Department of Physics
University of California San Diego

Modern Physics (4E)
Prof. V. Sharma
Final Exam, June 6, 2005

Problem 1: Mystery Metal [20 pts]

Photons of wavelength 450nm are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular arc of radius 20cm by a magnetic field whose strength is equal to 2.0×10^{-5} T. What is the work function of the metal?

Problem 2: Quantum Pool [25 pts]

In a Compton experiment, an x-ray photon scatters through an angle of 17.4^{o} from a free electron that is initially at rest. The electron recoils with a speed of 2180 km/s. Calculate (a) the wavelength of the incident photon and (b) the angle through which the electron scatters.

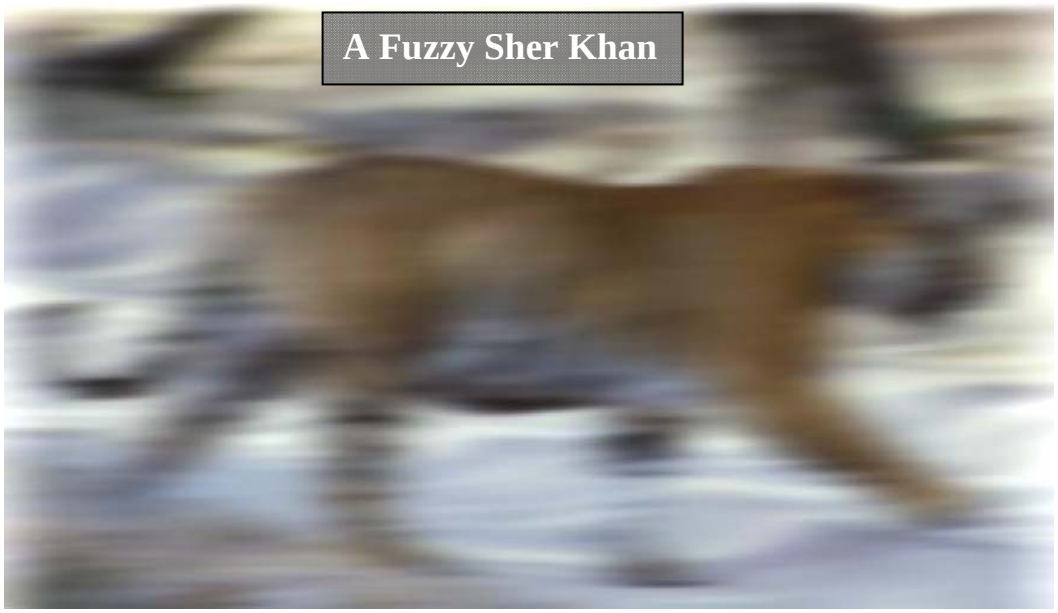
Problem 3: The Need For Speed ! [20 pts]

The radius of the gold (Au) atom has been measured by high energy electron scattering at 6.6fm. (a) α-particles (He^{++} ions) of what speed v_{α} would Rutherford have needed so that for a 180° scattering angle, the α-particle would just reach the nuclear surface before reversing direction? (b) Can you justify the usage of non-relativistic kinematics in calculating the speed of the alpha particle ?

Ask the proctor if you have difficulty understanding the problem or if you think you need some additional formula

Problem 4: Tiger Hunting in a Quantum Jungle ! [20 pts]

Somewhere in the Himalayan mountain range there are rumors of a mysterious Quantum jungle where the value of the Planck's constant h is much larger than our usual world. Imagine that you are in this quantum jungle where h=50 J.s !! **Sher Khan**, the tiger, runs past you in the bushes a few meters away. The tiger, weighing 100kg, is known to be in a region about 4m long. (a) What is the minimum uncertainty in his speed? (b) Assuming this uncertainty in his speed to prevail for 10 seconds, determine the uncertainty in his position after this time.



Problem 5: Guessing The H Atom With Uncertainty Principle [30 pts]

In a Hydrogen atom an electron of momentum p is at a distance x from a stationary proton. The bound state has kinetic energy $K = \frac{p^2}{2m_e}$ and potential energy $U = -\frac{ke^2}{x}$. Treat this atom as a one dimensional system in x and answer the following using order of magnitude argument and the Uncertainty principle. (a) What is the average position <x> of the electron? (b) What is the uncertainty in the location in <x> of the electron? (c) What is the average momentum <p> of the electron? (d) Estimate the uncertainty in the electron's momentum <p> in terms of x (e) Write the expression for the atom's kinetic, potential and total energies in terms of x (f) find the value of x that minimizes the energy of the atom. (g) In this way calculate the "ground state" energy of the stable Hydrogen atom and compare it with the prediction of Bohr's theory.

Problem 6: Classical & Quantum Oscillators Compared [20 pts]

Consider a one-dimensional quantum harmonic oscillator of mass m under the potential $U(x) = \frac{1}{2}m\omega^2 x^2$ where ω is the angular frequency of oscillation. Consider the oscillator in its ground state with energy $E = \frac{1}{2}\hbar\omega$ and wavefunction $\psi(x) = \left[\frac{m\omega}{\pi\hbar}\right]^{\frac{1}{4}} e^{\frac{-m\omega x^2}{2\hbar}}$. (a) Write an expression for the limit of vibration (or amplitude of oscillation **A**) for a classical oscillator having the same energy as the quantum oscillator in its ground state. In other words express A in terms of $\hbar, m$ and ω. (b) Can the classical oscillator be found in regions beyond x=±A? Why? (c) Calculate the probability (in %) that a quantum oscillator in its ground state will be found outside the range permitted for a classical oscillator with the same energy. Use :

$$\mathrm{erfc}(1) = \frac{2}{\sqrt{\pi}}\int_1^\infty e^{-z^2}dz = 0.157$$

Problem 7: Facts About the Hydrogen Atom in The 2p state [40 pts]

Consider a Hydrogen atom in the 2p state. The radial part of the wavefunction is $R_{2p}(r) = \left[\frac{1}{2a_0}\right]^{\frac{3}{2}} \left[\frac{r}{\sqrt{3}\, a_0}\right] e^{\frac{-r}{2a_0}}$ where a_0 is the Bohr radius.

Calculate (a) the average distance <r> of electron's location with respect to the nucleus (b) the most probable distance of the electron's location (c) the average potential energy <U> and (d) the average kinetic energy <KE>.

Problem 8: Spin-Orbit Coupling In Hydrogen [25 pts]

The prominent yellow doublet line in the spectrum of Sodium results from transition from the $3P_{3/2}$ and $3P_{1/2}$ states to the ground state. The wavelengths of the two lines are 589.6nm and 589.0nm. (a) Calculate the energies in eV of the photons corresponding to these wavelengths. (b) The difference in energy of these photons equals the difference in energy ΔE of the $3P_{3/2}$ and $3P_{1/2}$ states. As you know this energy is due to the spin-orbit effect. Calculate the value of this ΔE. (c) Next, calculate the magnetic field B (in units of Tesla) that the 3p electron in sodium experiences due to the proton's motion.

Good Luck &

Have A Great Summer !