Phys 4E Week 2 HW

Ch. 4: 5, 6, 10, 11, 16, 20, 24, 25, 41, 53.

5] $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, $R = 1.097 \times 10^7\ m^{-1}$

For $n_f = 1$, longest $\lambda = \left[R\left(1 - \frac{1}{2^2}\right)\right]^{-1} = 121.6\ nm$ No.

For $n_f = 2$, longest $\lambda = \left[R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\right]^{-1} = 656\ nm$. NO!

For $n_f = 3$ longest $\lambda = \left[R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)\right]^{-1} = 1875\ nm$. NO!!

$n_f = 4 \longrightarrow \lambda = 4051\ nm$

Any in here? $\lambda_{54} = 4051\ nm$

$\lambda_{64} = 2625\ nm$. NO.

$n_f = 5$, longest $\lambda = \left[R\left(\frac{1}{5^2} - \frac{1}{6^2}\right)\right]^{-1} = 7460\ nm$! ✓

$\lambda_{75} = \left[R\left(\frac{1}{5^2} - \frac{1}{7^2}\right)\right]^{-1} = 4653\ nm$! ✓

$\lambda_{85} = \left[R\left(\frac{1}{5^2} - \frac{1}{8^2}\right)\right]^{-1} =$ 3740 nm! ✓.

That's 3 of them – only one left is $\boxed{4103\ nm}$

4.6 This is just like Ex. 4-2 on p. 170:

a) fraction $= \pi b^2 n t$

$$n = \frac{\rho N_A}{M}, \quad b = \frac{k q_\alpha Q}{m_\alpha v^2} \cot\left(\frac{\theta}{2}\right)$$

So $f = \pi \left[\frac{k q_\alpha Q}{m v^2} \cot\left(\frac{\theta}{2}\right)\right]^2 \frac{\rho N_A}{M} t$

Now, use $K = 9\times10^9 \frac{N\cdot m^2}{C^2}$, $q_\alpha = 2e$, $Q = 79e$

$mv^2 = 2K_\alpha = 2(7\,MeV)$, $\theta = 90°$, $t = 2\mu m$,

$\rho = 19.3\,g/cm^3$, $M = 197\,g/mol$

So $b^2 = (1.63\times10^{-14}\,m)^2 = 2.64\times10^{-28}\,m^2$

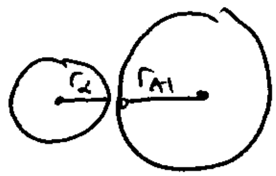
$$\frac{\rho N_A}{M} = 5.9\times10^{28}\,\frac{atom}{m^3}$$

$t = 2\mu m \Rightarrow \boxed{f = 9.8\times10^{-5}}$

b) Now just do $|f_{75} - f_{45}| = \boxed{4.04\times10^{-4}}$

$f_{75} \downarrow 1.66\times10^{-4}$ $\quad f_{45} \downarrow 5.7\times10^{-4}$

10]



$$\frac{k q_\alpha Q_{Al}}{r} = K, \text{ by energy conservation.}$$

$r = r_\alpha + r_{Al}$, and $r_{Al} = 10\text{ fm}$, $r_\alpha = 4\text{ fm}$.

$$\therefore\ K = \frac{2\cdot 13\cdot ke^2}{14\text{ fm}} = \frac{2\cdot 13\cdot(1.44\text{ eV}\cdot\text{nm})}{14\times 10^{-6}\text{ nm}}$$

$$= \frac{2\cdot 13\,(1.44\text{ MeV}\cdot\text{fm})}{14\text{ fm}}$$

$$= \boxed{2.67\text{ MeV}}$$

11] $10° = \sqrt{N}\,(0.01°) \Rightarrow N = 10^6$ collisions.

$$\#\text{layers} = \frac{\text{thickness total}}{\text{thickness per layer}} = \frac{10^{-6}\text{ m}}{10^{-10}\text{ m}} = 10^4 \text{ layers.}$$

10^4 layers isn't enough to get a $10°$ deflection!

16) $m_E v r = n\hbar$, $r = 1.5\times10^{11}$ m, $m_E = 5.98\times10^{24}$

So $n = \frac{m_E v r}{\hbar} = \frac{m_E r}{\hbar}\left(\frac{2\pi r}{1\,yr}\right) = \boxed{2.54\times10^{74}}$

Taking $n \to n-1$ doesn't really change anything, so compute $E = \frac{1}{2}m_E v^2 = \frac{1}{2}m_E\left(\frac{n\hbar}{m_E r}\right)^2$

$$= \frac{1}{2}\frac{n^2\hbar^2}{m_E r^2}$$

$$\Delta E = \frac{\cancel{2}n\Delta n\,\hbar^2}{\cancel{2}\,m_E r^2} - \underbrace{\frac{n^2\hbar^2}{m_E r^3}\Delta r}_{\text{Assume } \Delta r \text{ very small, ignore this.}}$$

$$\Rightarrow \Delta E = \frac{(2.54\times10^{74})(1)\hbar^2}{m_E r^2} = \boxed{2\times10^{-41}\text{ J}}$$

NOT detectable.

So if we ignore the Δn piece and set

$$\Delta E = \frac{-n^2\hbar^2}{m_E r^3}\Delta r \Rightarrow \Delta r = \boxed{5.6\times10^{-64}\text{ m}}$$ NOT detectable.

(This is a little non-rigorous, but it's correct).

20] $Z' = \left(\frac{E_n^2}{E_1}\right)^{1/2} = \boxed{1.26}$

24] Need reduced mass:

$$R = R_\infty \left(\frac{1}{1+\frac{m}{M}}\right) = \frac{R_\infty}{2}.$$

So $E = \frac{-hcR}{n^2} = \frac{-6.8\,eV}{n^2}$

$$\boxed{\begin{aligned} E_1 &= -6.8\,eV \\ E_2 &= -1.7\,eV \\ E_3 &= -0.75\,eV \end{aligned}}$$

b] Lyman: $n_f = 1$. Since $E \downarrow 2$, $\lambda \uparrow 2$

$$\Rightarrow \boxed{\begin{aligned} \lambda_\alpha &= 243\,nm \\ \lambda_\beta &= 205\,nm \end{aligned}}$$

25

a) $r_n = n^2 a_0 = 1.9\times10^5\,\text{Å} = 1.9\times10^{-5}\,m = \boxed{19\,\mu m}$

b) $\frac{mv^2}{r} = \frac{ke^2}{r^2} \Rightarrow v^2 = \frac{ke^2}{mr}$

$\Rightarrow \boxed{v = 3.65\times10^3\,m/s}$

c) In $n=1$, $v = 2\times10^6\,m/s$. The $n=600$ is very slower!

41 a) $f_{rev} = \frac{v}{2\pi r} = \frac{2\times10^6\,m/s}{2\pi a_0} = 6.5\times10^{15}\,Hz$

So $i = q f_{rev} = \boxed{1.052\times10^{-3}\,A}$

b) $A = \pi r_1^2 = \pi a_0^2$

So $\mu = i\pi a_0^2 = \boxed{9.25\times10^{-24}\,A\cdot m^2}$

S3] Assume no quantization; just do this classically.

$$\vec{F} = \frac{Zke^2}{r^2}\hat{r} = \frac{\partial \vec{p}}{\partial t}.$$

Now, $\frac{d\vec{p}}{dt} = \frac{d}{dt}\gamma m\vec{v} = \gamma m \frac{d\vec{v}}{dt} = \gamma m\vec{a}$

$$|\vec{F}| = \frac{Zke^2}{r^2} = |\vec{a}| = \frac{\gamma m v^2}{r} \quad \left(a = \frac{v^2}{r} \text{ b/c circular orbit}\right)$$

$$\Rightarrow \gamma v^2 = \frac{Zke^2}{mr} \Rightarrow \frac{v^2}{\sqrt{1 - v^2/c^2}} = \frac{Zke^2}{mr} \equiv A$$

So $v^4 = A^2\left(1 - \frac{v^2}{c^2}\right) \Rightarrow v^4 + \frac{A^2}{c^2}v^2 - A^2 = 0$

$$\Rightarrow v^2 = \frac{-(A^2/c^2) \pm \sqrt{\left(\frac{A^2}{c^2}\right)^2 + 4A^2}}{2}$$

$$v^2 = \frac{\sqrt{\left(\frac{A^2}{c^2}\right)^2 + 4A^2} - A^2/c^2}{2}$$

$$\Rightarrow \boxed{v = \left[\frac{\sqrt{(A^2/c^2)^2 + 4A^2} - A^2/c^2}{2}\right]^{1/2}}$$

For energies, Use $KE = \gamma mc^2 - mc^2$

$$\Rightarrow KE = (\gamma - 1) mc^2$$

And plug our v into γ.

Not a pretty sight!