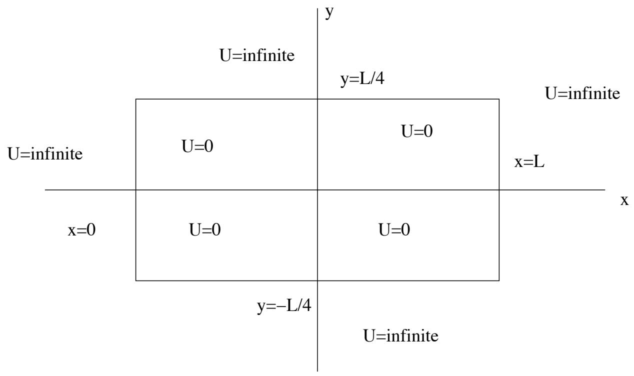
1. Problem 1

a)



b) For the full detail see last week's quiz. For these solutions I will use a shortcut. First we know $\Psi(x,y) = \psi_1(x)\psi_2(y)$. Also $\psi_1(x)$ is given on the formula sheet, $\psi_1(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$. To find $\psi_2(y)$ we just tweak this answer a bit. The length of the box in this direction is $L/2$ not L. So first we replace L in the given solution with $L/2$. Also our box now begins at $-L/4$, not 0. To compensate we subtract $-L/4$ from y. Thus our answer is: $\psi_2(y) = \sqrt{\frac{2}{L/2}} \sin\left(\frac{n\pi}{L/2}(y + L/4)\right)$. So the wavefunctions are:

$$\Psi(x,y) = \frac{2\sqrt{2}}{L} \sin\left(\frac{n_x\pi}{L}x\right) \sin\left(\frac{2n_y\pi}{L}(y + L/4)\right) \tag{1.1}$$

We can see that the normalization constant is $\frac{2\sqrt{2}}{L}$. The corresponding probability densities are:

$$P(x,y) = |\Psi(x,y)|^2 = \frac{8}{L^2} \sin^2\left(\frac{n_x\pi}{L}x\right) \sin^2\left(\frac{2n_y\pi}{L}(y + L/4)\right) \tag{1.2}$$

Now we must determine which are the three lowest energy states, what on the quiz was called "$n = 1$, $n = 2$ and $n = 3$." From part c) below we see that the three lowest energy states are $(n_x, n_y) = (1,1)$, $(2,1)$ and $(3,1)$. Plugging these values into (1.1) and (1.2) give the wavefunctions and probability densites for these states.

c) We have that $E_{n_x,n_y} = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2\right)$, where k is what multiplies x in the wavefunction. Therefore:

$$\begin{aligned} E_{n_x,n_y} &= \frac{\hbar^2}{2m}\left((\frac{n_x\pi}{L})^2 + (\frac{2n_y\pi}{L})^2\right) \\ &= \frac{\hbar^2\pi^2}{2mL^2}(n_x^2 + 4n_y^2) \end{aligned} \tag{1.3}$$

d) For $n = 2$, that is $(n_x, n_y) = (2, 1)$, $\Psi(x, y) = \frac{2\sqrt{2}}{L} \sin\left(\frac{2\pi}{L}x\right) \sin\left(\frac{2\pi}{L}(y + \frac{L}{4})\right)$. Thus:

$$\begin{aligned}
\langle y \rangle &= \int_0^L dx \int_{-L/4}^{L/4} dy y |\Psi(x, y)|^2 \\
&= \int dy y \frac{4}{L} \sin^2\left(\frac{2\pi}{L}(y + L/4)\right) \\
& \text{The } x \text{ part of the integral just cancelled} \\
& \text{the x part of the normalization constant} \\
\langle y \rangle &= 4/L \int_0^{L/2} (y' - L/4) \sin^2\left(\frac{2\pi}{L}y'\right) \\
&= 4/L \left(\int_0^\pi \left(\frac{L}{2\pi}\right)^2 y'' \sin^2 y'' dy'' - \frac{L^2}{8\pi} \int_0^\pi \sin^2 y'' dy'' \right) \\
&= \frac{4}{L} \left(\frac{L^2}{4\pi^2} \left((y'')^2/4 - \cos(2y'')/8 - y'' \sin(2y'')\right) |_0^\pi - \frac{L^2}{8\pi} \left(y''/2 - \sin(2y'')/4\right) |_0^\pi \right) \\
&= \frac{4}{L} \left(\frac{L^2}{4\pi^2} \left(\frac{\pi^2}{4}\right) - \frac{L^2}{8\pi} \left(\frac{\pi}{2}\right) \right) \\
&= 0
\end{aligned} \tag{1.4}$$

This is the answer we knew we had to get. Above $y' = (y + L/4)$ and $y'' = \frac{2\pi}{L}y'$.

2. Problem 2

a) In the $1s$ state $\ell = 0$, thus the total angular momentum is just from the spin. $S = \sqrt{s(s+1)}\hbar = \frac{\sqrt{3}}{2}\hbar$.

b) $S = I\omega$, so:

$$\omega = S/I = \frac{\frac{\sqrt{3}}{2}\hbar}{\frac{2}{5}mR^2} = \frac{5\sqrt{3}}{4} \frac{\hbar}{mR^2} \tag{2.1}$$

c)Since $v = \omega R$, we have $v = \frac{5\sqrt{3}}{4} \frac{\hbar}{mR} = 8.35 \times 10^{10} \frac{m}{s}$. That is $v = 278c$, where c is the speed of light.

d) Very much bogus. if something were to sit on the outside of the electron it would be traveling at many times the speed of light.