Atomic Excitation by Electrons: Franck-Hertz Expt

Other ways of Energy exchange are also quantized! Example:

- Transfer energy to atom by colliding electrons on it
  - Elastic and inelastic collisions with a heavy atom (Hg)
- Accelerate electrons, collide with Hg atoms, measure energy transfer in inelastic collision (by applying retarding voltage)
- Count how many electrons get thru and arrive at plate P
**Atomic Excitation by Electrons: Franck-Hertz Expt**

Plot # of electrons/time (current) overcoming the retarding potential (V)

Equally spaced Maxima in I-V curve

Atoms accept only discrete amount of Energy, no matter the fashion in which energy is transferred.
Bohr’s Explanation of Hydrogen like atoms

- Bohr’s Semiclassical theory explained some spectroscopic data → Nobel Prize: 1922
- The “hotch-potch” of classical & quantum attributes left many (Einstein) unconvinced
  - “appeared to me to be a miracle – and appears to me to be a miracle today ...... One ought to be ashamed of the successes of the theory”
- Problems with Bohr’s theory:
  - Failed to predict INTENSITY of spectral lines
  - Limited success in predicting spectra of multi-electron atoms (He)
  - Failed to provide “time evolution” of system from some initial state
  - Overemphasized Particle nature of matter-could not explain the wave-particle duality of light
  - No general scheme applicable to non-periodic motion in subatomic systems
- “Condemned” as a one trick pony! Without fundamental insight ...raised the question: Why was Bohr successful?
Prince Louise de Broglie & Matter Waves

- Key to Bohr atom was **Angular momentum quantization**
- Why this Quantization: \( mvr = |L| = \frac{\hbar}{2\pi} \) ?
- Invoking symmetry in nature, Prince Louise de Broglie conjectured:

Because photons have wave **and** particle like nature → particles may have wave **like** properties !!

Electrons have accompanying “pilot” wave (not EM) which guide particles thru spacetime.
• Matter Wave!
  – “Pilot wave” of $\lambda = h/p = h / (\gamma mv)$
  – Frequency of pilot wave $f = E/h$

• Consequence:
  – If matter has wave-like properties then there would be interference (destructive & constructive) of some kind!
    • Analogy of standing waves on a plucked string to explain the quantization condition of Bohr orbits
Matter Waves: How big, how small?

1. Wavelength of baseball, \( m = 140 \text{ g}, \ v = 27 \text{ m/s} \)

\[ \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J.s}}{(0.14 \text{ kg})(27 \text{ m/s})} = 1.75 \times 10^{-34} \text{ m} \]

\[ \Rightarrow \quad \lambda_{\text{baseball}} \ll \text{size of nucleus} \]

\[ \Rightarrow \quad \text{Baseball "looks" like a particle} \]

2. Wavelength of electron \( K = 120 \text{ eV} \) (assume NR)

\[ K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK} \]

\[ = \sqrt{2 \times (9.11 \times 10^{-31}) \times (120 \text{ eV}) \times (1.6 \times 10^{-19})} \]

\[ = 5.91 \times 10^{-24} \text{ Kg.m/s} \]

\[ \lambda_e = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J.s}}{5.91 \times 10^{-24} \text{ kg.m/s}} = 1.12 \times 10^{-10} \text{ m} \]

\[ \Rightarrow \quad \lambda_e \approx \text{Size of atom} \]
Models of Vibrations on a Loop: Model of e in atom

Modes of vibration when a integral # of $\lambda$ fit into loop (Standing waves) vibrations continue Indefinitely

Fractional # of waves in a loop can not persist due to destructive interference

Circumference = 2 wavelengths

Circumference = 4 wavelengths

Circumference = 8 wavelengths
De Broglie’s Explanation of Bohr’s Quantization

Standing waves in H atom:
Constructive interference when
\[ n\lambda = 2\pi r \]

Since \[ \lambda = \frac{h}{p} = \frac{h}{mv} \] ...... (NR)
\[ \Rightarrow \frac{nh}{mv} = 2\pi r \]
\[ \Rightarrow [nh = mvr] \]

Angular momentum
Quantization condition!

This is too intense! Must verify such “loony tunes” with experiment
Reminder: Light as a Wave : Bragg Scattering Expt

(a) X-ray scatter off a crystal sample

X-rays constructively interfere from certain planes producing bright spots

Interference \( \Rightarrow \) Path diff = \( 2dsin\theta = n\lambda \)
If electrons have associated wave like properties → expect interference pattern when incident on a layer of atoms (reflection diffraction grating) with inter-atomic separation \(d\) such that

\[
\text{path diff } AB = d \sin \theta = n\lambda
\]
Electrons Diffract in Crystal, just like X-rays

Diffraction pattern produced by 600eV electrons incident on a Al foil target

Notice the waxing and waning of scattered electron Intensity.

What to expect if electron had no wave like attribute
**Davisson-Germer Experiment: 54 eV electron Beam**

Polar graphs of DG expt with different electron accelerating potential when incident on same crystal (d = const)

Peak at $\Phi = 50^\circ$
when $V_{\text{acc}} = 54$ V
Analyzing Davisson-Germer Expt with de Broglie idea

de Broglie $\lambda$ for electron accelerated thru $V_{\text{acc}} = 54V$

$$\frac{1}{2}mv^2 = K = \frac{p^2}{2m} = eV \Rightarrow v = \sqrt{\frac{2eV}{m}} ; \quad p = mv = m\sqrt{\frac{2eV}{m}}$$

If you believe de Broglie

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2meV}} = \lambda_{\text{predict}}$$

For $V_{\text{acc}} = 54$ Volts $\Rightarrow \lambda = 1.67 \times 10^{-10} m$ (de Broglie)

Exptal data from Davisson-Germer Observation:

d_{\text{nickel}} = 2.15 Å = 2.15 \times 10^{-10} m$ (from Bragg Scattering)

$\theta_{\text{diff}}^{\text{max}} = 50^\circ$ (observation from scattering intensity plot)

Diffraction Rule : $d \sin \phi = n \lambda$

$\Rightarrow$ For Principal Maxima ($n=1$); $\lambda_{\text{meas}} = (2.15 \text{ Å})(\sin 50^\circ)$

$\lambda_{\text{predict}} = 1.67 \text{ Å}$

$\lambda_{\text{observ}} = 1.65 \text{ Å}$

Excellent agreement!
Davisson Germer Experiment: Matter Waves!

\[ \frac{h}{\sqrt{2meV}} = \lambda^{\text{predict}} \]

Excellent Agreement
Practical Application: Electron Microscope

- Electron gun
- Cathode
- Anode
- Electromagnetic lens
- Electromagnetic condenser lens
- Vacuum
- Core
- Coil
- Electron beam
- Specimen goes here
- Specimen chamber door
- Screen
- Visual transmission
- Projector lens
- Photo chamber
Electron Microscope: Excellent Resolving Power

Electron Micrograph showing Bacteriophage viruses in E. Coli bacterium

The bacterium is $\approx 1\mu$ size
West Nile Virus extracted from a crow brain
Just What is Waving in Matter Waves?

For waves in an ocean, it’s the water that “waves”
For sound waves, it’s the molecules in medium
For light it’s the $\mathbf{E}$ & $\mathbf{B}$ vectors that oscillate

Just What’s “waving” in matter waves?

- It’s the PROBABILITY OF FINDING THE PARTICLE that waves!
- Particle can be represented by a wave packet
  - At a certain location ($x$)
  - At a certain time ($t$)
  - Made by superposition of many sinusoidal waves of different amplitudes, wavelengths $\lambda$ and frequency $f$
  - It’s a “pulse” of probability in spacetime
**What Wave Does Not Describe a Particle**

- What wave form can be associated with particle’s pilot wave?
- A traveling sinusoidal wave? \( y = A \cos (kx - \omega t + \Phi) \)
- Since de Broglie “pilot wave” represents particle, it must travel with same speed as particle ……(like me and my shadow)

\[
y = A \cos (kx - \omega t + \Phi)
\]

- Phase velocity \((v_p)\) of sinusoidal wave: \( v_p = \frac{\lambda f}{2} \)

In Matter:

\[
\begin{align*}
(a) \quad \lambda &= \frac{h}{p} = \frac{h}{\gamma mv} \\
(b) \quad f &= \frac{E}{h} = \frac{\gamma mc^2}{h}
\end{align*}
\]

\[\Rightarrow v_p = \lambda f = \frac{E}{p} = \frac{\gamma mc^2}{\gamma mv} = \frac{c^2}{v} > c\!\]

Conflicts with Relativity \(\rightarrow\) Unphysical

Single sinusoidal wave of infinite extent does not represent particle localized in space

Need “wave packets” localized Spatially (x) and Temporally (t)
Superposition of two sound waves of slightly different frequencies \( f_1 \) and \( f_2 \), \( f_1 \approx f_2 \)

Pattern of beats is a series of wave packets

Beat frequency \( f_{\text{beat}} = f_2 - f_1 = \Delta f \)

\( \Delta f = \) range of frequencies that are superimposed to form the wave packet
Resulting wave's "displacement" $y = y_1 + y_2$:

$$y = A\left[\cos(k_1 x - w_1 t) + \cos(k_2 x - w_2 t)\right]$$

Trigonometry: $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$\therefore y = 2A\left[\left(\cos\left(\frac{k_2 - k_1}{2} x - \frac{w_2 - w_1}{2} t\right)\right)\left(\cos\left(\frac{k_2 + k_1}{2} x - \frac{w_2 + w_1}{2} t\right)\right)\right]$$

since $k_2 \approx k_1 \approx k_{ave}$, $w_2 \approx w_1 \approx w_{ave}$, $\Delta k \ll k$, $\Delta w \ll w$

$$\therefore y = 2A\left[\left(\cos\left(\frac{\Delta k}{2} x - \frac{\Delta w}{2} t\right)\right)\cos(kx - wt)\right] = y = A' \cos(kx - wt)$, $A'$ oscillates in x,t

$$A' = 2A\left(\cos\left(\frac{\Delta k}{2} x - \frac{\Delta w}{2} t\right)\right) = \text{modulated amplitude}$$

**Phase Vel** $V_p = \frac{w_{ave}}{k_{ave}}$

**Group Vel** $V_g = \frac{\Delta w}{\Delta k}$

$V_g : \text{Vel of envelope} = \frac{dw}{dk}$

**Wave Group Or packet**

**Addition of 2 Waves with slightly different wavelengths and slightly different frequencies**
Non-repeating wave packet can be created thru superposition of many waves of similar (but different) frequencies and wavelengths.

Waves to be added span the frequency range from $f_0 - \frac{1}{2}\Delta f$ to $f_0 + \frac{1}{2}\Delta f$.

The waves are all in phase at this instant of time.

The superposition of the many waves spanning a range of frequencies is a wave packet.
Finite # of diff. Monochromatic waves always produce INFINITE
sequence of repeating wave groups → can’t describe (localized) particle
To make localized wave packet, add “infinite” # of waves with
Well chosen Amplitude A, Wave number k and ang. frequency ω

\[
\psi(x,t) = \int_{-\infty}^{\infty} A(k) e^{i(kx-wt)} dk
\]

\[A(k) = \text{Amplitude Fn}
\]

\[\Rightarrow \text{diff waves of diff } k
\]

\[\text{have different amplitudes } A(k)
\]

\[w = w(k), \text{depends on type of wave, media}
\]

Group Velocity \[V_g = \frac{dw}{dk}\bigg|_{-k_0}
\]
In a Wave Packet: \( w = w(k) \)

Group Velocity \( V_g = \frac{dw}{dk} \bigg|_{k=k_0} \)

Since \( V_p = wk \) (def) \( \Rightarrow w = kV_p \)

\[ V_g = \frac{dw}{dk} = V_p \bigg|_{k=k_0} + k \frac{dV_p}{dk} \bigg|_{k=k_0} \]

usually \( V_p = V_p(k \text{ or } \lambda) \)

Material in which \( V_p \) varies with \( \lambda \) are said to be Dispersive

Individual harmonic waves making a wave pulse travel at different \( V_p \) thus changing shape of pulse and become spread out.

In non-dispersive media, \( V_g = V_p \)

In dispersive media \( V_g \neq V_p \), depends on \( \frac{dV_p}{dk} \)

1ns laser pulse disperse
By x30 after travelling 1km in optical fiber
Consider An Electron:

\[ \text{mass} = m \quad \text{velocity} = v, \quad \text{momentum} = p \]

Energy \( E = hf = \gamma mc^2 \); \( \omega = 2\pi f = \frac{2\pi}{h} \gamma mc^2 \)

Wavelength \( \lambda = \frac{h}{p} \); \( k = \frac{2\pi}{\lambda} \Rightarrow k = \frac{2\pi}{h} \gamma mv \)

**Group Velocity:** \( V_g = \frac{dw}{dk} = \frac{dw}{dv} / \frac{dv}{dk} \)

\[
\frac{dw}{dv} = \frac{d}{dv} \left[ \frac{2\pi mc^2}{h [1-(\frac{v}{c})^2]^{1/2}} \right] = \frac{2\pi mv}{h[1-(\frac{v}{c})^2]^{3/2}} \quad \text{and} \quad \frac{dk}{dv} = \frac{d}{dv} \left[ \frac{2\pi}{h[1-(\frac{v}{c})^2]^{1/2}} \right] = \frac{2\pi}{h[1-(\frac{v}{c})^2]^{3/2}}
\]

\[ V_g = \frac{dw}{dk} = \frac{dw}{dv} / \frac{dv}{dk} = v \Rightarrow \text{Group velocity of electron Wave packet "pilot wave"}

is same as electron's physical velocity

But velocity of individual waves making up the wave packet \( V_p = \frac{w}{k} = \frac{c^2}{v} > c \) (not physical)
Wave Packets & Uncertainty Principles

- Distance $\Delta X$ between adjacent minima = $(X_2)_{\text{node}} - (X_1)_{\text{node}}$
- Define $X_1=0$ then phase diff from $X_1 \rightarrow X_2 = \pi$ (similarly for $t_1 \rightarrow t_2$)

Node at $y = 0 = 2A \cos \left( \frac{\Delta w}{2} t - \frac{\Delta k}{2} x \right)$, Examine x or t behavior

$\Rightarrow$ in $x$: $\Delta k \Delta x = \pi \Rightarrow$ Need to combine many waves of diff. $k$ to make small $\Delta x$ pulse

$\Delta x = \frac{\pi}{\Delta k}$, for small $\Delta x \rightarrow 0 \Rightarrow \Delta k \rightarrow \infty$ & Vice Versa

and in $t$: $\Delta w \Delta t = \pi \Rightarrow$ Need to combine many waves of diff $\omega$ to make small $\Delta t$ pulse

$\Delta t = \frac{\pi}{\Delta \omega}$, for small $\Delta t \rightarrow 0 \Rightarrow \Delta \omega \rightarrow \infty$ & Vice Versa
Signal Transmission and Bandwidth Theory

- Short duration pulses are used to transmit digital info
  - Over phone line as brief tone pulses
  - Over satellite link as brief radio pulses
  - Over optical fiber as brief laser light pulses
- Regardless of type of wave or medium, any wave pulse must obey the fundamental relation
  \[ \Delta \omega \Delta t \cong \pi \]
- Range of frequencies that can be transmitted are called bandwidth of the medium
- Shortest possible pulse that can be transmitted thru a medium is \( \Delta t_{\text{min}} \cong \pi/\Delta \omega \)
- Higher bandwidths transmits shorter pulses & allows high data rate
Wave Packets & Uncertainty Principles of Subatomic Physics

in space x: $\Delta k \cdot \Delta x = \pi \Rightarrow$ since $k = \frac{2\pi}{\lambda}$, $p = \frac{\hbar}{\lambda}$

$\Rightarrow \Delta p \cdot \Delta x = \hbar / 2$

usually one writes $\Delta p \cdot \Delta x \geq \hbar / 2$ approximate relation

In time t: $\Delta \omega \cdot \Delta t = \pi \Rightarrow$ since $\omega = 2\pi f$, $E = hf$

$\Rightarrow \Delta E \cdot \Delta t = \hbar / 2$

usually one writes $\Delta E \cdot \Delta t \geq \hbar / 2$ approximate relation

What do these inequalities mean physically?