4E : The Quantum Universe

Lecture 8, April 12

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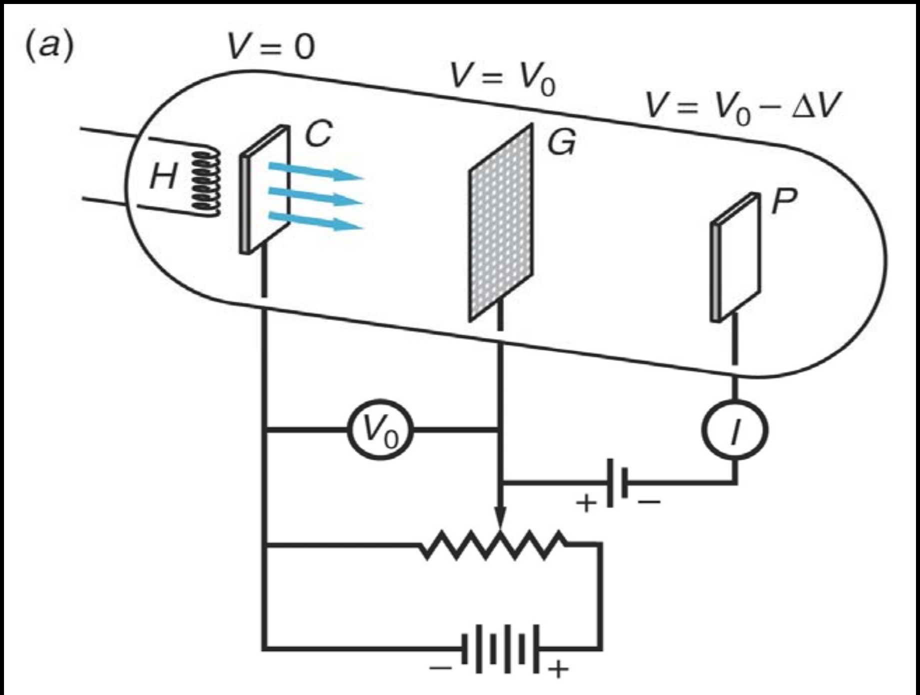
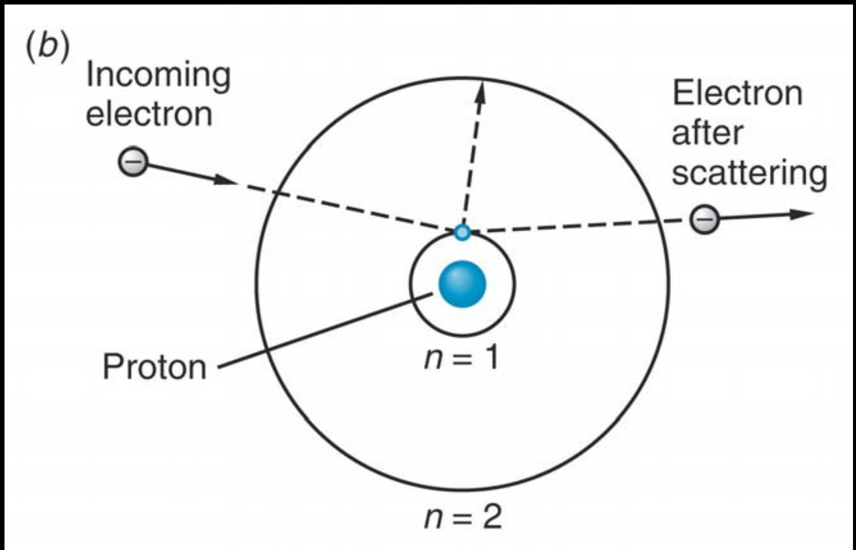
Atomic Excitation by Electrons: Franck-Hertz Expt

Other ways of Energy exchange are also quantized ! Example:

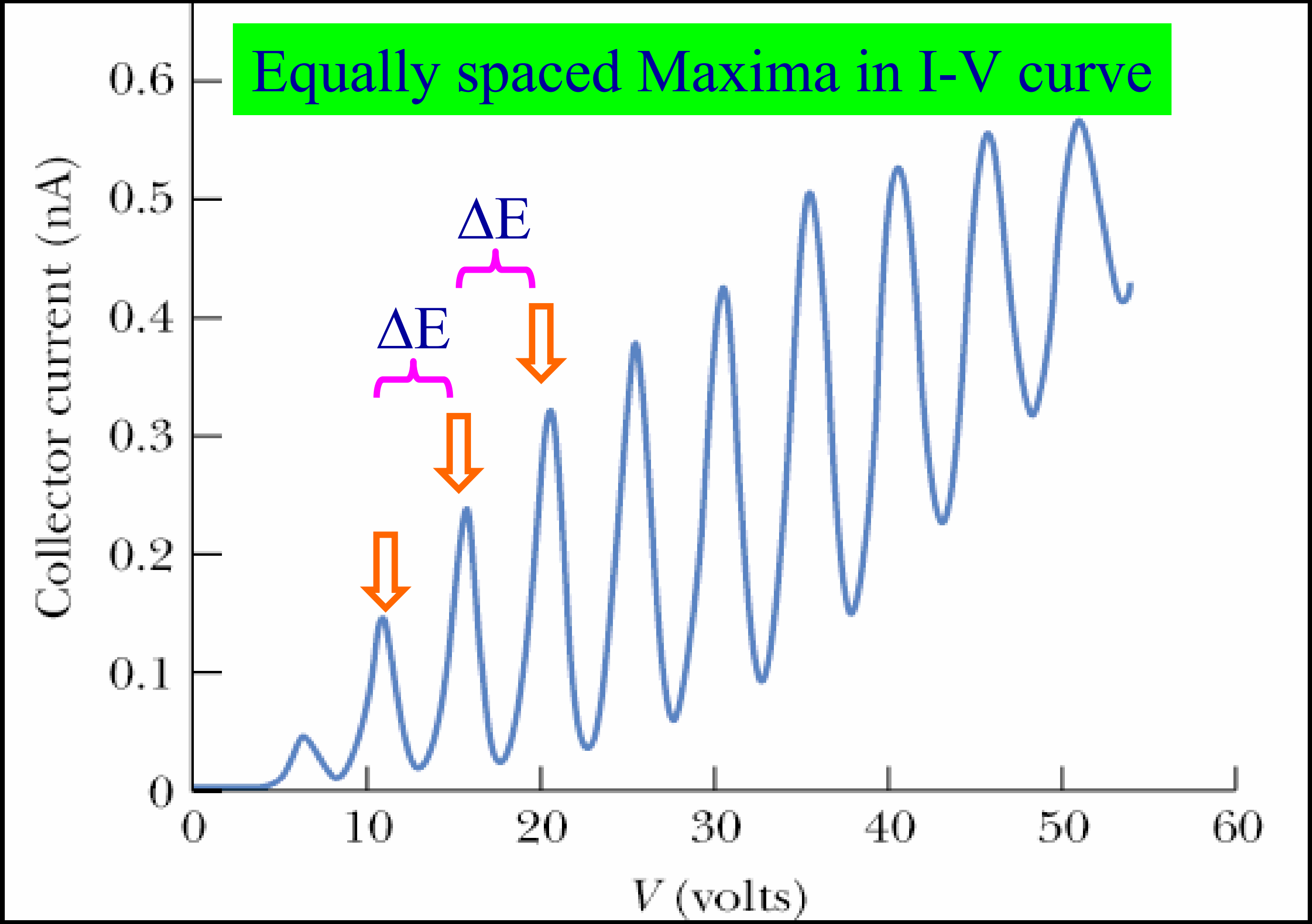
- Transfer energy to atom by colliding electrons on it
  - Elastic and inelastic collisions with a heavy atom (Hg)
- Accelerate electrons, collide with Hg atoms, measure energy transfer in inelastic collision (by applying retarding voltage)
- Count how many electrons get thru and arrive at plate P





Atomic Excitation by Electrons: Franck-Hertz Expt

Plot # of electrons/time (current) overcoming the retarding potential (V)



Atoms accept only discrete amount of Energy,
no matter the fashion in which energy is transfered

Bohr's Explanation of Hydrogen like atoms

- Bohr's Semiclassical theory explained some spectroscopic data → Nobel Prize : 1922
- The "hotch-potch" of clasical & quantum attributes left many (Einstein) unconvinced
  - "appeared to me to be a miracle – and appears to me to be a miracle today ...... One ought to be ashamed of the successes of the theory"
- Problems with Bohr's theory:
  - Failed to predict INTENSITY of spectral lines
  - Limited success in predicting spectra of multi-electron atoms (He)
  - Failed to provide "time evolution " of system from some initial state
  - Overemphasized Particle nature of matter-could not explain the wave-particle duality of light
  - No general scheme applicable to non-periodic motion in subatomic systems
- "Condemned" as a one trick pony ! Without fundamental insight …raised the question : Why was Bohr successful?

Prince Louise de Broglie & Matter Waves

- Key to Bohr atom was <u>Angular momentum quantization</u>
- Why this Quantization: $mvr = |L| = nh/2\pi$?
- Invoking symmetry in nature, Prince Louise de Broglie conjectured:

Because photons have wave <u>and</u> particle like nature → particles may have <u>wave</u> like properties !!

Electrons have accompanying "pilot" wave (not EM) which guide particles thru spacetime.

A PhD Thesis Fit For a Prince !

- Matter Wave !
  - "Pilot wave" of $\lambda = h/p = h / (\gamma m v)$
  - Frequency of pilot wave $f = E/h$

- Consequence:
  - If matter has wave like properties then there would be interference (destructive & constructive) of some kind!
    - Analogy of standing waves on a plucked string to explain the quantization condition of Bohr orbits

Matter Waves : How big, how small ?

1.Wavelength of baseball, m=140g, v=27m/s

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{h}{mv} = \frac{6.63\times10^{-34}\,J.s}{(.14kg)(27m/s)} = 1.75\times10^{-34}\,m$$

$\Rightarrow$ $\lambda_{baseball}$ <<< size of nucleus

$\Rightarrow$ Baseball "looks" like a particle

2. Wavelength of electron K=120eV (assume NR)

$$\mathrm{K} = \frac{\mathrm{p}^2}{2\mathrm{m}} \Rightarrow p = \sqrt{2mK}$$

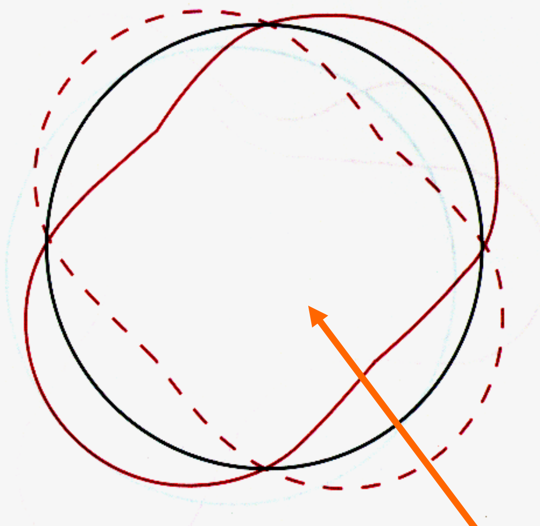
$$= \sqrt{2(9.11\times10^{-31})(120eV)(1.6\times10^{-19})}$$

$$= 5.91\times 10^{-24}\,Kg.m/s$$

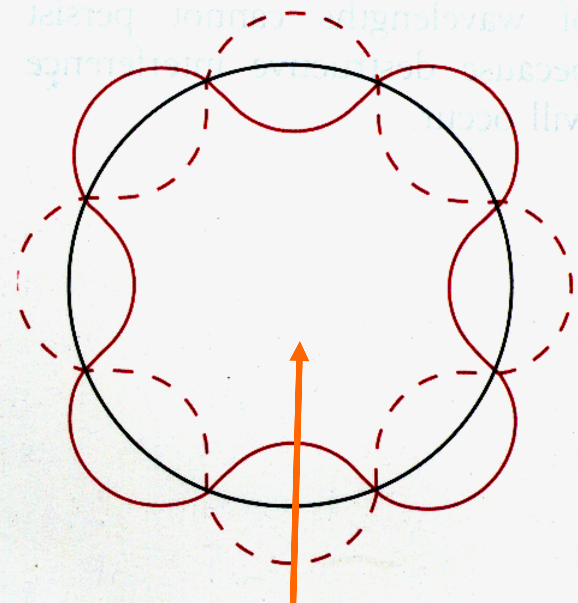
$$\lambda_e = \frac{h}{p} = \frac{6.63\times10^{-34}\,J.s}{5.91\times10^{-24}\,kg.m/s} = 1.12\times10^{-10}\,m$$

$\Rightarrow$ $\lambda_e \simeq$ Size of atom !!

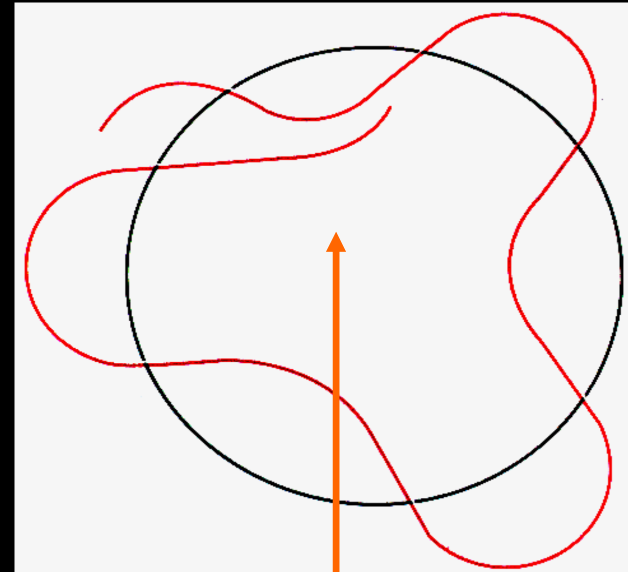
Models of Vibrations on a Loop: Model of e in atom

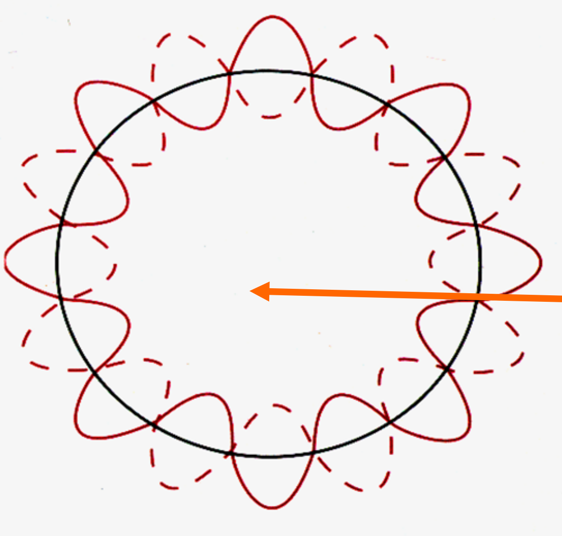


Circumference = 2 wavelengths



Circumference = 4 wavelengths





Circumference = 8 wavelengths

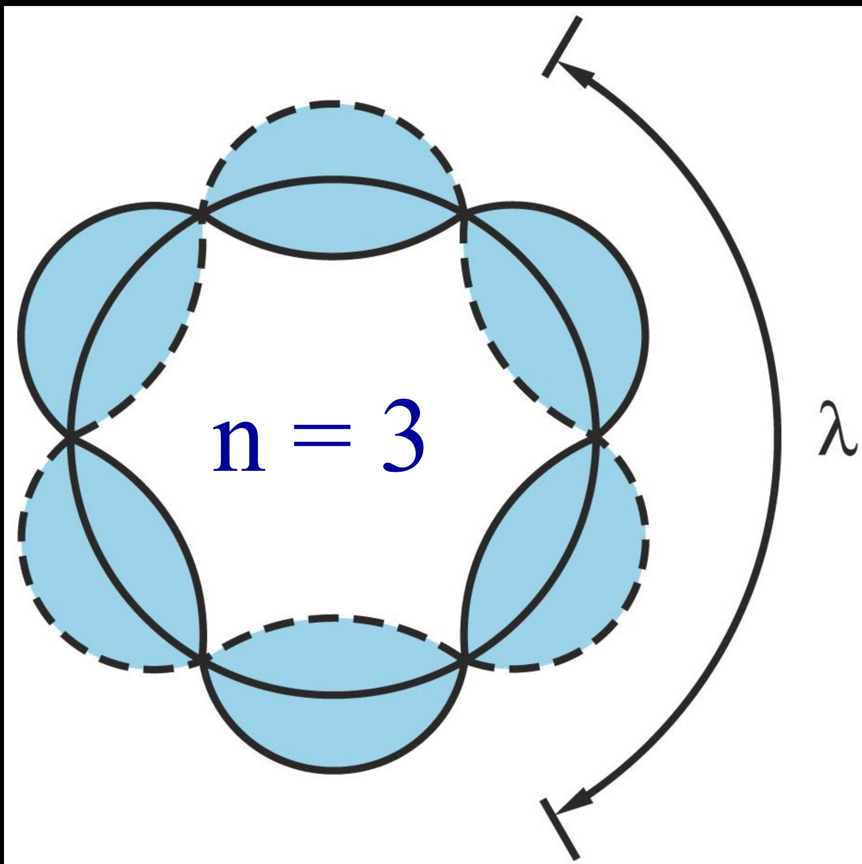
Modes of vibration when a integral # of λ fit into loop

(Standing waves)

vibrations continue Indefinitely

Fractional # of waves in a loop can not persist due to destructive interference

De Broglie's Explanation of Bohr's Quantization



Standing waves in H atom:

Constructive interference when

$n\lambda = 2\pi r$

since $\lambda = \frac{h}{p} = \frac{h}{mv}$ ......(*NR*)

$\Rightarrow \frac{nh}{mv} = 2\pi r$

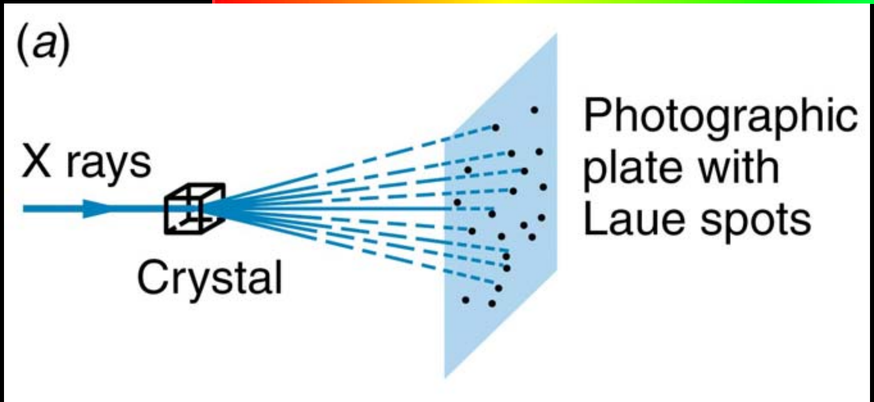
$\Rightarrow \boxed{n\hbar = mvr}$

Angular momentum

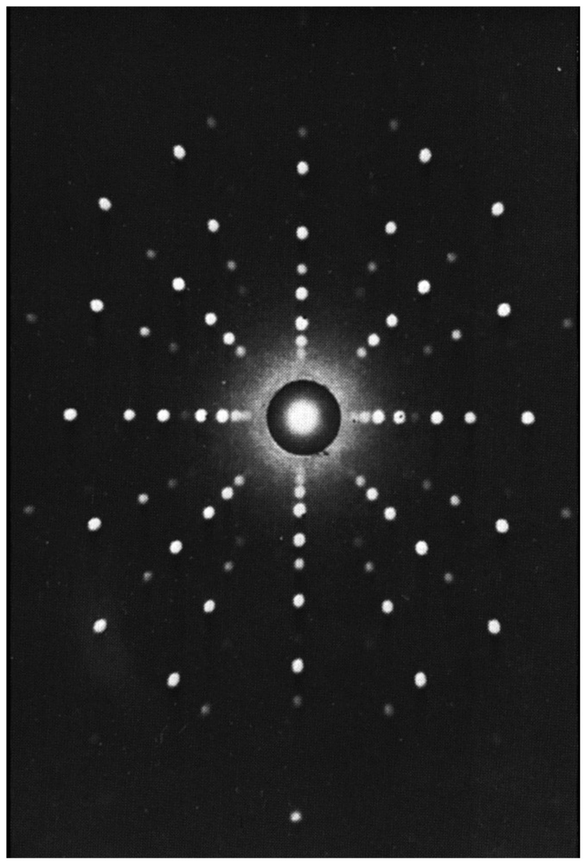
Quantization condition!

This is too intense ! Must verify such "loony tunes" with experiment

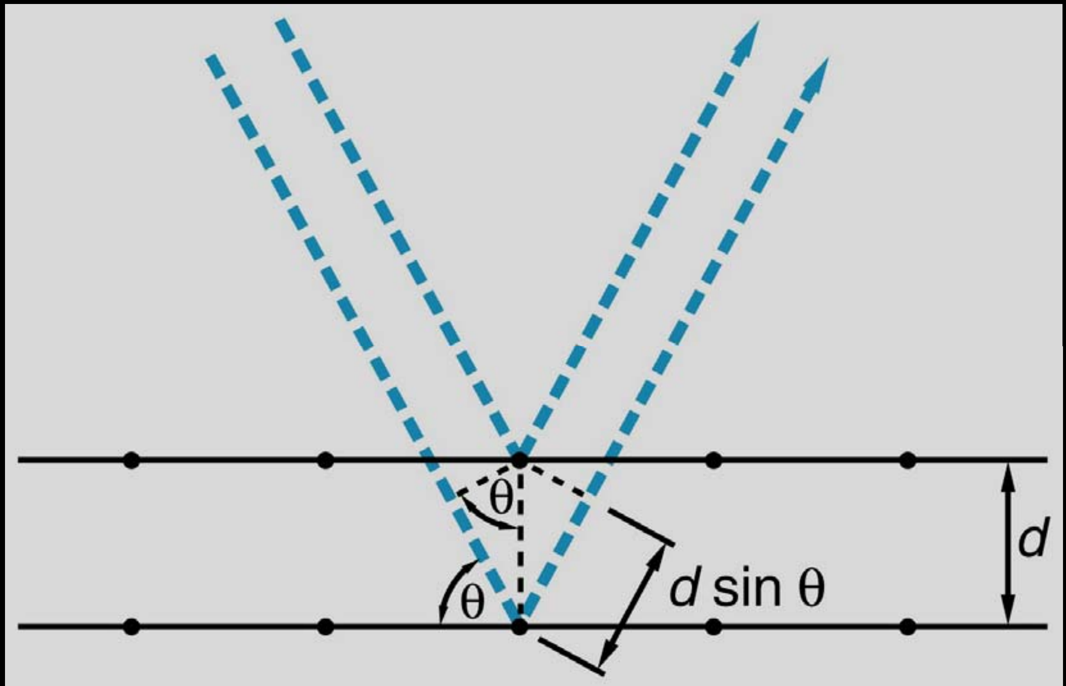
Reminder: Light as a Wave : Bragg Scattering Expt



X-ray scatter off a crystal sample
X-rays constructively interfere from certain planes producing bright spots



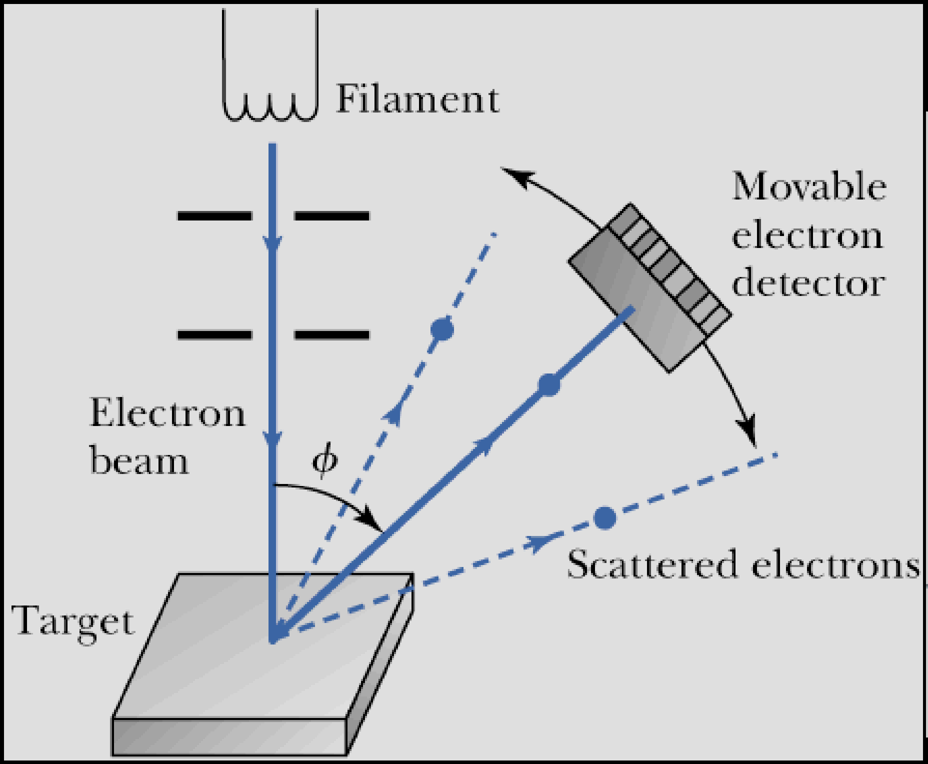
Interference → Path diff=2dsinϑ = nλ



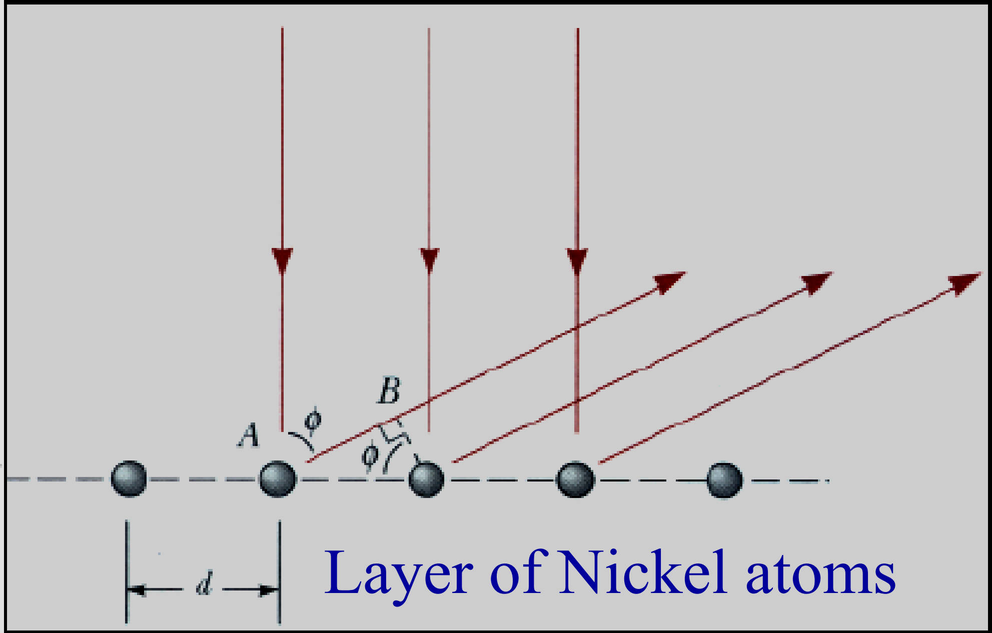
Verification of Matter Waves: Davisson & Germer Expt

If electrons have associated wave like properties → expect interference pattern when incident on a layer of atoms (reflection diffraction grating) with inter-atomic separation d such that

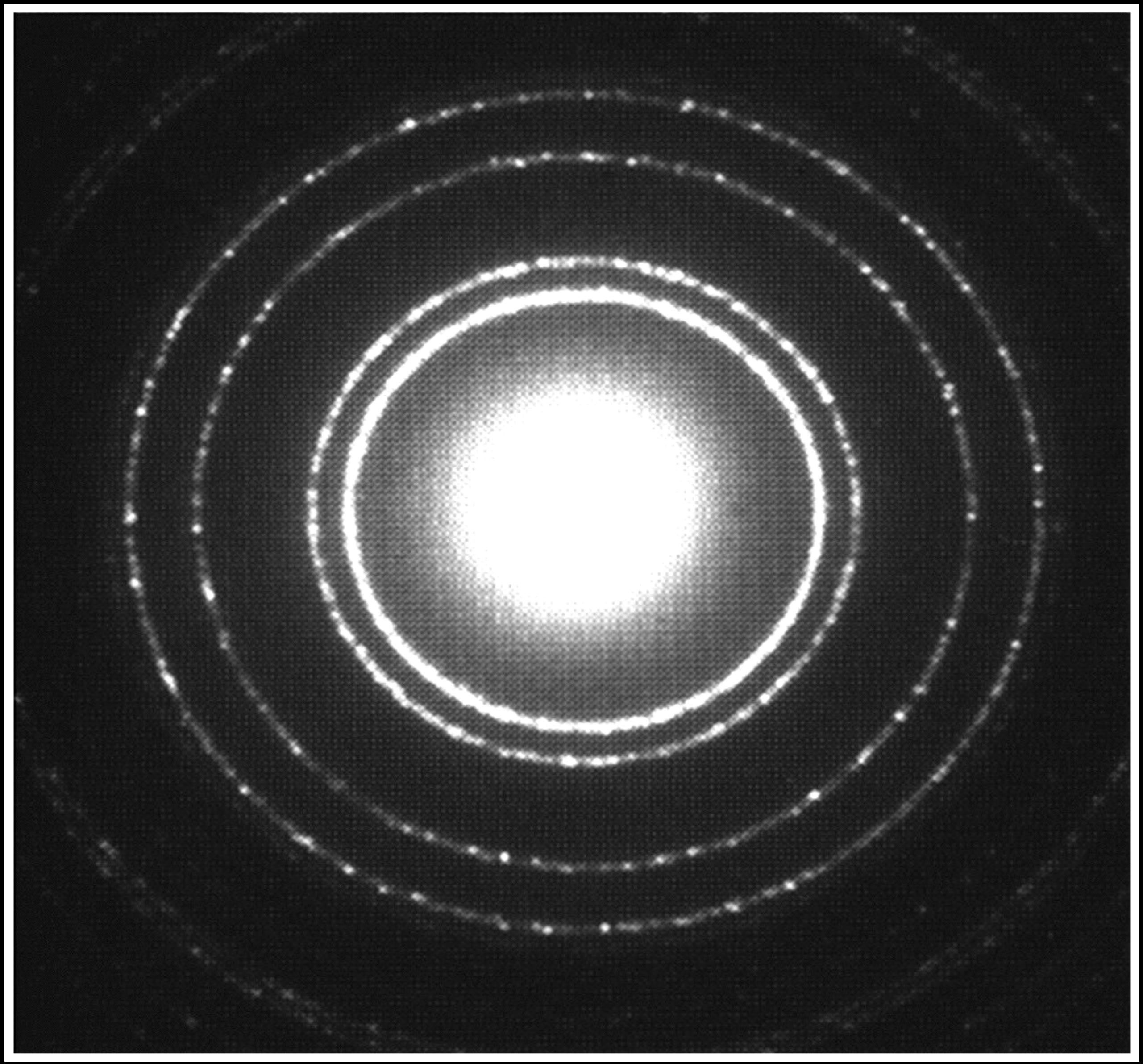
path diff AB= $d \sin\vartheta = n\lambda$



Atomic lattice as diffraction grating



Electrons Diffract in Crystal, just like X-rays

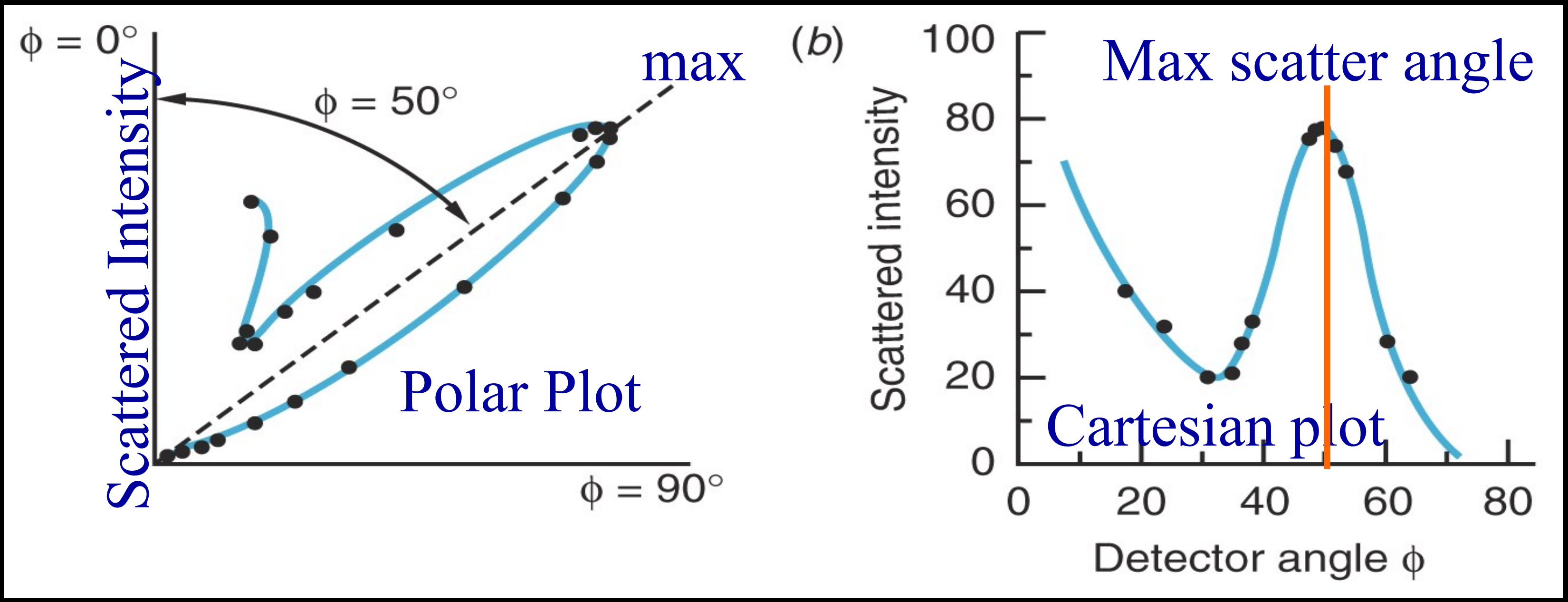


Diffraction pattern produced by 600eV electrons incident on a Al foil target

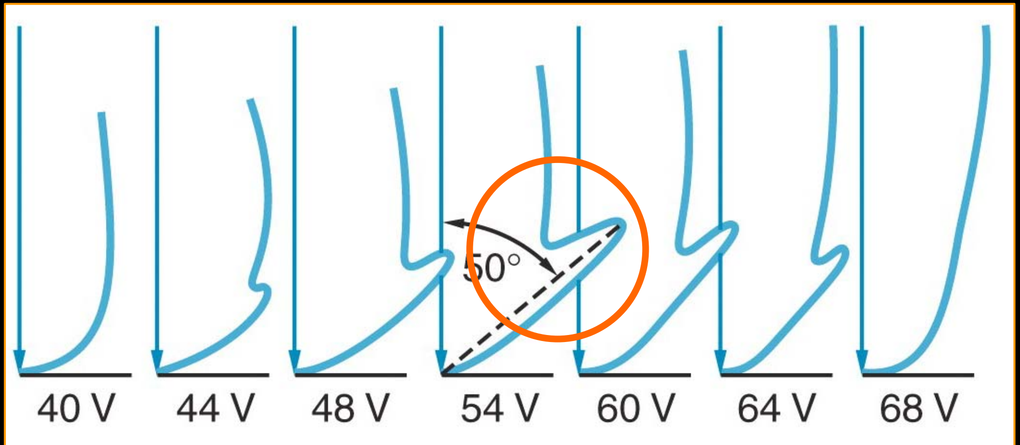
Notice the waxing and waning of scattered electron Intensity.

What to expect if electron had no wave like attribute

Davisson-Germer Experiment: 54 eV electron Beam



Polar graphs of DG expt with different electron accelerating potential when incident on same crystal (d = const)



Peak at $\Phi = 50^o$ when $V_{acc} = 54$ V

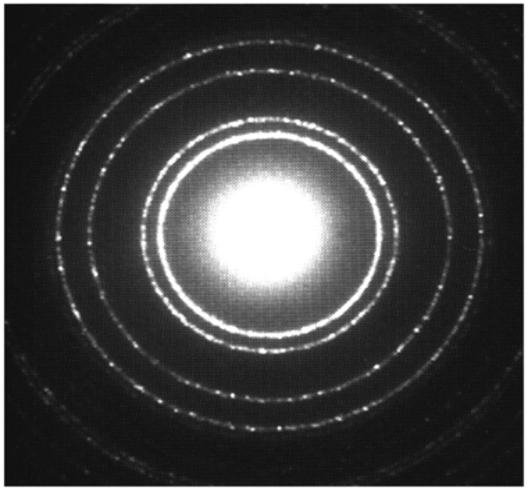
Analyzing Davisson-Germer Expt with de Broglie idea

de Broglie λ for electron accelerated thru V_{acc} =54V

$$\frac{1}{2}mv^2 = K = \frac{p^2}{2m} = eV \Rightarrow v = \sqrt{\frac{2eV}{m}} \quad ; \quad p = mv = m\sqrt{\frac{2eV}{m}}$$

If you believe de Broglie

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \boxed{\frac{h}{\sqrt{2meV}} = \lambda^{predict}}$$



For $V_{acc} = 54$ Volts $\Rightarrow \lambda = 1.67 \times 10^{-10} m$ (de Broglie)

Exptal data from Davisson-Germer Observation:

d_{nickel} =2.15 Å =$2.15 \times 10^{-10} m$ (from Bragg Scattering)

$\theta_{diff}^{max} = 50^o$ (observation from scattering intensity plot)

$$\boxed{\text{Diffraction Rule : d } \sin\phi = \text{n}\lambda}$$

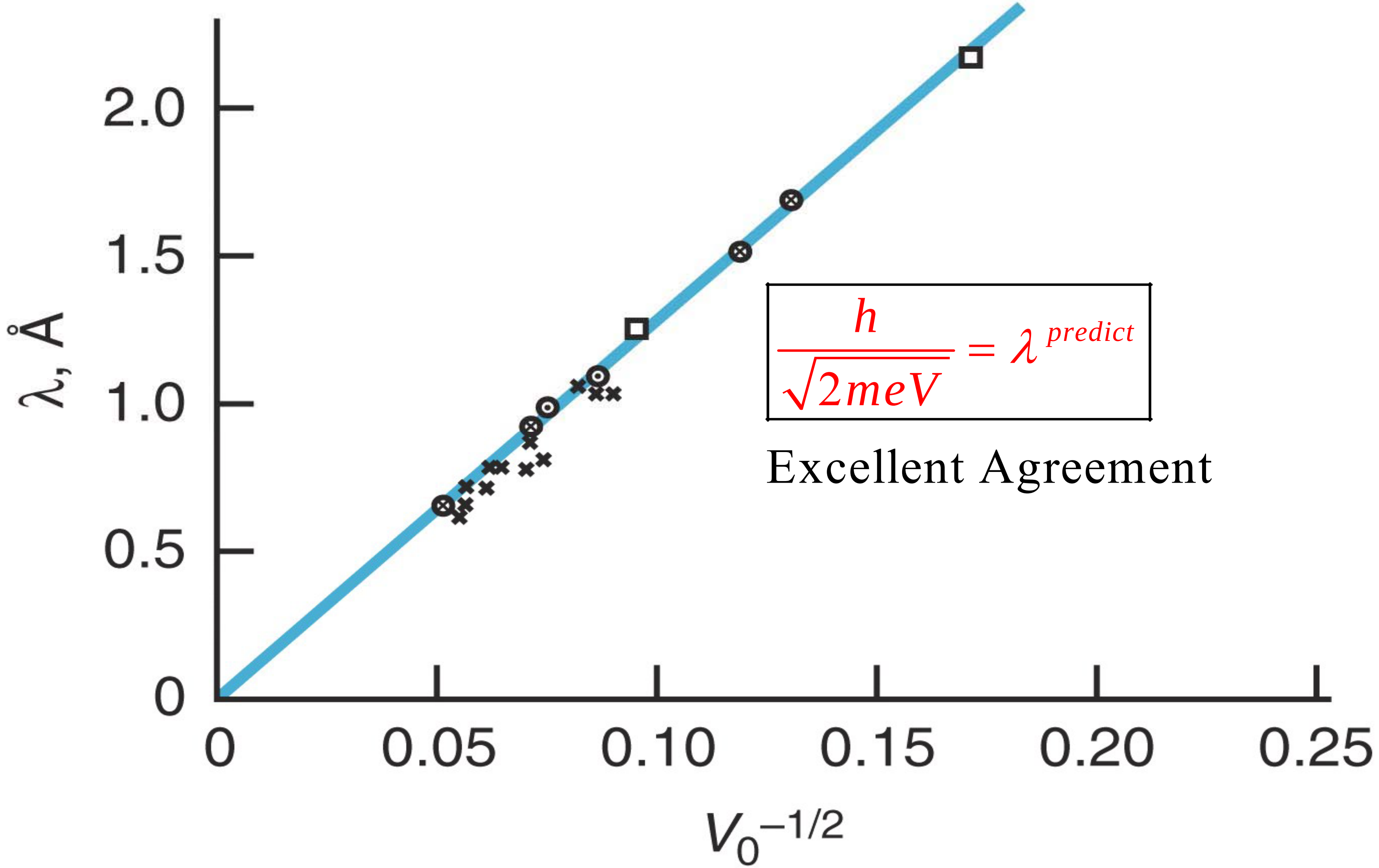
$\Rightarrow$ For Principal Maxima (n=1); λ^{meas} =(2.15 Å)(sin 50°)

$\lambda^{predict}$ =1.67Å

λ^{observ} =1.65 Å

Excellent agreement!

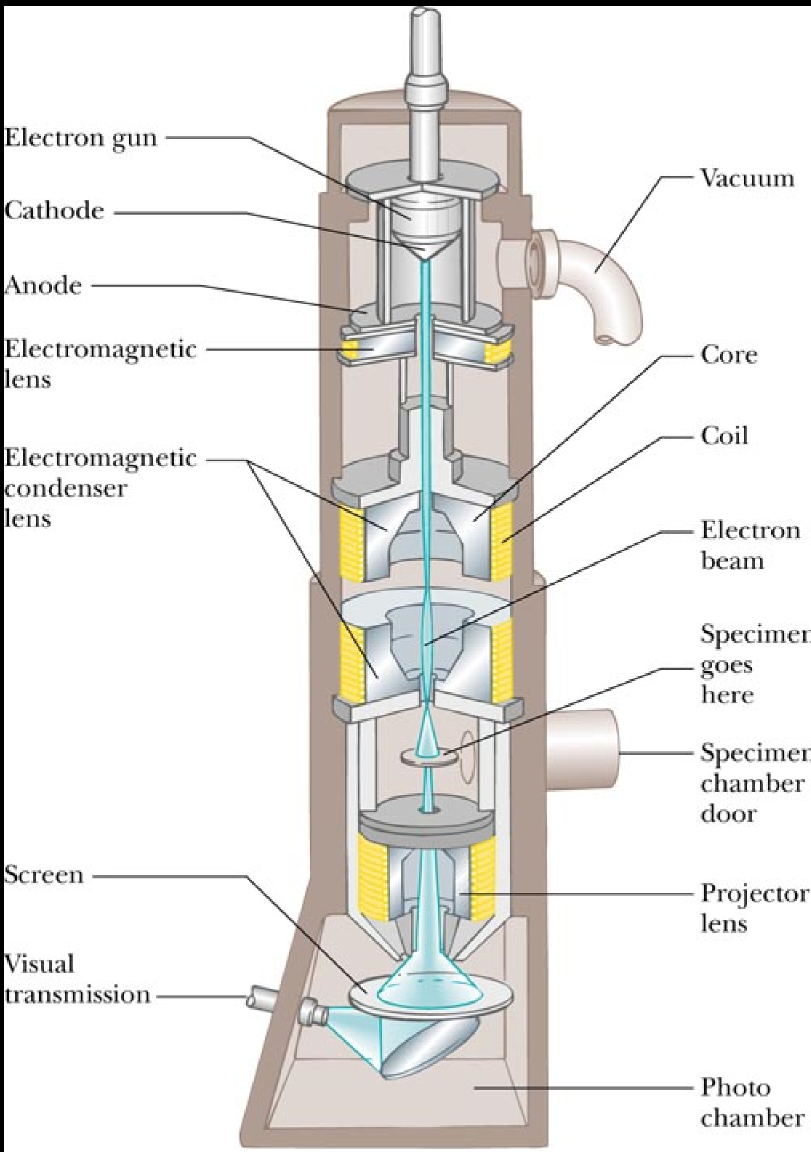
Davisson Germer Experiment: Matter Waves !

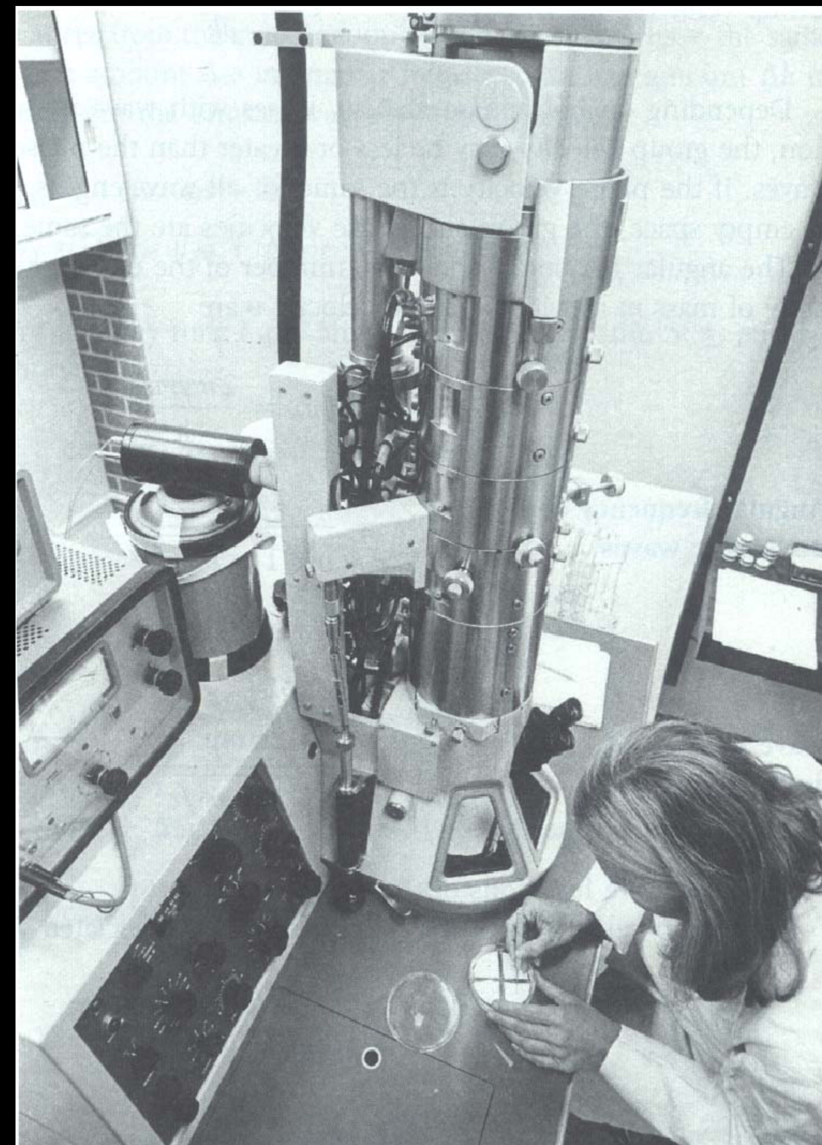


$$\frac{h}{\sqrt{2meV}} = \lambda^{predict}$$

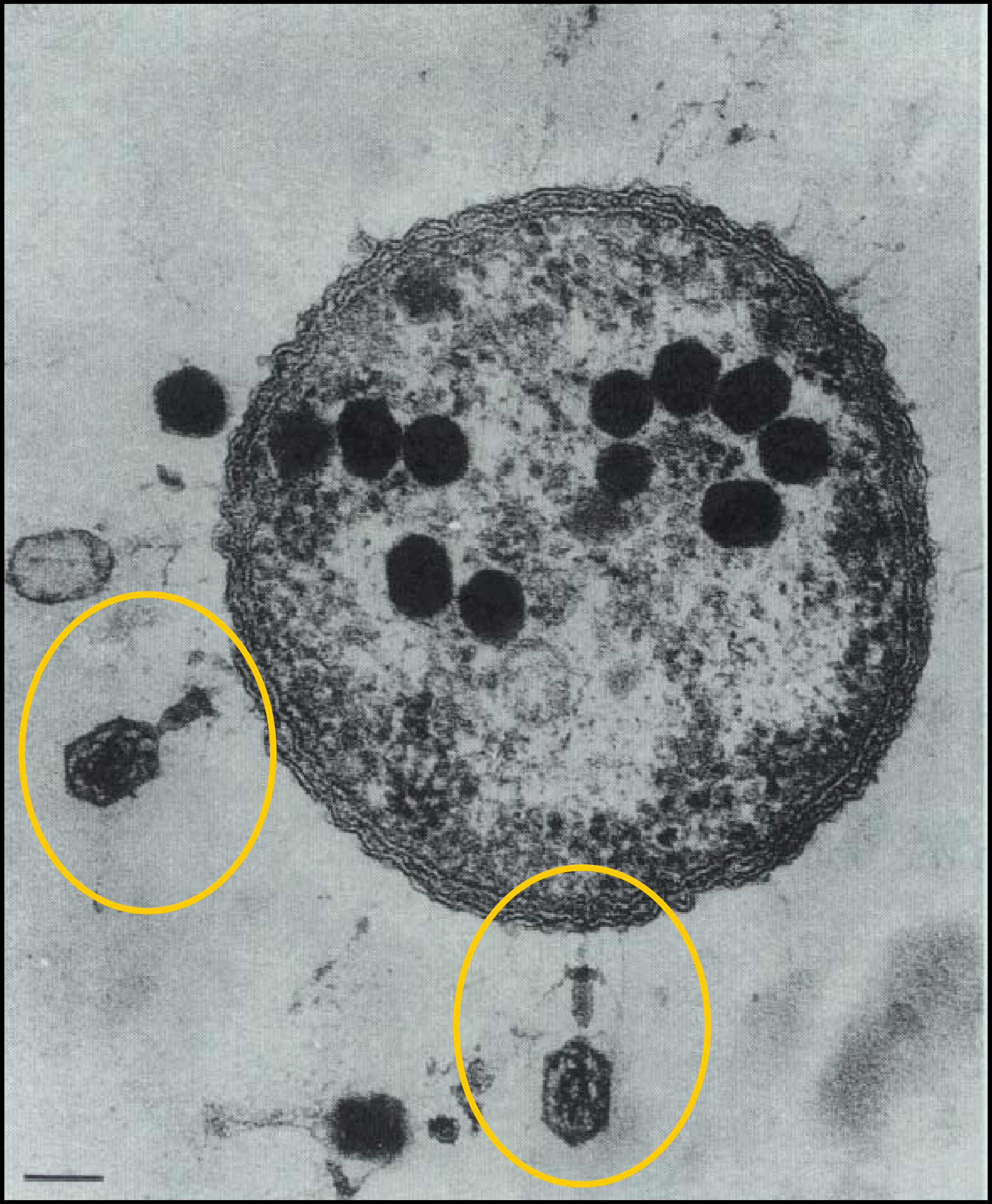
Excellent Agreement

Practical Application : Electron Microscope





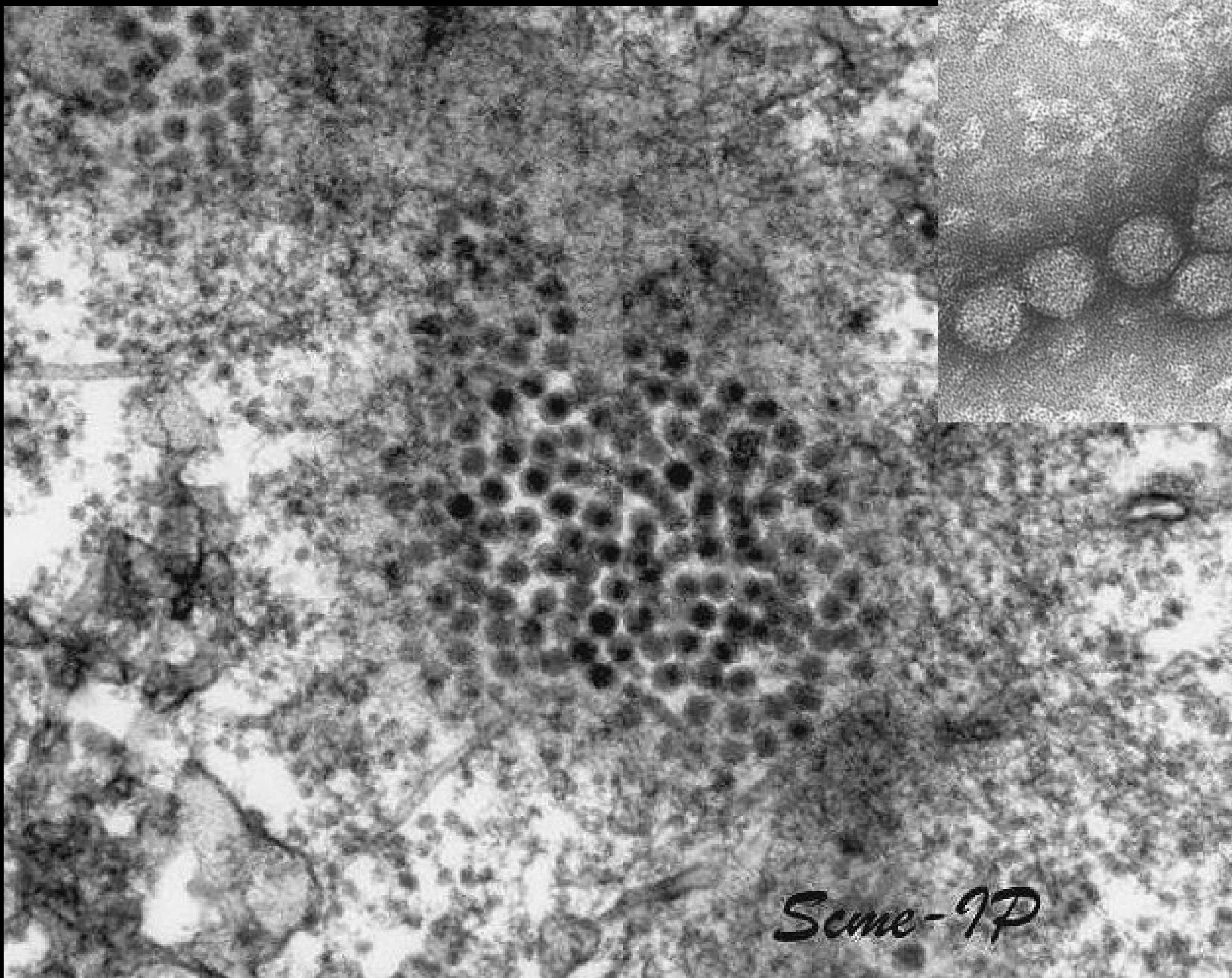
Electron Microscope : Excellent Resolving Power

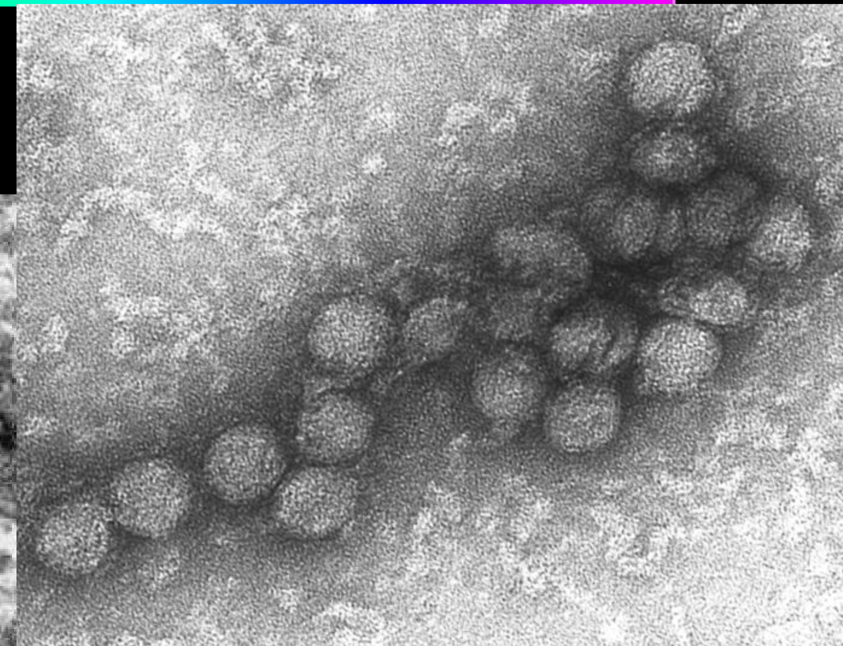


Electron Micrograph showing Bacteriophage viruses in E. Coli bacterium

The bacterium is $\simeq 1\mu$ size

West Nile Virus extracted from a crow brain





Just What is Waving in Matter Waves ?

For waves in an ocean, it's the water that "waves"
For sound waves, it's the molecules in medium
For light it's the **E** & **B** vectors that oscillate

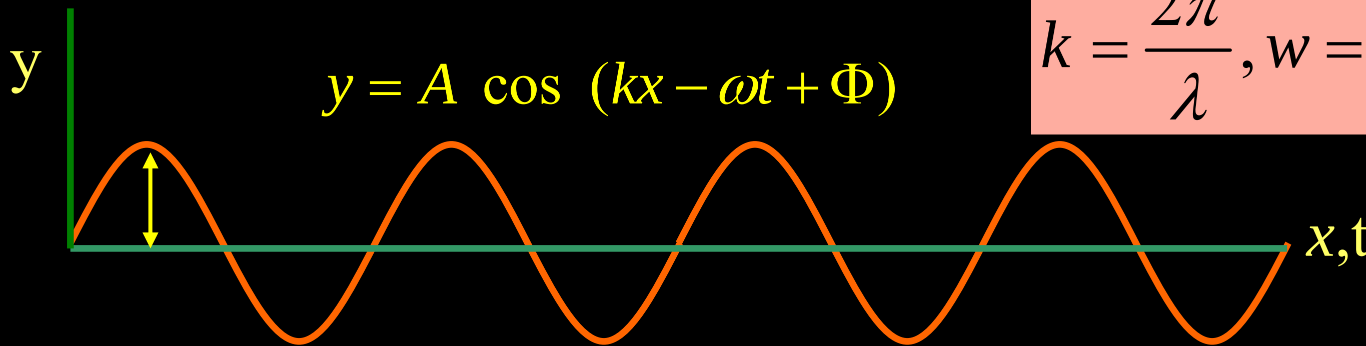
Just What's "**waving**" in matter waves ?

- **It's the PROBABLILITY OF FINDING THE PARTICLE that waves !**
- **Particle can be represented by a wave packet**
  - At a certain location (x)
  - At a certain time (t)
  - **Made by superposition of many sinusoidal waves of different amplitudes, wavelengths λ and frequency f**
  - **It's a "pulse" of probability in spacetime**

What Wave Does Not Describe a Particle

$$k = \frac{2\pi}{\lambda}, w = 2\pi f$$

$$y = A\ \cos\ (kx - \omega t + \Phi)$$



- What wave form can be associated with particle's pilot wave?
- A traveling sinusoidal wave? $y = A\ \cos\ (kx - \omega t + \Phi)$
- Since de Broglie "pilot wave" represents particle, it must travel with same speed as particle ……(like me and my shadow)

Phase velocity (v_p) of sinusoidal wave: $v_p = \lambda f$

In Matter:

$$(a)\quad \lambda = \frac{h}{p} = \frac{h}{\gamma m v}$$

$$(b)\quad f = \frac{E}{h} = \frac{\gamma m c^2}{h}$$

$$\Rightarrow v_p = \lambda f = \frac{E}{p} = \frac{\gamma m c^2}{\gamma m v} = \frac{c^2}{v} > c!$$

Conflicts with Relativity → Unphysical

Single sinusoidal wave of infinite extent does not represent particle localized in space

Need "wave packets" localized Spatially (x) and Temporally (t)

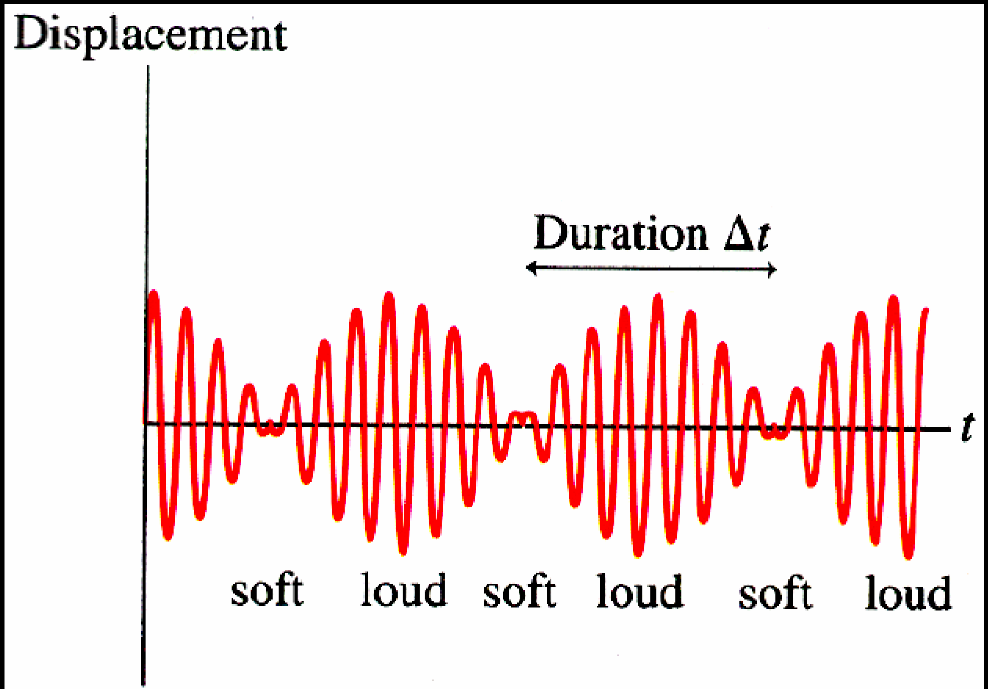
How To Make Wave Packets : Just Beat it !

Superposition of two sound waves of slightly different frequencies f_1 and f_2 , $f_1 \cong f_2$

Pattern of beats is a series of wave packets

Beat frequency $f_{beat} = f_2 - f_1 = \Delta f$

Δf = range of frequencies that are superimposed to form the wave packet



Addition of 2 Waves with slightly different wavelengths and slightly different frequencies

Resulting wave's "displacement" $y = y_1 + y_2$:

$$y = A\left[\cos(k_1 x - w_1 t) + \cos(k_2 x - w_2 t)\right]$$

Trignometry : $\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$

$$\therefore y = 2A\left[\left(\cos(\frac{k_2 - k_1}{2}x - \frac{w_2 - w_1}{2}t)\right)\left(\cos(\frac{k_2 + k_1}{2}x - \frac{w_2 + w_1}{2}t)\right)\right]$$

since $k_2 \cong k_1 \cong k_{ave}$, $w_2 \cong w_1 \cong w_{ave}, \Delta k \ll k, \Delta w \ll w$

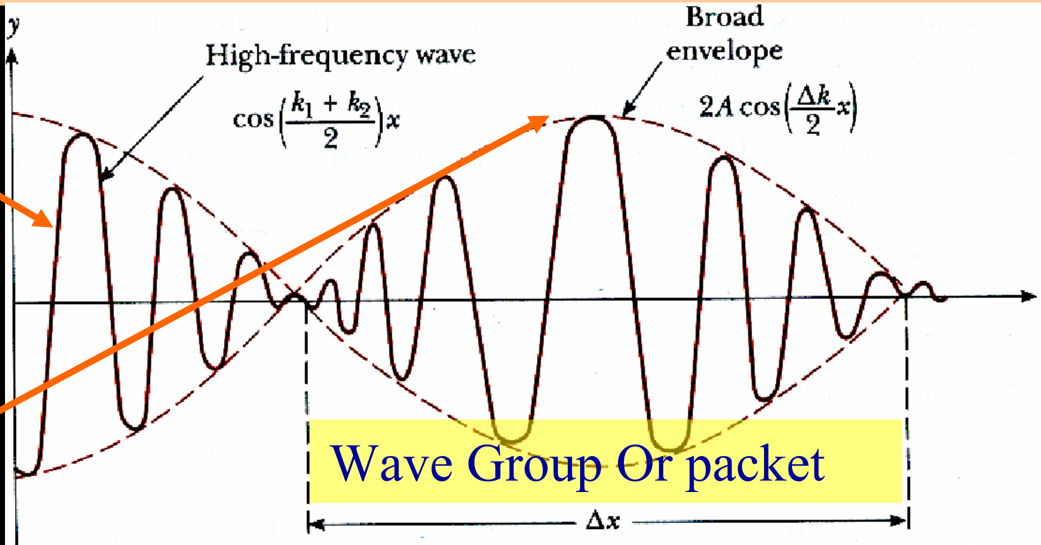
$$\therefore \boxed{y = 2A\left[\left(\cos(\frac{\Delta k}{2}x - \frac{\Delta w}{2}t)\right)\cos(kx - wt)\right]} \equiv y = A' \cos(kx - wt), \text{ A' oscillates in x,t}$$

$$A' = 2A\left(\cos(\frac{\Delta k}{2}x - \frac{\Delta w}{2}t)\right) = \text{modulated amplitude}$$

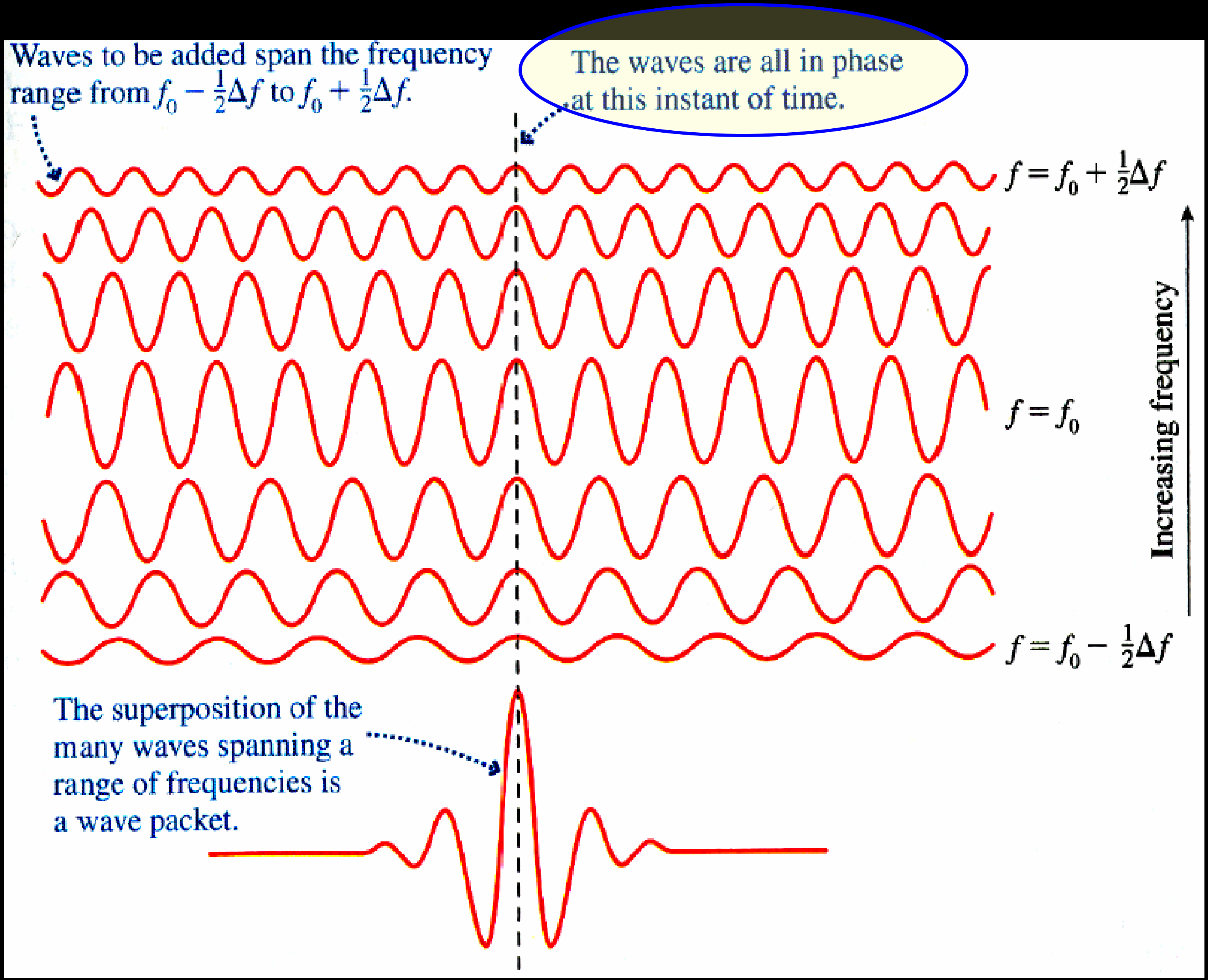
Phase Vel $V_p = \frac{w_{ave}}{k_{ave}}$

Group Vel $V_g = \frac{\Delta w}{\Delta k}$

V_g : Vel of envelope $= \frac{dw}{dk}$



Non-repeating wave packet can be created thru superposition Of many waves of similar (but different) frequencies and wavelengths



Wave Packet : Localization

Finite # of diff. Monochromatic waves always produce INFINTE sequence of repeating wave groups → can't describe (localized) particle

To make localized wave packet, add " infinite" # of waves with Well chosen Amplitude A, Wave number k and ang. f requency ω

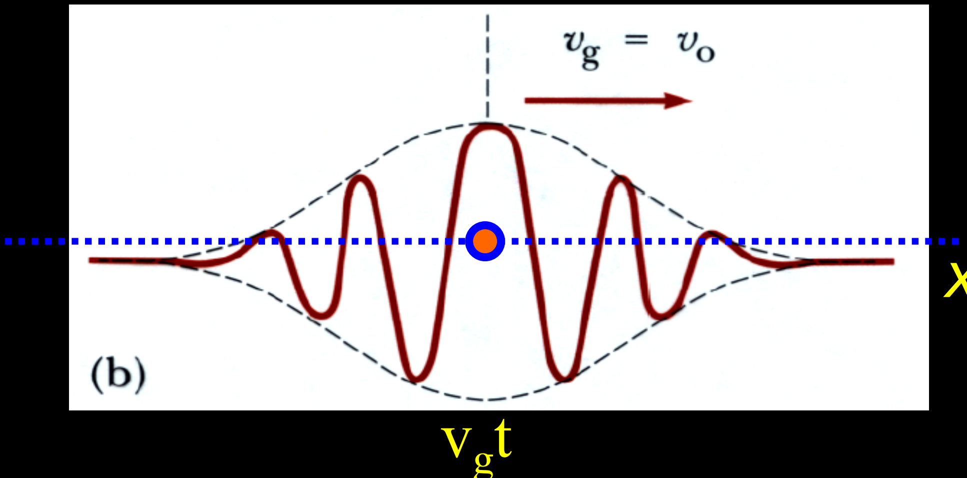
$$\psi(x,t) = \int_{-\infty}^{\infty} A(k)\, e^{i(kx-wt)} dk$$

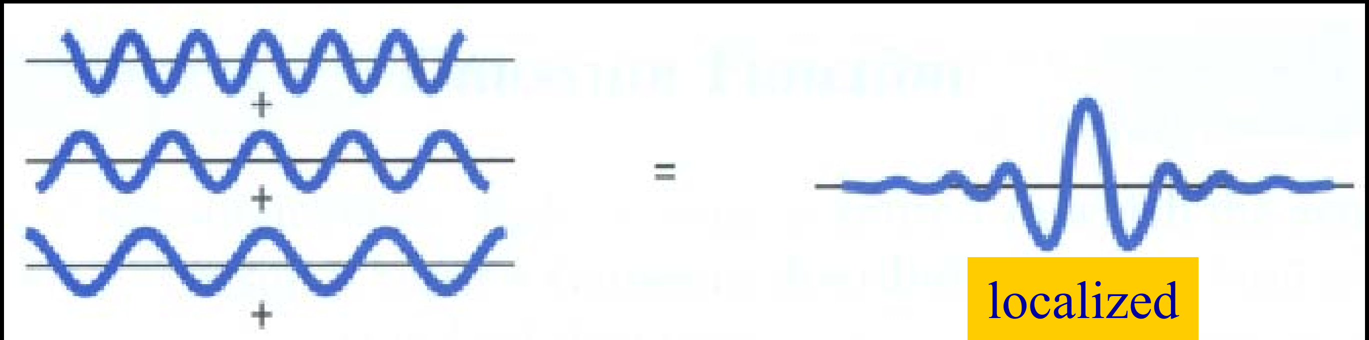
$A(k)$ = Amplitude Fn

⇒ diff waves of diff k have different amplitudes A(k)

w = w(k), depends on type of wave, media

Group Velocity $V_g = \left.\frac{dw}{dk}\right|_{k=k_0}$





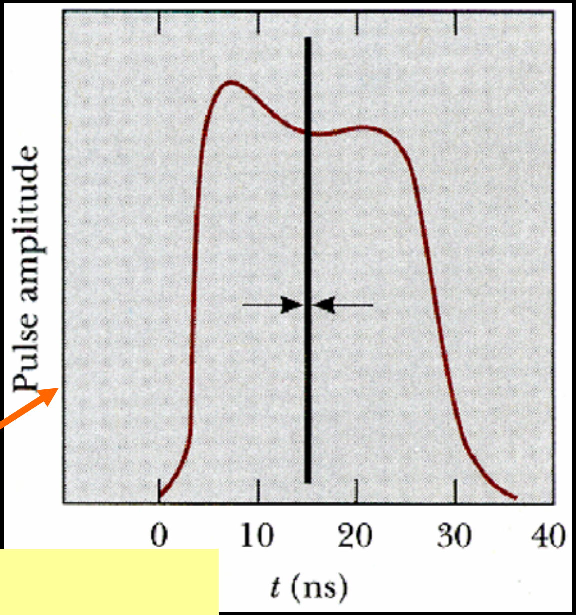
Group, Velocity, Phase Velocity and Dispersion

In a Wave Packet: $w = w(k)$

Group Velocity $V_g = \left.\frac{dw}{dk}\right|_{k=k_0}$

Since $\mathrm{V_p} = wk \quad (def) \Rightarrow w = kV_p$

$$\therefore \boxed{V_g = \left.\frac{dw}{dk}\right| = V_p\Big|_{k=k_0} + k\left.\frac{dV_p}{dk}\right|_{k=k_0}}$$



usually $\mathrm{V_p} = V_p(k \ or \lambda)$

Material in which $\mathrm{V_p}$ varies with λ are said to be Dispersive

Individual harmonic waves making a wave pulse travel at different $\mathrm{V_p}$ thus changing shape of pulse and become spread out

In non-dispersive media, $\mathrm{V_g} = V_p$

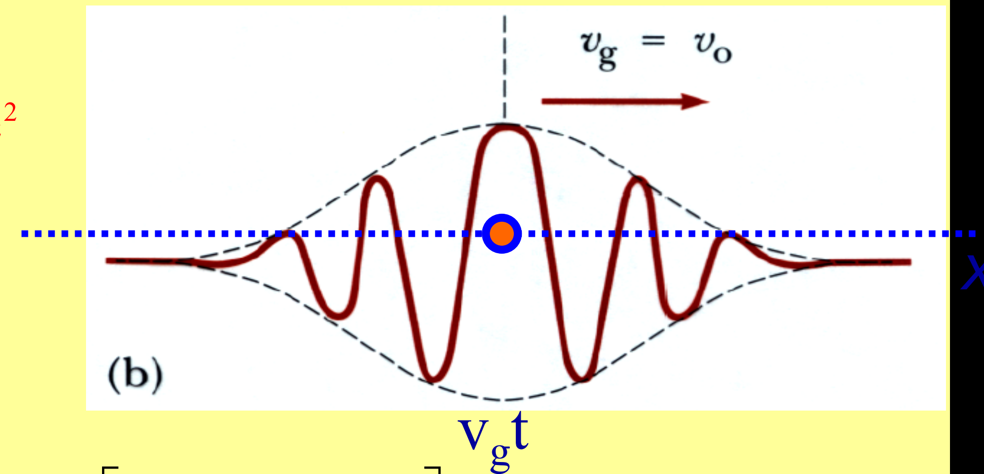
In dispersive media $\mathrm{V_g} \neq V_p$, depends on $\frac{dV_p}{dk}$

1ns laser pulse disperse By x30 after travelling 1km in optical fiber

Group Velocity of Wave Packets: v_g

Consider An Electron:

mass = m velocity = v, momentum = p



Energy $E = hf = \gamma mc^2$; $\omega = 2\pi f = \frac{2\pi}{h}\gamma mc^2$

Wavelength $\lambda = \frac{h}{p}$; $k = \frac{2\pi}{\lambda} \Rightarrow k = \frac{2\pi}{h}\gamma mv$

Group Velocity : $V_g = \frac{dw}{dk} = \frac{dw/dv}{dk/dv}$

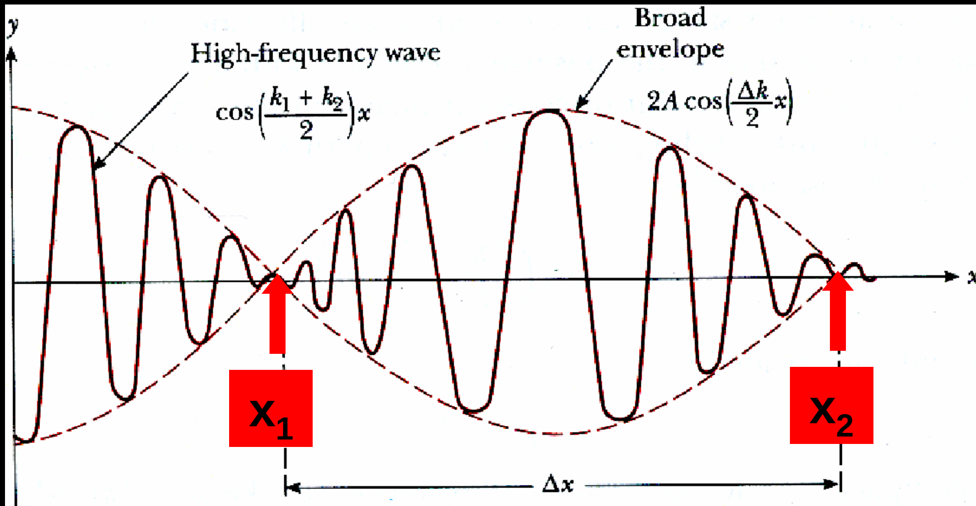
$$\frac{dw}{dv} = \frac{d}{dv}\left[\frac{\frac{2\pi}{h}mc^2}{[1-(\frac{v}{c})^2]^{1/2}}\right] = \frac{2\pi mv}{h[1-(\frac{v}{c})^2]^{3/2}} \quad \& \quad \frac{dk}{dv} = \frac{d}{dv}\left[\frac{2\pi}{h[1-(\frac{v}{c})^2]^{1/2}}mv\right] = \frac{2\pi m}{h[1-(\frac{v}{c})^2]^{3/2}}$$

$V_g = \frac{dw}{dk} = \frac{dw/dv}{dk/dv} = v \Rightarrow$ Group velocity of electron Wave packet "pilot wave"

is same as electron's physical velocity

But velocity of individual waves making up the wave packet $V_p = \frac{w}{k} = \frac{c^2}{v} > c!$ (not physical)

Wave Packets & Uncertainty Principles



$$y = 2A\left[\left(\cos\left(\frac{\Delta k}{2}x - \frac{\Delta w}{2}t\right)\right)\cos(kx - wt)\right]$$

- Distance ΔX between adjacent minima = $(X_2)_{node} - (X_1)_{node}$
- Define $X_1=0$ then phase diff from $X_1 \rightarrow X_2 = \pi$ (similarly for $t_1 \rightarrow t_2$)

Node at y = 0 = 2A cos $\left(\frac{\Delta w}{2}t - \frac{\Delta k}{2}x\right)$, Examine x or t behavior

$\Rightarrow$ in x: $\boxed{\Delta k.\Delta x = \pi}$ $\Rightarrow$ Need to combine many waves of diff. k to make small Δx pulse

$$\Delta x = \frac{\pi}{\Delta k}, \text{ for small } \Delta x \to 0 \Rightarrow \Delta k \to \infty \text{ \& Vice Verca}$$

and In t : $\boxed{\Delta w.\Delta t = \pi}$ $\Rightarrow$ Need to combine many waves of diff ω to make small Δt pulse

$$\Delta t = \frac{\pi}{\Delta \omega}, \text{ for small } \Delta t \to 0 \Rightarrow \Delta \omega \to \infty \text{ \& Vice Verca}$$

Signal Transmission and Bandwidth Theory

- Short duration pulses are used to transmit digital info
  - Over phone line as brief tone pulses
  - Over satellite link as brief radio pulses
  - Over optical fiber as brief laser light pulses
- Ragardless of type of wave or medium, any wave pulse must obey the fundamental relation
  - $\Delta\omega\Delta t \simeq \pi$
- Range of frequencies that can be transmitted are called bandwidth of the medium
- Shortest possible pulse that can be transmitted thru a medium is $\Delta t_{min} \simeq \pi/\Delta\omega$
- Higher bandwidths transmits shorter pulses & allows high data rate

in space x: $\boxed{\Delta k.\Delta x = \pi}$ $\Rightarrow$ since $k = \dfrac{2\pi}{\lambda}, p = \dfrac{h}{\lambda}$

$$\Rightarrow \quad \boxed{\Delta p.\Delta x = h/2}$$

usually one writes $\boxed{\Delta p.\Delta x \geq \hbar/2}$ approximate relation

In time t : $\boxed{\Delta w.\Delta t = \pi}$ $\Rightarrow$ since $\omega = 2\pi f, E = hf$

$$\Rightarrow \quad \boxed{\Delta E.\Delta t = h/2}$$

usually one writes $\boxed{\Delta E.\Delta t \geq \hbar/2}$ approximate relation

What do these inequalities mean physically?