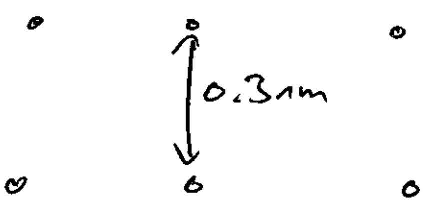
Phys 4E Week 3 HW

S: 13, 19, 22, 26, 30, 31, 38, 43, 48, 49, 50, 52

13| It's ambiguous, but say the distance between the horizontal planes is $D = 0.3$ nm.

Then $n\lambda = D\sin\phi$



$$\Rightarrow \lambda = .3\sin(42^\circ) = \boxed{.2\text{ nm}}$$

So $$p = \frac{h}{\lambda} = \frac{hc}{\lambda c} = \frac{1240\text{ eV}\cdot\text{nm}}{(.2\text{nm})\cdot c}$$

$$= 6.2\text{ keV}/c$$

This is small for neutrons, so use non-relativistic

$$KE = \frac{p^2}{2m} \Rightarrow \frac{(pc)^2}{2mc^2} = \frac{(6.2\text{keV})^2}{2(939\text{MeV})} = \boxed{2\times10^{-2}\text{ eV}}$$

1. 5-17

(a)

$$\begin{aligned} y(x,t) &= y_1(x,t) + y_2(x,t) \\ &= 2(A)\cos\left(\tfrac{1}{2}(k_2-k_1)x - \tfrac{1}{2}(\omega_2-\omega_1)t\right)\cos\left(\frac{k_1+k_2}{2}x - \frac{\omega_1+\omega_2}{2}t\right) \\ &= (0.004m)\cos\left(0.2x/m - 10t/s\right)\cos\left(7.8x/m - 390t/s\right) \end{aligned} \tag{1.1}$$

(b)

$$\begin{aligned} v &= \frac{\bar{\omega}}{\bar{k}} = \frac{\omega_1+\omega_2}{k_1+k_2} \\ &= 50\frac{m}{s} \end{aligned} \tag{1.2}$$

(c)

$$\begin{aligned} v_g &= \frac{\Delta\omega}{\Delta k} \\ &= \frac{\omega_1-\omega_2}{k_1-k_2} \\ &= 50\frac{m}{s} \end{aligned} \tag{1.3}$$

(d) Successive zeros requires $0.2\Delta x = \pi m$, thus $\Delta x = \frac{\pi}{0.2}m = 5\pi m$, and $Deltak = 0.4m^{-1}$

2. 5-18

(a) We start with $v = f\lambda$ and differentiate:

$$\begin{aligned} \frac{dv}{d\lambda} &= f + \lambda\frac{df}{d\lambda} \\ \lambda\frac{dv}{d\lambda} &= f\lambda + \lambda^2\frac{df}{d\lambda} \\ &= v + \frac{\lambda^2}{2\pi}\frac{d\omega}{d\lambda} \\ \frac{-\lambda^2}{2\pi}\frac{d\omega}{d\lambda} &= v - \lambda\frac{dv}{d\lambda} \\ &\text{Using } k = 2\pi/\lambda \\ \frac{d\omega}{dk} = v_g &= v - \lambda\frac{dv}{d\lambda} \end{aligned} \tag{2.1}$$

(b) v decreases as λ decreases, so $\frac{dv}{d\lambda}$ is positive.

19]

a] The wave travels out at speed c, for .25 μs

∴ Length of packet = $(.25\,\mu s) \times c = \boxed{75\ m}$

b] $f = \frac{c}{\lambda} = \boxed{1.5 \times 10^{10}\ Hz}$ This is the peak wavelength in the packet.

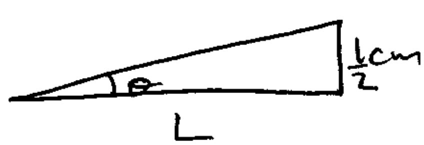
c] $\Delta\omega \Delta t \approx 1$, and since $\Delta t \approx .25\,\mu s$

$$\Rightarrow \Delta f \approx \frac{1}{2\pi(.25\,\mu s)} = \boxed{637\ kHz}$$

22] $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{hc}{\sqrt{2mc^2E}} = \frac{1240\ eV\cdot nm}{\sqrt{2(511\ keV)(5\ eV)}}$$

$$= 0.549\ nm$$

So $d\sin\theta = \frac{\lambda}{2} \Rightarrow \boxed{d = 3.15\ nm}$

b] 

$\Rightarrow \tan\theta = \frac{\frac{1}{2}\ cm}{L}$ for $\theta = 5°$

$$\Rightarrow \boxed{L = 5.7\ cm}$$

26]

a] Don't really know when to stop counting - how small does it have to get?

$$f = \frac{1}{T} \Rightarrow T = \frac{\Delta t}{N}, \text{ so } f = \frac{N}{\Delta t}.$$

$$\therefore \Delta f = \frac{\Delta N}{\Delta t} \approx \boxed{\frac{1}{\Delta t}}$$

b] $$\lambda = \frac{\Delta x}{N} \Rightarrow k = \frac{2\pi N}{\Delta x} \Rightarrow \Delta k = \frac{2\pi \Delta N}{\Delta x} = \boxed{\frac{2\pi}{\Delta x}}$$

30] $$\Delta x \Delta p \approx \frac{\hbar}{2} \Rightarrow \lambda \Delta p \approx \frac{\hbar}{2} \Rightarrow \Delta p \approx \left(\frac{h}{\lambda}\right)\frac{1}{4\pi}$$

$$\Rightarrow \boxed{\Delta p \approx \frac{p}{4\pi}}$$

31] Set $\Delta p \approx p = 900 \frac{kg\,m}{s}$

So $$\Delta x = \frac{\hbar}{2(\Delta p)} = \frac{50}{2(900)} = \boxed{2.8 \text{ cm}}$$

You could "see" the uncertainty - it would be fuzzy!

38] Set $\Delta x = 1\ \text{fm}$

$$p \simeq \Delta p \approx \frac{\hbar}{2(1\text{fm})} = 98.5\ \frac{\text{MeV}}{c}$$

for neutron, not relativistic $\Rightarrow E = \frac{p^2}{2m} = \boxed{5\ \text{MeV}}$

Electron is relativistic: $E = \sqrt{p^2c^2 + m^2c^4}$

So $KE = \sqrt{p^2c^2+m^2c^4} - mc^2 = \boxed{98.0\ \text{MeV}}$

42] $\lambda = \frac{2L}{n}$, so $p = \frac{h}{\lambda} = \frac{hn}{2L}$

Now, $E = \frac{p^2}{2m} = \frac{h^2 n^2}{8mL^2} = \boxed{n^2\left(\frac{h^2}{8mL^2}\right) \equiv n^2 E_1}$

b] $E_1 = 37.6\ \text{eV}$, so $\boxed{E_n = n^2(37.6\ \text{eV})}$

c] $f = 3(37.6\text{eV})/h \Rightarrow \lambda = \frac{hc}{3(37.6\text{eV})} = \boxed{11\ \text{nm}}$

d] $\lambda = hc/5(37.6\ \text{eV}) = \boxed{6.6\ \text{nm}}$

e] $\lambda = hc/24(37.6\text{eV}) = \boxed{1.4\ \text{nm}}$

3. 5-39

(a) We will need: $E = hf = \hbar\omega$ and $p = \frac{h}{\lambda} = \hbar k$.

$$\begin{aligned} E^2 &= p^2c^2 + m^2c^4 \\ \hbar^2\omega^2 &= \hbar^2k^2c^2 + m^2c^4 \\ v &= \frac{\omega}{k} = \frac{\hbar\omega}{\hbar k} \\ &= \frac{\sqrt{\hbar^2k^2c^2 + m^4c^2}}{\hbar k} \\ &= c\sqrt{1 + \frac{m^2c^2}{\hbar^2k^2}} > c \end{aligned} \tag{3.1}$$

(b)

$$\begin{aligned} v_g &= \frac{d\omega}{dk} \\ &= \frac{d}{dk}\sqrt{k^2c^2 + m^2c^4/\hbar^2} \\ &= \frac{c^2k}{\sqrt{k^2c^2 + m^2c^4/\hbar^2}} = \frac{c^2k}{\omega} = \frac{c^2p}{E} = u \end{aligned} \tag{3.2}$$

4. 5-47

(a) The particles are moving at $0.01c$, so we can ignore relativity.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = 2.43 \times 10^{-10} \tag{4.1}$$

(b) Since the photons have equal energy, they also have equal momentum and wavelength. The incoming energy of the electron and positron are $E = mc^2 + \frac{1}{2}mv^2$, and since v is much smaller than c, we can ignore the second term. Thus the incoming energy of the electron and positron are both $mc^2 = 0.511MeV$. This is conserved, so the photons must also both have $E = 0.511MeV$.

(c) $p = E/c = 0.511MeV/c$

(d) $\lambda = \frac{h}{p} = 2.43 \times 10^{-12}m$

48] a] $m_{\pi^+} + m_n - m_p = 141.3 \text{ MeV}/c^2$

$\Rightarrow \boxed{\Delta E = 141.3 \text{ MeV}}$

b] $\Delta E \Delta t \approx \frac{\hbar}{2} \Rightarrow \boxed{\Delta t = 2.3 \times 10^{-24} s}$

c] $c\Delta t = \boxed{7 \times 10^{-16} m}$, about 1 fm

49] $hf = \gamma mc^2 \Rightarrow \gamma = \frac{hf}{mc^2} \Rightarrow \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{hf}{mc^2}$

So $1 - v^2/c^2 = \left(\frac{mc^2}{hf}\right)^2 \Rightarrow \frac{v^2}{c^2} = 1 - \left(\frac{mc^2}{hf}\right)^2$

$\Rightarrow \frac{v}{c} = \left(1 - \left(\frac{mc^2}{hf}\right)^2\right)^{1/2} \approx 1 - \frac{1}{2}\left(\frac{mc^2}{hf}\right)^2$

So using $v > .99c$, $v/c \geq 0.99$

$\Rightarrow 1 - \frac{1}{2}\left(\frac{mc^2}{hf}\right)^2 \geq 0.99 \Rightarrow 0.01 \geq \frac{1}{2}\left(\frac{mc^2}{hf}\right)^2$

So use $\lambda = 30\,m$ to get

$$1.08 \times 10^{-88} \geq m^2 \Rightarrow \boxed{m < 10^{-44}}$$

5c] $\Delta v_x \simeq \frac{\hbar}{2m\Delta x}$. And $y_0 = \frac{1}{2}gt^2 \Rightarrow t = \left(\frac{2y_0}{g}\right)^{1/2}$

$$\therefore \Delta \underline{X} = \Delta v_x t = \boxed{\left(\frac{2y_0}{g}\right)^{1/2}\left(\frac{\hbar}{2m\Delta x}\right)}$$

6] Now say $\Delta y \Delta p_y \simeq \frac{\hbar}{2} \Rightarrow \Delta v_y \simeq \frac{\hbar}{2m\Delta y}$

So now $y_0 = \frac{1}{2}gt^2 + \Delta v_y t$

$$\Rightarrow t = \frac{-\Delta v_y \pm \sqrt{(\Delta v_y)^2 + 2gy_0}}{g}$$

Take the + root b/c $\Delta v_y = 0$ should give $\left(\frac{2y_0}{g}\right)^{1/2}$

$$\therefore \boxed{\Delta \underline{X} = \left(\frac{\hbar}{2m\Delta x}\right)\left[\frac{-\Delta v_y + \sqrt{(\Delta v_y)^2 + 2gy_0}}{g}\right]}$$

where $\Delta v_y \simeq \frac{\hbar}{2m\Delta y}$.

52

a) $p_\gamma = \frac{E_\gamma}{c} = \frac{1eV}{c}$

So $\frac{p^2}{2m} = \frac{(1eV)^2}{2mc^2}$. Use $m_{Fe} = 56 \, u$.

$= 56\left(931.5 \frac{MeV}{c^2}\right)$

$\Rightarrow E = \frac{p^2}{2m} = 9.6 \times 10^{-12}$

The line width is $h\Delta f \approx 10^{-8}$, so this is $\boxed{10^{-3}}$ times that.

b) Now use $p = 1MeV$ to get $\boxed{E = 9.6 \, eV}$

This is $10^8 \times$ the line width.