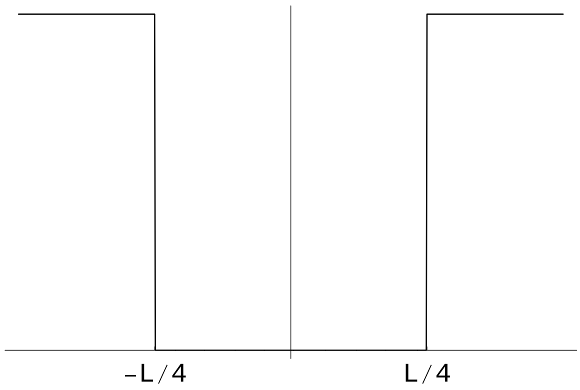
1. Problem 1

a)



b) Outside the well $\psi = 0$, and inside: $\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi$

Our boundary conditions are that $\psi(-L/4) = \psi(L/4) = 0$. $\psi = A\cos(kx) + B\sin(kx)$. Using our B.C.'s we know that $k_1 L/4 = \frac{2n-1}{2}\pi$ for the cos term. For the sin term we have: $k_2 L/4 = n\pi$. So $k_1 = \frac{2(2n-1)\pi}{L}$ and $k_2 = \frac{4n\pi}{L}$. These are not compatible, so our wavefunctions are either $A\cos(k_1 x)$ or $B\sin(k_2 x)$. These give energies of $E_n^{(2)} = \frac{\hbar^2 k_2^2}{2m} = 8\frac{\hbar^2 n^2 \pi^2}{mL^2}$ and $E_n^{(1)} = 2\frac{(2n-1)^2\hbar^2\pi^2}{mL^2}$. The first few values of these are (in terms of $E_0 = \hbar^2\pi^2/mL^2$) :

$$\begin{aligned} E_1^{(2)} &= 8E_0 \\ E_2^{(2)} &= 32E_0 \\ E_3^{(2)} &= 72E_0 \\ E_1^{(1)} &= 2E_0 \\ E_2^{(1)} &= 18E_0 \\ E_3^{(1)} &= 50E_0 \end{aligned} \tag{1.1}$$

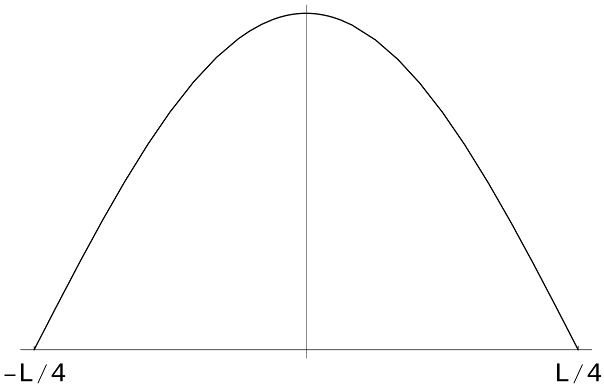
For the normalization:

$$
\begin{aligned}
1 &= \int |\psi|^2 dx \\
&= A^2 \int_{-L/4}^{L/4} \cos^2(\frac{2(2n-1)\pi}{L}x) \\
&= A^2 \int (1 - \sin^2(\frac{2(2n-1)\pi}{L}x) \\
&= A^2[x - (\frac{x}{2} - \frac{1}{4k}\sin(2kx)]|_{-L/4}^{L/4} \\
&= A^2(\frac{L}{4}) \\
A &= \frac{2}{\sqrt{L}}
\end{aligned}
\tag{1.2}
$$

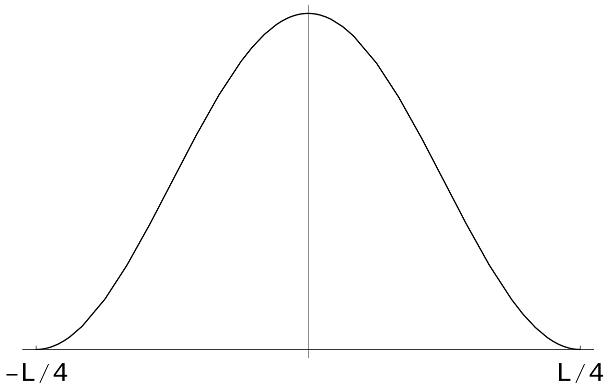
So the normalization constant is $\frac{2}{\sqrt{L}}$. We will find the same thing for B. The first three wave functions with associated probability densities are:

$$
\begin{aligned}
\psi_1 &= \frac{2}{\sqrt{L}}\cos(\frac{2\pi x}{L}) \qquad & |\psi_1|^2 &= \frac{4}{L}\cos^2(\frac{2\pi x}{L}) \\
\psi_2 &= \frac{2}{\sqrt{L}}\sin(\frac{4\pi x}{L}) \qquad & |\psi_2|^2 &= \frac{4}{L}\sin^2(\frac{4\pi x}{L}) \\
\psi_3 &= \frac{2}{\sqrt{L}}\cos(\frac{6\pi x}{L}) \qquad & |\psi_3|^2 &= \frac{4}{L}\cos^2(\frac{6\pi x}{L})
\end{aligned}
\tag{1.3}
$$

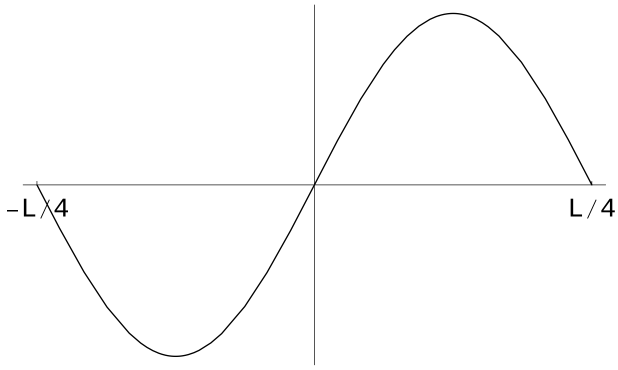
c)ψ_1



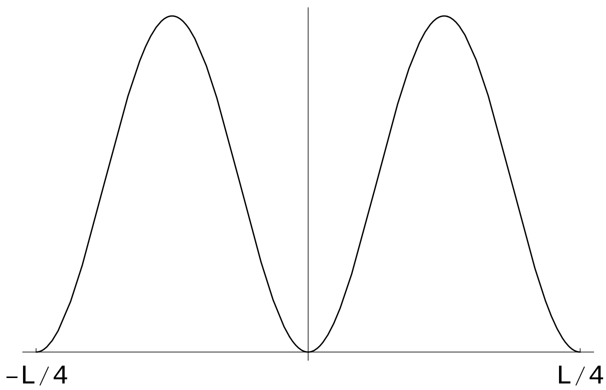
$|\psi_1|^2$



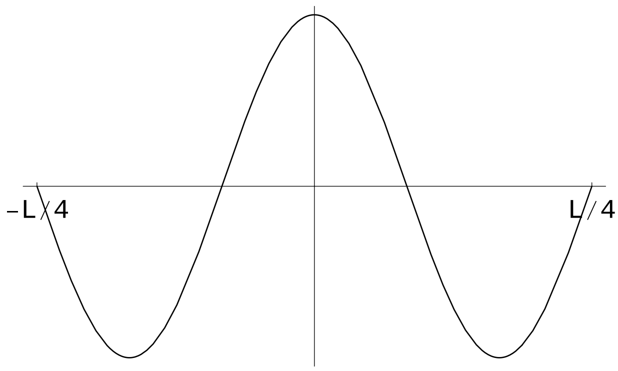
ψ_2



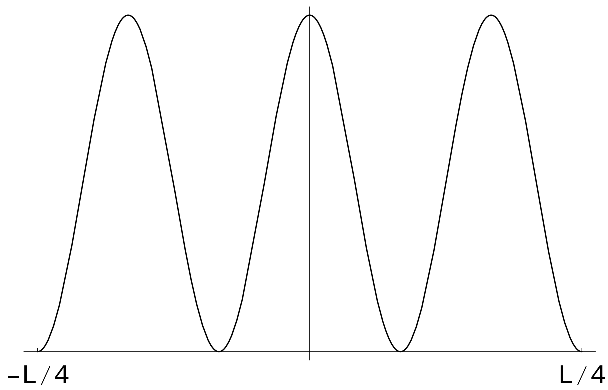
$|\psi_2|^2$



ψ_3

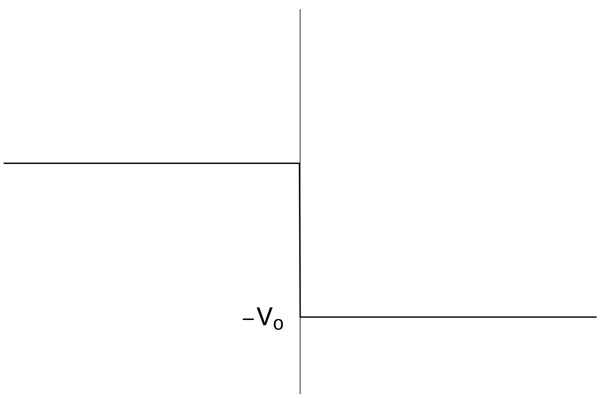


$|\psi_3|^2$



2. Problem 2

a)



b) Speeds up. The total energy is the same, but the potential energy decreases, so KE must increase.

c) From the S. Equation:

$$\begin{aligned}\frac{\hbar^2 k_2^2}{2m} - V_0 &= 2V_0 \\ k_2^2 &= \frac{6mV_0}{\hbar^2} \\ k_2 &= \sqrt{\frac{6mV_0}{\hbar^2}} = \sqrt{\frac{3k_1^2}{2}} = \sqrt{\frac{3}{2}}k_1\end{aligned} \tag{2.1}$$

d) For ψ we have:

$$\begin{aligned}\psi_{x<0} &= Ae^{ik_1x} + Be^{-ik_1x} \\ \psi_{x>0} &= Ce^{ik_2x}\end{aligned} \tag{2.2}$$

Continuity of ψ and ψ' gives:

$$\begin{aligned}A + B &= C \\ ik_1(A - B) &= ik_2C \\ A + B = C &= \frac{k_1}{k_2}(A - B) \\ 1 + \frac{B}{A} &= \frac{k_1}{k_2}(1 - \frac{B}{A}) \\ \frac{B}{A} &= \frac{k_1 - k_2}{k_1 + k_2} \\ R = \frac{|B|^2}{|A|^2} &= \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2 \\ &= \left(\frac{1 - \sqrt{3/2}}{1 + \sqrt{3/2}}\right)^2 = 0.0102\end{aligned} \tag{2.3}$$

e) $T = 1 - R = 0.9898$

f) $10^6 \times T = 989,800$

g) Classically all 10^6 would continue.