Phys 4E Week 6 HW

Ch. 6 Problems: See last week's HW sols.

Ch. 7: 1–8

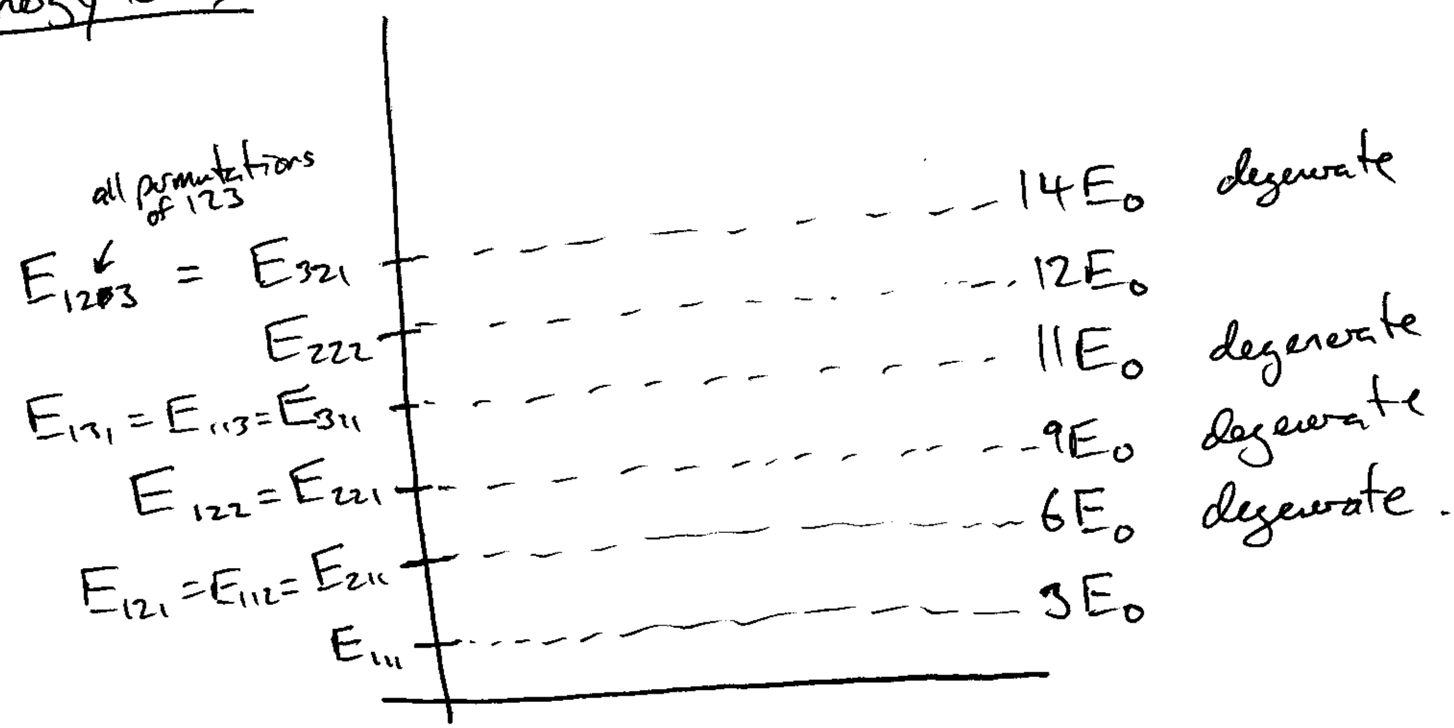
1) $E = \frac{\hbar^2\pi^2}{2mL^2}(n_x^2+n_y^2+n_z^2)$, so

$$E_{311} = \frac{11\hbar^2\pi^2}{2mL^2} \equiv 11E_0, \quad E_0 = \frac{\hbar^2\pi^2}{2mL^2}$$

$$E_{222} = \frac{6\hbar^2\pi^2}{mL^2} = 12E_0$$

$$E_{321} = \frac{7\hbar^2\pi^2}{mL^2} = 14E_0$$

Energy levels



2] $$E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n_x}{L_x^2} + \frac{n_y}{L_y^2} + \frac{n_z}{L_z^2}\right)$$

$$= \frac{\hbar^2 \pi^2}{2mL^2}\left(n_x^2 + \frac{n_y^2}{4} + \frac{n_z^2}{9}\right)$$

Lowest energy: $E_{111} = \frac{49}{36} E_0$

Now it's a matter of playing around to get the next 9 levels, in order:

$E_{112} = 1.694 E_0$

$E_{121} = 2.111 E_0$

$E_{113} = 2.25 E_0$

$E_{122} = 2.44 E_0$

$E_{123} = 3 E_0$

$E_{114} = 3.03 E_0$

$E_{131} = 3.36 E_0$

$E_{132} = 3.69 E_0$

$E_{124} = 3.78 E_0$

3] a] It's a sin in the x-direction, and then cos in y & z (just translate sin).

$$\boxed{\psi_{111} = \left(\frac{2}{L}\right)^{3/2} \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi y}{L}\right) \cos\left(\frac{\pi z}{L}\right)}$$

b] They're exactly the same!

4] Let's take all sides to start at 0 in the appropriate coordinate.

Then $\psi_{n_x n_y n_z} = \left(\frac{2}{L_x}\cdot\frac{2}{L_y}\cdot\frac{2}{L_z}\right)^{1/2} \sin\left(\frac{n_x \pi x}{L_x}\right) \sin\left(\frac{n_y \pi y}{L_y}\right) \sin\left(\frac{n_z \pi z}{L_z}\right)$

First, note that $\left(\frac{2}{L_x}\cdot\frac{2}{L_y}\cdot\frac{2}{L_z}\right) = \frac{8}{L\cdot 2L\cdot 3L} = \frac{4}{3L^3}$

So $\psi_{111} = \left(\frac{4}{3L^3}\right)^{1/2} \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi y}{2L}\right) \sin\left(\frac{\pi z}{3L}\right)$

$\psi_{112} = \left(\frac{4}{3L^3}\right)^{1/2} \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi y}{2L}\right) \sin\left(\frac{2\pi z}{3L}\right)$

etc

⋮

5] $E_{n_x n_y n_z} = \underbrace{\frac{\hbar^2 \pi^2}{2mL^2}}_{E_0}\left(n_x^2 + \frac{n_y^2}{4} + \frac{n_z^2}{16}\right)$

$E_{111} = 1.31 E_0$, $E_{112} = 1.5 E_0$, $E_{113} = 1.813 E_0$,
$E_{121} = 2.063 E_0$, $\boxed{E_{114} = E_{122} = 2.25 E_0}$ degenerate!
$E_{123} = 2.56 E_0$, $E_{115} = 2.813 E_0$,
$E_{124} = 3 E_0$, $E_{116} = 3.5 E_0$

6] First one:

$$\psi_{111} = \left(\frac{2\cdot 2\cdot 2}{L\cdot 2L\cdot 4L}\right)^{1/2} \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi y}{2L}\right)\sin\left(\frac{\pi z}{4L}\right)$$

$$\boxed{\psi_{111} = L^{-3/2}\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi y}{2L}\right)\sin\left(\frac{\pi z}{4L}\right)}$$

etc.

7] $E_{311} - E_{111} = 11E_0 - 3E_0 = 8E_0$.

Now, $E_0 = \dfrac{\hbar^2 c^2 \pi^2}{2mc^2 L^2} = \dfrac{(197.3\,\text{eV}\cdot\text{nm})^2 \pi^2}{2(511\times 10^3\,\text{eV})(.1\,\text{nm})} = 3.76\,\text{eV}$

So $8E_0 = \boxed{30\,\text{eV}}$

$E_{222} - E_{111} = 9E_0 = \boxed{33.8\,\text{eV}}$

$E_{321} - E_{111} = 11E_0 = \boxed{41.3\,\text{eV}}$

8] a] Just like 3D, multiply!

$$\boxed{\psi(x,y) = \left(\frac{2}{L}\cdot\frac{2}{L}\right)^{1/2}\sin\left(\frac{n_x \pi x}{L}\right)\sin\left(\frac{n_y \pi y}{L}\right)}$$

b] $$\boxed{E_{n_x n_y} = \frac{\hbar^2\pi^2}{2mL^2}(n_x^2 + n_y^2)}$$

c] $$\boxed{E_{12} = E_{21}}$$