1. Problem 1

a) A probe which can localize an electron within $0.1a_0$, must be at least this small. We will take this to be Δx. From the uncertainty principle the momentum of this probe must have uncertainty of $\Delta p \approx \hbar/\Delta x$ ($\hbar/2\Delta x$ is also acceptable). Thus $\Delta p = 10\hbar/a_0$. This is approximately how much momentum that is transferred to the electron. The angular momentum is quantized: $mvr = n\hbar$. So the momentum of an electron in the hydrogen will have a momentum $p = n\hbar/r_n = \hbar/na_0$. So we have:

$$\begin{aligned}
KE &= \frac{p^2}{2m} \\
\Delta KE &= \frac{(p+\Delta p)^2}{2m} - \frac{p^2}{2m} \\
&= \frac{1}{2m}(\Delta p^2 + 2p\Delta p) \\
&= \frac{1}{2m}(100\frac{\hbar^2}{a_0^2} + 20\frac{\hbar^2}{na_0^2}) \\
&= 10\frac{\hbar^2}{ma_0^2}(5+\frac{1}{n}) \\
&= 305eV(5+\frac{1}{n})
\end{aligned} \tag{1.1}$$

If you used $\Delta p = \hbar/2\Delta x$, you would find: $\Delta KE = 76.3eV(5+\frac{2}{n})$.

b) This is much larger than the ionization energy, so if we did localize the electron to a region this small, it would be ripped from the atom.

c) The electron can not be localized to a small band, so the solar system type structure of an atom is not a reasonable model.

2. Problem 2

a) For a spring the potential energy is $\frac{1}{2}kx^2$, in terms of ω:

$$E = \frac{p^2}{2m} + \tfrac{1}{2}m\omega^2x^2 \tag{2.1}$$

The minimum classical energy E_c is zero. This is because both p and x can be exactly zero classically.

b) From the uncertainty principle we know $\Delta p = \hbar/2\Delta x$. Taking $p = \Delta p$ and $x = \Delta x$, (2.1) becomes:

$$
\begin{aligned}
E &= \frac{\Delta p^2}{2m} + \tfrac{1}{2} m\omega^2 \Delta x^2 \\
&= \frac{\hbar^2}{8m\Delta x^2} + \tfrac{1}{2} m\omega^2 \Delta x^2 \\
&\text{differentiating w.r.t } \Delta x \text{ and setting to 0 to minimize, we find:} \\
0 &= \frac{-\hbar^2}{4m\Delta x^3} + m\omega^2 \Delta x \\
\Delta x &= \sqrt{\frac{\hbar}{2m\omega}} \qquad (2.2) \\
&\text{plugging back in:} \\
E_q &= \frac{\hbar^2}{8m(\frac{\hbar}{2m\omega})} + \tfrac{1}{2} m\omega^2 (\frac{\hbar}{2m\omega}) \\
&= \frac{\hbar\omega}{4} + \frac{\hbar\omega}{4} \\
E_q &= \tfrac{1}{2}\hbar\omega
\end{aligned}
$$

c) This is larger, a quantum oscillator cannot have zero energy.

d) Planck's oscillator had energy $E_n = nhf = n\hbar\omega$ for $n = 1, 2..., \infty$. The mimimum energy of the Planck oscillator is twice E_q. If you had used $\Delta x \Delta p = \hbar$, you would have found that $E_q = E_1$.