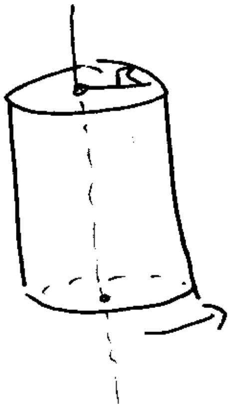
4E Week 8 HW

7: 29, 30, 32, 33, 36, 40, 63, 64, 68, 71, 73, 75

29



Say, the cylinder rotates around the axis w/ frequency $f = \frac{\omega}{2\pi}$.

$$\mu = \underset{\uparrow \atop \text{current}}{I} \cdot \underset{\uparrow \atop \text{Area.}}{A}$$

Well, $A = \pi R^2$. $I = \frac{\partial Q}{\partial t}$, where Q is the charge that passes through some slice. In a time t, the cylinder rotates through an angle $\theta = \omega t$. The amount of charge on that much cylinder is $\frac{Q}{2\pi R \underset{\uparrow \atop \text{height}}{h}} \cdot \theta R h = \frac{Q\omega t}{2\pi}$

$\therefore \frac{\partial Q}{\partial t} = \frac{Q\omega}{2\pi}$, so $I \cdot A = Qf\pi R^2 = \mu$

$$\text{Now, } \mu = g\frac{Q}{2M}L = g\frac{Q}{2\cancel{M}} \cdot \overbrace{\frac{1}{\cancel{2}}\cancel{M}R^2}^{I}\overbrace{(\cancel{2}\pi f)}^{\omega} = g\frac{QR^2\pi f}{2}$$

$$\text{So } g\left(\frac{QR^2\pi f}{2}\right) = Qf\pi R^2 \Rightarrow \boxed{g = 2}.$$

b] As before, $I = Q \cdot f$, and $A = \pi R^2$ (equator)

So $g\left(\frac{Q}{2M}\right)L = g\left(\frac{Q}{2M}\right)\frac{2}{5}MR^2 \cdot 2\pi f = Q \cdot f \pi R^2$

$$\Rightarrow \boxed{g = \frac{5}{2}}$$

30] $S = I\omega$, so $\omega = \frac{S}{I}$

$v = r\omega \Rightarrow \quad v = \frac{rS}{I} = \frac{r\sqrt{\frac{3}{4}}\hbar}{\frac{2}{5}Mr^2} = \frac{\sqrt{\frac{3}{4}}\hbar}{\frac{2}{5}Mr}$

Plug in: $\boxed{v = 2.5 \times 10^{11}\,\frac{m}{s} = 835c}$

32]

$$F_z = \underset{\underset{\frac{1}{2}}{\uparrow}}{m_s}\, g_s \mu_B \left|\frac{\partial B}{\partial z}\right| = \underset{\underset{\text{mass of Silver.}}{\uparrow}}{m_{Ag}}\, a_z$$

The v_z will be small, so say $|v| = 250\,m/s$.

$\therefore$ It takes $\frac{1\,m}{250\,m/s} = \frac{1}{250}\,s$ to go through each region.

So in the field, $\Delta v_z = a_z \cdot \frac{1}{250}$ and $\Delta z = \frac{1}{2} a_z \left(\frac{1}{250}\right)^2$

Outside the field, $\Delta z = v_z t = a_z \left(\frac{1}{250}\right)^2$

So $\Delta z_{TOT} = \frac{3}{2} a_z \left(\frac{1}{250}\right)^2 = \frac{3}{2}\left[\frac{m_s g_s \mu_B \left|\frac{\partial B}{\partial z}\right|}{m_{Ag}}\right]\left(\frac{1}{250}\right)^2$

Setting $\Delta z = .5\,mm$, we get

$$\left|\frac{\partial B}{\partial z}\right| = \frac{(5\times10^{-4}\,m)(250\,s)^2 \cdot 2\, m_{Ag}}{3\, m_s g_s \mu_B}$$

Using $m_s = \frac{1}{2}$, $g_s = 2$, $\mu_B = 9.27\times10^{-24}\,\frac{J}{T}$, $m_{Ag} = 1.79\times10^{-25}\,kg$

We get $\left|\frac{\partial B}{\partial z}\right| = \boxed{0.4\,\frac{T}{m}}$

33] Since $m=-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$, $\boxed{\text{four lines}}$

b] $\boxed{\text{Three: } m_\ell = \pm 1, 0}$

36] a] $\ell = 2$, $s = \frac{1}{2}$.

$\therefore \boxed{j = \frac{5}{2}, \frac{3}{2}}$

b] $|J| = \hbar\sqrt{j(j+1)} = \boxed{\frac{\sqrt{35}}{2}\hbar, \frac{\sqrt{15}}{2}\hbar}$

c] $m = -j, -j+1, \cdots j-1, j$

So $\boxed{J_z = -\frac{5}{2}\hbar, -\frac{3}{2}\hbar, -\frac{1}{2}\hbar, \frac{1}{2}\hbar, \frac{3}{2}\hbar, \frac{5}{2}\hbar.}$

All for $j=\frac{5}{2}$, middle 4 for $j=\frac{3}{2}$.

40] a] $E = \frac{hc}{\lambda} \Rightarrow E = \frac{1240\,\text{eV}\cdot\text{nm}}{\lambda}$

$\Rightarrow E =$ $\boxed{2.103\,\text{eV}, 2.105\,\text{eV}}$

b] $\Delta E = \boxed{2.14\times10^{-3}\,\text{eV}}$

c] $B = \frac{\Delta E}{2\mu_B} = \boxed{18.5\,\text{T}}$

63

$$\psi_{100} = R_{10} Y_{00}$$

$$= \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$$

$$\text{So } \langle r \rangle = \int_0^\infty dr \int_0^\pi d\theta \int_0^{2\pi} d\phi \; r^3 \sin\theta \, \frac{1}{\pi}\left(\frac{Z}{a_0}\right)^3 e^{-2Zr/a_0}$$

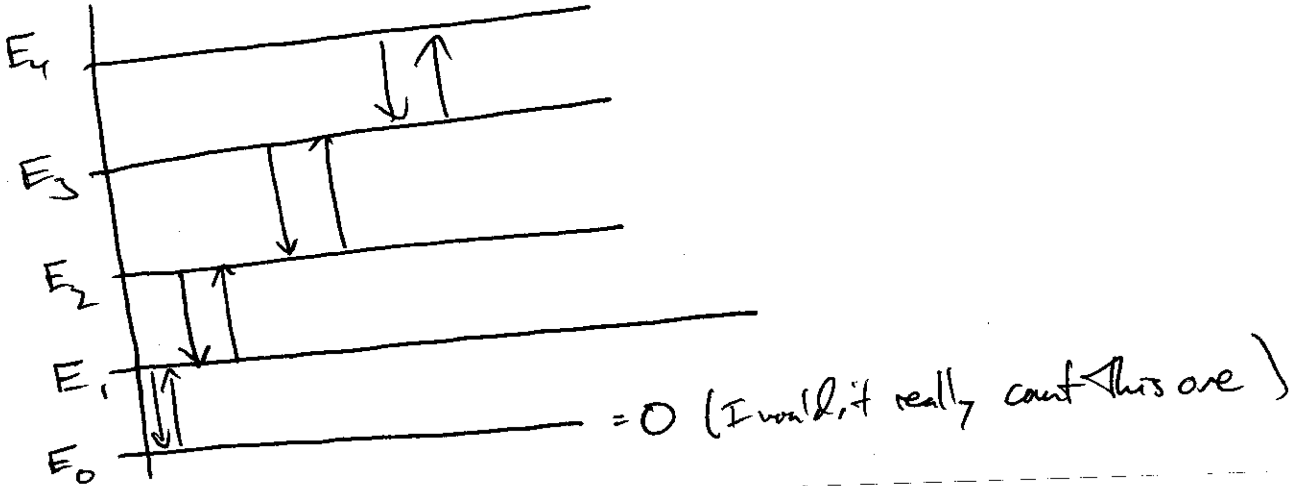
$$= 4\left(\frac{Z}{a_0}\right)^3 \int_0^\infty dr \; r^3 e^{-2Zr/a_0}$$

$$= 4\left(\frac{Z}{a_0}\right)^3 \left(\frac{a_0}{2Z}\right)^4 \underbrace{\int_0^\infty du \; u^3 e^{-u}}_{3!}$$

$$= \frac{a_0}{Z}\left(\frac{4}{16} \cdot 3!\right) = \boxed{\frac{3}{2}\frac{a_0}{Z}}$$

64

$$E_1 = 2\left(\frac{\hbar^2}{2I}\right) \quad E_2 = 6\left(\frac{\hbar^2}{2I}\right), \; E_3 = 12\left(\frac{\hbar^2}{2I}\right), \; E_4 = 20\left(\frac{\hbar^2}{2I}\right)$$



b] Since we can have $\Delta l = \pm 1$, look at

$$E_{l+1} - E_l = (l+1)(l+2)\left(\frac{\hbar^2}{2I}\right) - l(l+1)\left(\frac{\hbar^2}{2I}\right)$$

$$= (l+1)\left(\frac{2\hbar^2}{2I}\right) = \boxed{(l+1)\left(\frac{\hbar^2}{I}\right)}$$

c] $I = \frac{1}{2} m_p r^2$

$$E_1 = \frac{\hbar^2}{I} = \frac{2\hbar^2}{m_p r^2} = \boxed{0.015\,\text{eV}}$$

d] $\lambda = \frac{hc}{0.015\,\text{eV}} = \boxed{82\,\mu\text{m}}$

68] Last week's HW - see that.

71] a] s ⇒ $l=0$, so no effect

p ⇒ $l=1$, so splits into 3 levels ($m_l = \pm 1, 0$).

b] Three lines, as in pt. a

c] The energies of these lines differ by $\mu_z B$ [illegible] B.

$$= \frac{e\hbar}{2m_K} B$$

$$= \mu_B \left(\frac{m_e}{m_K}\right) B$$

So this number is $\Delta E = \mu_B \left(\frac{m_e}{m_K}\right) B$

$$= 6\times10^{-8}\,\text{eV}.$$

Now, since $E = \frac{hc}{\lambda}$, $\Delta E = -\frac{hc}{\lambda^2}\Delta\lambda$

$$\Rightarrow \left|\frac{\Delta\lambda}{\lambda}\right| = \left|\frac{\Delta E}{hc}\lambda\right| = \left|\frac{\Delta E}{E}\right|$$

E is the difference between $n=2$ & $n=1$ states — but we have to be careful, since the e^- is a K^- now.

$$E_n = -\frac{\mu k^2 e^4}{2\hbar^2 n^2} = -\frac{\mu}{m_e}\frac{(13.6\,\text{eV})}{n^2}.$$

$$\mu = \frac{m_p m_K}{m_p + m_K} = 323.5\ \text{MeV}/c^2$$

So $E_n = -\left(\frac{323.5}{.511}\right)\left(\frac{13.6\,\text{eV}}{n^2}\right) = \frac{-8609\,\text{eV}}{n^2}$

So $|E_2 - E_1| = \frac{3}{4}\cdot 8609\,\text{eV} = 6457\,\text{eV}$

$$\frac{\Delta\lambda}{\lambda} = \frac{6\times10^{-8}\,\text{eV}}{6457\,\text{eV}} = \boxed{9.3\times10^{-12}}$$

73] $\vec{\mu} = \frac{-e}{2m}(2\vec{S} + \vec{L})$

and $J = (\vec{S} + \vec{L})$

So $\mu \cdot J = \frac{-e}{2m}(2\vec{S} + \vec{L}) \cdot (\vec{S} + \vec{L})$

$$= \frac{-e}{2m}(2S^2 + 3\vec{S} \cdot \vec{L} + L^2)$$

So $\boxed{\frac{\mu \cdot J}{J} = -\frac{\mu_B}{\hbar J}(L^2 + 2S^2 + 3\vec{S} \cdot \vec{L})}$

b] $J^2 = (\vec{L} + \vec{S})^2 = L^2 + S^2 + 2\vec{S} \cdot \vec{L}$

$$\rightarrow \boxed{\vec{S} \cdot \vec{L} = \frac{1}{2}(J^2 - L^2 - S^2)}$$

c] So $\mu_J = -\frac{\mu_B}{\hbar J}\left(L^2 + 2S^2 + \frac{3}{2}J^2 - \frac{3L^2}{2} - \frac{3S^2}{2}\right)$

$$= \boxed{-\frac{\mu_B}{2\hbar J}(3J^2 - L^2 + S^2)}$$

d] $\mu_z = -\frac{\mu_B J_z}{2\hbar J^2}(3J^2 - L^2 + S^2) = -\mu_B \frac{J_z}{\hbar}\left(\frac{3}{2} + \frac{S^2 - L^2}{2J^2}\right)$

$$\boxed{= -\mu_B \frac{J_z}{\hbar}\left(1 + \frac{J^2 + S^2 - L^2}{2J^2}\right)}$$

75]

a] $$B = \frac{2K_m\mu}{r^3} = \frac{2(10^{-7})e\hbar}{2m_p a_0^3} \cdot 2.8$$

$$= \frac{2(10^{-7})2.8}{a_0^3}\mu_B\left(\frac{m_e}{m_p}\right)$$

$$= \boxed{0.02\ T}$$

b] $$\Delta E \approx 2\mu_B B = \boxed{2.21\times10^{-6}\ eV}$$

c] $$\lambda \sim \frac{hc}{\Delta E} = \boxed{0.56\ m}$$