4E Week 9 HW

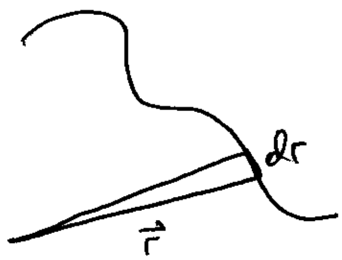
Serway Ch. 8: 1, 2, 8, 9, 13, 15, 17, 18, 23

1] This is just the difference in energy between the states. Recall

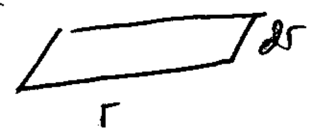
$$U = -\mu \cdot B = g\mu_B m_s B$$

So $\Delta U = g\mu_B B = (2)\mu_B \cdot (0.35T) = 6.5\times10^{-24} J$

So since $E = hf$, $f = \frac{E}{h} = \boxed{9.79\times10^{9}\ Hz}$

2] 

Area of triangle $= \frac{1}{2}|\vec{r}\times d\vec{r}| = dA$

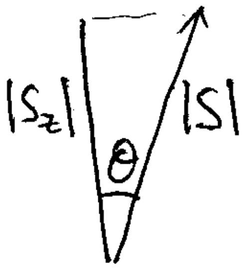
(since $|\vec{r}\times d\vec{r}|$ = Area of 

)

So $\frac{dA}{dt} = \frac{1}{2}\left|\vec{r}\times\frac{d\vec{r}}{dt}\right| = \frac{1}{2m}|\vec{r}\times\vec{p}| = \boxed{\frac{|\vec{L}|}{2m}}$

8] This is 7-29a from last week; see those solutions.

9] $s = \frac{3}{2}$, so $|\vec{S}| = \sqrt{\frac{3 \cdot 5}{4}}\,\hbar = \boxed{\frac{\sqrt{15}}{2}\hbar}$



$\cos\theta = \frac{|S_z|}{|S|} = \frac{m_s \hbar}{|S|} = \frac{2 m_s}{\sqrt{15}}$

So $\cos\theta = \frac{3}{\sqrt{15}}, \frac{1}{\sqrt{15}}, \frac{-1}{\sqrt{15}}, \frac{-3}{\sqrt{15}}$

or $\theta = \boxed{39°, 75°, 105°, 141°}$

$\boxed{\text{Yes,}}$ Ω^- does obey The Pauli ex. principle b/c it's a spin-$\frac{3}{2}$ particle

13] a] $4F_{5/2}$ means $\boxed{n=4,\ \ell=3,\ j=\frac{5}{2}.}$

b] $|J| = \hbar\sqrt{j(j+1)} = \boxed{\frac{\sqrt{35}}{2}\hbar}$

c] $\boxed{\frac{5}{2}\hbar, \frac{3}{2}\hbar, \frac{1}{2}\hbar, -\frac{1}{2}\hbar, -\frac{3}{2}\hbar, -\frac{5}{2}\hbar}$

Chapter 8

J_z can be $-(5/2)\hbar$, $-(3/2)\hbar$, $-(1/2)\hbar$, $(1/2)\hbar$, $(3/2)\hbar$, or $(5/2)\hbar$

8-14. (a) $\mathbf{J}\cdot\mathbf{J} = (\mathbf{L}+\mathbf{S})\cdot(\mathbf{L}+\mathbf{S}) \quad \Rightarrow \quad J^2 = L^2 + S^2 + 2\mathbf{L}\cdot\mathbf{S}$

$\mathbf{L}\cdot\mathbf{S} = (1/2)(J^2 - L^2 - S^2) = (\hbar^2/2)[j(j+1) - \ell(\ell+1) - s(s+1)]$

(b) $\mathbf{L}\cdot\mathbf{S} = |\mathbf{L}|\,|\mathbf{S}|\cos\theta = \hbar^2[\ell(\ell+1)]^{1/2}[s(s+1)]^{1/2}\cos\theta$ so

$\cos\theta = (1/2)[j(j+1) - \ell(\ell+1) - s(s+1)] / [\ell(\ell+1)]^{1/2}[s(s+1)]^{1/2}$

since $s = 1/2$,

$\cos\theta = [j(j+1) - \ell(\ell+1) - 3/4]/(3)^{1/2}[\ell(\ell+1)]^{1/2}$

(1) $P_{1/2}$ has $\ell = 1$, $j = 1/2$

$\cos\theta = [(1/2)(3/2) - 1(1+1) - 3/4] / (\sqrt{3}\sqrt{2})$

$= -(2/3)^{1/2} = -0.816,$

$\theta = 144.7°$

$P_{3/2}$ has $\ell = 1$, $j = 3/2$

$\cos\theta = [(3/2)(5/2) - 1(1+1) - 3/4] / (\sqrt{3}\sqrt{2}) = 1/\sqrt{6} = 0.408$

$\theta = 65.9°$

(2) $H_{9/2}$ has $\ell = 5$, $j = 9/2$

$\cos\theta = [(9/2)(11/2) - 5(6) - 3/4] / \{[5(6)]^{1/2}\sqrt{3}\}$

$= -6/\sqrt{90} = -2/\sqrt{10}$

$\cos\theta = -0.632, \qquad \theta = 129.2°$

$H_{11/2}$ has $\ell = 5$, $j = 11/2$

$\cos\theta = [(11/2)(13/2) - 5(6) - 3/4] / \{[5(6)]^{1/2}\sqrt{3}\}$

$= 5/\sqrt{90} = 0.527$

$\theta = 58.2°$

8-15. The spin of the atomic electron has a magnetic energy in the field of the orbital moment given by Equations 8.6 and 8.12 with a g-factor of 2, or

$$U = -\boldsymbol{\mu}_S\cdot\mathbf{B} = 2(e/2m_e)S_zB = 2\mu_B m_s B$$

The magnetic field $\mathbf{B}$ originates with the orbiting electron. To estimate $\mathbf{B}$, we adopt the equivalent viewpoint of the atomic nucleus (proton) circling the electron, and borrow a result from classical electromagnetism for the $\mathbf{B}$ field at the center of a circular current loop with radius r:

$$B = 2k_m\mu / r^3$$

Here k_m is the magnetic constant and $\mu = i\pi r^2$ is the magnetic moment of the loop, assuming it carries a current i. In the atomic case, we identify r with the orbit radius and the current i with the proton charge $+e$ divided by the orbital period $T = 2\pi r/v$. Then

$$\mu = evr/2 = (e/2m_e)L$$

where $L = m_e vr$ is the orbital angular momentum of the electron. For a p electron $\ell = 1$ and $L = [\ell(\ell+1)]^{1/2}\hbar = \sqrt{2}\hbar$, so

$$\mu = (e\hbar/2m_e)\sqrt{2} = \mu_B\sqrt{2} = 1.31 \times 10^{-23}\ \text{J/T}$$

For r we take a typical atomic dimension, say $4a_0$ ($= 2.12 \times 10^{-10}$ m) for a $2p$ electron, and find

$B = 2(10^{-7}\ \text{N/A}^2)(1.31 \times 10^{-23}\ \text{J/T})/(2.12 \times 10^{-10}\ \text{m})^3 = 0.276\ \text{T}$

Since m_s is $\pm 1/2$, the magnetic energy of the electron spin in this field is

$U = \pm \mu_B B = \pm (9.27 \times 10^{-24}\ \text{J/T})(0.276\ \text{T}) = \pm 2.56 \times 10^{-24}\ \text{J}$

$= \pm 1.59 \times 10^{-5}\ \text{eV}$

The up spin orientation (+) has the higher energy; the predicted energy difference between the up (+) and down (-) spin orientations is twice this figure, or about

3.18×10^{-5} eV, a result which compares favorably with the measured value, 5×10^{-5} eV.

8-16. In the symmetric combination

$$\psi_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) + \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)$$

interchange $\mathbf{r}_1$ and $\mathbf{r}_2$ to get

$$\psi_{ab}(\mathbf{r}_2, \mathbf{r}_1) = \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1) + \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)$$

Comparing with the original shows clearly that $\psi_{ab}(\mathbf{r}_2, \mathbf{r}_1) = \psi_{ab}(\mathbf{r}_1, \mathbf{r}_2)$, and therefore $\psi_{ab}(\mathbf{r}_1, \mathbf{r}_2)$ is a symmetric wavefunction as required for bosons. For two bosons in the same state, we set a = b. Because the resulting wavefunction is nonzero, two bosons may indeed occupy the same quantum state.

8-17. From Equation 7.9 we have $E = (\hbar^2\pi^2 / 2mL^2)(n_x^2 + n_y^2 + n_z^2)$

$E = (1.054 \times 10^{-34})^2(\pi^2)(n_x^2 + n_y^2 + n_z^2) / [2\ (9.11 \times 10^{-31})(2 \times 10^{-10})^2]$

$= (1.5 \times 10^{-18}\ \text{J})(n_x^2 + n_y^2 + n_z^2) = (9.4\ \text{eV})(n_x^2 + n_y^2 + n_z^2)$

(a) 2 electrons per state. The lowest states have

$(n_x, n_y, n_z) = (1, 1, 1) \Rightarrow E_{111} = (9.4\ \text{eV})(1^2 + 1^2 + 1^2)\text{eV} = 28.2\ \text{eV}$

For $(n_x, n_y, n_z) = (1, 1, 2)$ or $(1, 2, 1)$ or $(2, 1, 1)$,

$E_{112} = E_{121} = E_{211} = (9.4\ \text{eV})(1^2 + 1^2 + 2^2) = 56.4\ \text{eV}$

$E_{min} = 2 \times (E_{111} + E_{112} + E_{121} + E_{211}) = 2(28.2 + 3 \times 56.5) = 394.8\ \text{eV}$

(b) All 8 particles go into the $(n_x, n_y, n_z) = (1, 1, 1)$ state, so

$$E_{min} = 8 \times E_{111} = 225.6\ \text{eV}$$

8-18. Classically, $|\mathbf{L}| = |\mathbf{r}|p_\perp$, where $p_\perp$ is the component of particle momentum perpendicular to $\mathbf{r}$. Remembering that $\mathbf{p}$ is tangent to the orbit at every point, we see that a highly eccentric orbit is one for which $\mathbf{p}$ and $\mathbf{r}$ are nearly collinear over most of the orbit, making $p_\perp$ small almost everywhere. The exceptions occur at the perigee (nearest point), where $\mathbf{r}$ and $\mathbf{p}$ are perpendicular but $|\mathbf{r}|$ is small, and apogee (farthest point), where $\mathbf{r}$ and $\mathbf{p}$ are again perpendicular but now $|\mathbf{p}|$ is small. Thus, we expect that $|\mathbf{L}|$ will be smaller for the more eccentric orbits. The extreme case is that for $L = 0$, where the classical orbit degenerates to a straight line.

The quantum probabilities are found by integrating the radial probability density for each state, $P(r)$, from $r = 0$ to $r = a_0$. For the $2s$ state we find from Table 7.4 (with Z = 1 for hydrogen)

$$P_{2s}(r) = |rR_{2s}(r)|^2 = (8a_0)^{-1} (r / a_0)^2(2 - r / a_0)^2 e^{-r/a_0}$$

and

$$P = (8a_0)^{-1} \int_0^{a_0} (r / a_0)^2(2 - r / a_0)^2 e^{-r/a_0}\, dr$$

Changing variables from r to $z = r / a_0$ gives

$$P = 8^{-1}\int_0^1 (4z^2 - 4z^3 + z^4)e^{-z}\, dz$$

Repeated integration by parts gives

$$P = 8^{-1}\left\{- (4z^2 - 4z^3 + z^4) - (8z - 12z^2 + 4z^3) - (8 - 24z + 12z^2) - (-24 + 24z) - (24)\right\} e^{-z}\Big|_0^1$$

$$= 8^{-1}\left\{- (1 + 0 - 4 + 0 + 24)e^{-1} + 8\right\} = 0.034$$

For the $2p$ state of hydrogen

$$P_{2p}(r) = |rR_{2p}(r)|^2 = (24a_0)^{-1} (r / a_0)^4 e^{-r/a_0}$$

and

$$P = (24a_0)^{-1}\int_0^{a_0} (r / a_0)^4 e^{-r/a_0}\, dr = 24^{-1}\int_0^1 z^4 e^{-z}\, dz$$

Again integrating by parts, we get

$$P = 24^{-1}\left\{-z^4 - 4z^3 - 12z^2 - 24z - 24\right\} e^{-z}\Big|_0^1 = 24^{-1}\left\{- 65e^{-1} + 24\right\}$$

$$= 0.0037$$

The probability for the $2p$ electron is nearly ten times smaller, suggesting that this electron spends more of its time in the outer regions of the atom where it is screened more effectively by the inner shell electrons.

8-19. Choose the task *EigenStates- 3D* and load the potential file *EX8-19. U3* to give the Thomas-Fermi potential for an electron in an atom, with distance measured in bohrs (a_0) and energy in rydbergs (Ry). The defaults $\Gamma = 79$ and $\alpha = 0.39$ describe the gold (Au) atom with screening length of 0.39 a_0.

To model the valence electron in Au we require the $6s$ state, which exhibits (6 - 1) = 5 nodes (not counting the one at $r = 0$). Using AutoSearch, we find that the s state ($\ell = 0$) with 5 nodes has energy $E = -0.635$ Ry ($= -8.64$ eV). The negative of this, 8.64 eV, is the ionization energy predicted by this model (compared to the experimental value 9.22 eV). The $6s$ pseudo-wavefunction $g(r)$

23] All spins paired in $[Kr]\,4d^{10}$

All but 2 spins paired in $[Kr]\,4d^{9}5s^{1}$.

The filled subshell is energetically more favorable.

This is Palladium (Pd).