave number k₁ is traveling in the right direction. Its energy \( E = \frac{\hbar^2 k_1^2}{2m} = 2V_0 \). At x=0, the potential step jumps from zero to \(-V_0\) and remains at this value for all positive x. (a) draw the potential (b) does the particle slow down or speed up in region x>0? (c) What is the wave number k₂ of the particle in x>0? (d) Calculate the reflection coefficient R at the potential step. (e) What is the transmission coefficient T? (f) If one million such particles with wave number k₁ are incident on the potential step, how many particles are expected to continue along the positive x direction? (g) How does this answer compare with the classical prediction?