Department of Physics
University of California San Diego

Quantum Universe (4E)
Prof. V. Sharma
Quiz # 3 (May 13, 2005)

Some Useful Numbers, Equations and Identities

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2} \quad ; \quad \Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

$$\text{TDSE}: \ -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

$$\text{TISE}: \ -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x) = E\psi(x)$$

$$\text{Operator}: \ [\hat{p}] = \frac{\hbar}{i}\frac{\partial}{\partial x}, \quad [\hat{p^2}] = -\hbar^2\frac{\partial^2}{\partial x^2}, \quad [\hat{E}] = i\hbar\frac{\partial}{\partial t}$$

$$\text{Uncertainty in observable Q}: \ \Delta Q = \sqrt{\langle Q^2\rangle - \langle Q\rangle^2}$$

$$\text{Expectation Value} < Q > \ = \int \psi(x)^*[Q]\psi(x)dx$$

$$\int_0^{+\infty} e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}, \ a > 0$$

$$\int_0^{+\infty} x^2 e^{-ax^2}\,dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}, \ a > 0$$

$$\int_0^{+\infty} x^{(2n)} e^{-ax^2}\,dx = \frac{1.3..(2n-1)}{2^{(n+1)}}\sqrt{\frac{\pi}{a^{2n+1}}}, \ a > 0$$

$$\int_0^{+\infty} x^{(2n+1)} e^{-ax^2}\,dx = \frac{n!}{2a^{(n+1)}}, \ a > 0$$

For a Quantum oscillator under $U(x) = \frac{1}{2}m\omega^2x^2$ the ground state wave-function is: $\psi_0(x) = (\frac{m\omega}{\pi\hbar})^{1/4} \times e^{-\frac{m\omega x^2}{2\hbar}}$ and energy is $E = (1/2)\,\hbar\omega$

Please consult the proctor if you don’t understand any part of the questions

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Problem 1 : Déjà Vu all over again ! [12 pts]

(a) Calculate $<x>, <x^2>$ and Δx for a quantum oscillator of mass m and angular frequency ω in its ground state. (b) What is the average potential energy for this system. Calculate (c) the average momentum <p>, (d) the average kinetic energy<KE> , (e) $<p^2>$ and (f) the product $\Delta x.\Delta p$.

Problem 2: A Step in the Right Direction [8 pts]

A particle is incident on a potential step characterized by

$$U(x) = 0 \quad \text{for } x < 0$$
$$U(x) = U \quad \text{for } x > 0$$

with a kinetic energy E < U. It is described by the wavefunction

$$\psi(x) = \frac{1}{2}\left\{(1+i)e^{ikx} + (1-i)e^{-ikx}\right\} \quad \text{for } x \le 0$$
$$\psi(x) = e^{-kx} \quad \text{for } x \ge 0$$

(a) verify by direct calculation that the reflection coefficient **R** is unity (b) How must *k* be related to E in order for $\psi(x)$ to solve Schrodinger equation in the region to the left of the step ($x \le 0$) **and** to the right of the step. Calculate the relation between energy E and the potential U.