Some Useful Numbers, Equations and Identities

Speed of Light, \( c = 2.998 \times 10^8 \text{m/s} \)

Planck’s Constant, \( h = 6.626 \times 10^{-34} \text{J} \times \text{S} = 4.136 \times 10^{-15} \text{eV} \times \text{S} \)

\( 1 \text{ eV} = 1.60 \times 10^{-19} \text{J}; \quad 1 \text{ MeV/c} = 5.344 \times 10^{-22} \text{Kg.m/s} \)

Coulomb’s Constant, \( k = 8.99 \times 10^9 \text{N} \times \text{m}^2/\text{C}^2 \)

Electron Mass = \( 9.11 \times 10^{-31} \text{Kg}; \quad \) Electron Charge = \( 1.602 \times 10^{-19} \text{C} \)

Atomic Mass Unit \( u = 1.6606 \times 10^{-27} \text{Kg} \) or \( 931.5 \text{ MeV/c}^2 \)

Proton Mass = \( 1.673 \times 10^{-27} \text{Kg}; \quad \) Neutron Mass = \( 1.675 \times 10^{-27} \text{Kg} \)

Electron Rest Energy = \( 0.511 \text{ MeV/c}^2; \quad \) Proton Rest Energy = \( 938 \text{ MeV/c}^2 \)

Force on a charged particle in \( B \) field: \( \vec{F} = q \vec{v} \times \vec{B} \)

Centripetal Acc. = \( v^2/R \) where \( v \) and \( R \) are velocity and radius of orbit

Bohr Radius \( a_0 = 0.529 \times 10^{-10} \text{m} \)

\( \Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}; \quad \Delta E \cdot \Delta t \geq \frac{\hbar}{2} \)

Construct. interfer. when path diff between two adjacent rays is \( d \sin \phi = n \lambda \)

\[ \int u \cdot dv = uv - \int v \cdot du \]

Please consult the proctor if you don’t understand any part of the questions
Problem 1: Seeing Is Believing [8 pts]
Suppose one undertook a study of the orbits in Hydrogen. According to the Bohr theory, the circular orbits have radii \( r_n = n^2 a_0 \), where \( a_0 \approx 0.05\text{nm} \) is the Bohr radius. To map the orbits with some precision, it is necessary to localize the electrons with a precision of say \( 0.1a_0 \). (a) What amount of kinetic energy (in eV) is likely to be transferred to the electron that is under observation with a probe that can localize the electrons with this precision? (b) Will the Hydrogen atom remain relatively undisturbed by the observation? (c) What do you think of the idea of atomic orbits?

Problem 2: Planck’s Oscillators [12 pts]
A subatomic particle is connected to a spring and undergoes one-dimensional motion. (a) Write an expression for the total energy in terms of its position \( x \), its mass \( m \), its momentum \( p \) and the angular frequency of oscillation \( \omega = \sqrt{\frac{k}{m}} \). What is the smallest energy \( E_c \) this oscillator can have according to classical physics? (b) Now treat the particle as a wave. Assume that the product of the uncertainties in position and momentum is governed by the Uncertainly Principle. Also assume that since \( x \) and \( p \) are on average zero, the uncertainty \( \Delta x \) and \( \Delta p \) are typical value of the particle’s position and momentum respectively. Calculate the minimum energy \( E_q \) of this oscillator. (c) How does it compare with \( E_c \)? (d) How does \( E_q \) compare with minimum energy of Planck’s oscillator?