Bragg Scattering

X-rays

Crystal

Photographic film

X-ray tube

Collimator

Photographic film

$\alpha$
Constructive Interference when net phase difference is 0, \(2\pi\) etc. This implied path difference traveled by two waves must be integral multiple of wavelength: \(n\lambda = 2ds\sin\theta\)

*From X Ray (EM Wave) Scattering data, size of atoms was known to be about \(10^{-10}\) m*
X-Ray Picture of a DNA Crystal and Discovery of DNA Structure!

Rosalind Franklin
Where are the electrons inside the atom?

Early Thought: “Plum pudding” model → Atom has a homogenous distribution of Positive charge with electrons embedded in them

How to test these hypotheses? → Shoot “bullets” at the atom and watch their trajectory. What Kind of bullets?

- Indestructible charged bullets → Ionized He$$^{++}$$ atom = $$\alpha^{++}$$ particles
- $$Q = +2e$$, Mass $$M_{\alpha} = 4\text{amu} > > m_e$$, $$V_{\alpha} = 2 \times 10^7 \text{ m/s}$$ (non-relativistic)
  [charged to probe charge & mass distribution inside atom]
Plum Pudding Model of Atom

- Non-relativistic mechanics ($V_\alpha/c = 0.1$)
- In Plum-pudding model, $\alpha$-rays hardly scatter because
  - Positive charge distributed over size of atom ($10^{-10}$m)
  - $M_\alpha \gg M_e$ (like moving truck hits a bicycle)
  - $\Rightarrow$ predict $\alpha$-rays will pass thru array of atoms with little scatter

Need to test this hypothesis $\Rightarrow$ Ernest Rutherford
“Rutherford Scattering” discovered by his PhD Student (Marsden)
Force on $\alpha$-particle due to heavy Nucleus

- Outside radius $r = R$, $F \propto Q/r^2$
- Inside radius $r < R$, $F \propto q/r^2 = Qr/R^2$
- Maximum force at radius $r = R$

\[ F \propto Q/r^2 \]

$\alpha$ particle trajectory is hyperbolic

Scattering angle is related to impact par.

Impact Parameter $b = \left( \frac{kq_\alpha Q}{m_\alpha v_\alpha^2} \right) \left( \cot \frac{\theta}{2} \right)$
Rutherford Scattering: Prediction and Experimental Result

\[ \Delta n = \frac{k^2 Z^2 e^4 N n A}{4 R^2 \left( \frac{1}{2} m_\alpha v_\alpha^2 \right)^2 \sin^4 \left( \phi / 2 \right)} \]

# scattered Vs \( \phi \) depends on:

- \( n \) = # of incident alpha particles
- \( N \) = # of nuclei/area of foil
- \( Z e \) = Nuclear charge
- \( K_\alpha \) of incident alpha beam
- \( A \) = detector area
Rutherford Scattering & Size of Nucleus

Distance of closest approach $\propto r$ size of nucleus

Kinetic energy of $\alpha = K_\alpha = \frac{1}{2} m_\alpha v_\beta^2$

$\alpha$ particle will penetrate thru a radius $r$ until all its kinetic energy is used up to do work AGAINST the Coulomb potential of the Nucleus:

$$K_\alpha = \frac{1}{2} m_\alpha v_\beta^2 = 8\text{MeV} = k \frac{(Ze)(2e)}{r}$$

$$\Rightarrow r = \frac{2kZe^2}{K_\alpha}$$

For $K_\alpha = 7.7\text{MeV}$, $Z_{\text{Al}} = 13$

$$\Rightarrow r = \frac{2kZe^2}{K_\alpha} = 4.9 \times 10^{-15} m$$

Size of Nucleus = $10^{-15} m$

Size of Atom = $10^{-10} m$
Dimension Matters!

Size of Nucleus = 10^{-15} m
Size of Atom = 10^{-10} m

How are the electrons located inside an atom? How are they held in a stable fashion? necessary condition for us to exist!

All these discoveries will require new experiments and observations.
Where are the Electrons in an Atom?
Clues: Continuous & Discrete spectra of Elements

- a Continuous spectrum
- b Absorption line spectrum
- c Emission line spectrum
Visible Spectrum of Sun Through a Prism
Emission & Absorption Line Spectra of Elements.

(b) Diagram showing a prism and a lens with a slit and a source of wavelengths $\lambda_1$ and $\lambda_2$ ($\lambda_2 > \lambda_1$), and a screen.

(a) Image of a spectrum of hydrogen with lines at 656.3, 486.1, 434.0, 410.2, 397.0, 389.0, and 364.6 nm.
Kirchhoff’ Experiment: “D” Lines in Na

D lines **darken** noticeably when Sodium vapor introduced Between slit and prism
Emission & Absorption Line Spectrum of Elements

Emission line appear dark because of photographic exposure

Absorption spectrum of Na
While light passed thru Na vapor is absorbed at specific $\lambda$
Spectral Observations: series of lines with a pattern

- Empirical observation (by trial & error)
- All these series can be summarized in a simple formula

\[
\frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \quad n_f > n_i, \quad n_i = 1, 2, 3, 4\ldots
\]

Fitting to spectral line series data

\[ R = 1.09737 \times 10^7 m^{-1} \]

How does one explain this?
The Rapidly Vanishing Atom: A Classical Disaster!

Not too hard to draw analogy with dynamics under another Central Force

Think of the Gravitational Force between two objects and their circular orbits.

Perhaps the electron rotates around the Nucleus and is bound by their electrical charge

\[ F = G \frac{M_1 M_2}{r^2} \implies k \frac{Q_1 Q_2}{r^2} \]

Laws of E&M destroy this equivalent picture: Why?
Bohr’s Bold Model of Atom: Semi Quantum/Classical

1. Electron in circular orbit around proton with vel=v

2. Only stationary orbits allowed. Electron does not radiate when in these stable (stationary) orbits

3. Orbits quantized:
   \[ M_e v r = n \frac{h}{2\pi} \quad (n=1,2,3\ldots) \]

4. Radiation emitted when electron “jumps” from a stable orbit of higher energy to stable orbit of lower energy: \( E_f - E_i = hf = \frac{hc}{\lambda} \)

5. Energy change quantized
   • \( f = \) frequency of radiation

\[ U(r) = -k \frac{e^2}{r} \]
\[ KE = \frac{1}{2} m_e v^2 \]
Reduced Mass of 2-body system

General two body motion under a central force

- Both Nucleus & $e^-$ revolve around their common center of mass (CM)
- Such a system is equivalent to single particle of “reduced mass” $\mu$ that revolves around position of Nucleus at a distance of $(e^- - N)$ separation
  - $\mu = \frac{m_e M}{m_e + M}$, when $M \gg m$, $\mu = m$ (Hydrogen atom)
  - Not so when calculating Muon ($m_\mu = 207 m_e$) or equal mass charges rotating around each other (similar to what you saw in gravitation)
Allowed Energy Levels & Orbit Radii in Bohr Model

E = KE + U = \frac{1}{2} m_e v^2 - k \frac{e^2}{r}

Force Equality for Stable Orbit
⇒ Coulomb attraction = CP Force

\[
\frac{k e^2}{r^2} = \frac{m_e v^2}{r}
\]

⇒ KE = \frac{m_e v^2}{2} = k \frac{e^2}{2r}

Total Energy

E = KE + U = - k \frac{e^2}{2r}

Negative E ⇒ Bound system

This much energy must be added to the system to break up the bound atom

Radius of Electron Orbit:

\[mvr = n\hbar\]

⇒ \[v = \frac{n\hbar}{mr}\]

substitute in \[KE = \frac{1}{2} m_e v^2 = \frac{ke^2}{2r}\]

⇒ \[r_n = \frac{n^2 \hbar^2}{mke^2}, n = 1, 2, \ldots, \infty\]

n = 1 ⇒ Bohr Radius \(a_0\)

\[a_0 = \frac{1^2 \hbar^2}{mke^2} = 0.529 \times 10^{-10} \text{ m}\]

In general \(r_n = n^2 a_0; n = 1, 2, \ldots, \infty\)

Quantized orbits of rotation
Energy Level Diagram and Atomic Transitions

\[ E_n = K + U = \frac{-ke^2}{2r} \]

since \( r_n = a_0n^2 \), \( n = \) quantum number

\[ E_n = \frac{-ke^2}{2a_0n^2} = -\frac{13.6}{n^2} \text{eV}, \quad n = 1, 2, 3, \ldots \infty \]

Interstate transition: \( n_i \rightarrow n_f \)

\[ \Delta E = hf = E_i - E_f \]

\[ = -\frac{ke^2}{2a_0} \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right) \]

\[ f = \frac{ke^2}{2hca_0} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

\[ \frac{1}{\lambda} = \frac{f}{c} = \frac{ke^2}{2hca_0} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

\[ = \frac{1}{\lambda} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]
Hydrogen Spectrum: as explained by Bohr

\[ E_n = -\left(\frac{ke^2}{2a_0}\right)\frac{Z^2}{n^2} \]

Bohr’s “R” same as Rydberg Constant \( R \) derived empirically from spectral series.
A Look Back at the Spectral Lines With Bohr’s Optic

\[ E_n = -\left(\frac{ke^2}{2a_0}\right) \frac{Z^2}{n^2} \]

Rydberg Constant
Bohr’s Atom: Emission & Absorption Spectra

a  Absorption

b  Emission

\( n = 2 \)

\( n = 3 \)

photon

photon
Some Notes About Bohr Like Atoms

• Ground state of Hydrogen atom (n=1) $E_0 = -13.6\ \text{eV}$
• Method for calculating energy levels etc applies to all Hydrogen-like atoms $\rightarrow -1e$ around $+Ze$
  – Examples: $\text{He}^+, \text{Li}^{++}$
• Energy levels would be different if replace electron with Muons
• Bohr’s method can be applied in general to all systems under a central force (e.g. gravitational instead of Coulombic)

$$U(r) = k \frac{Q_1 Q_2}{r} \rightarrow G \frac{M_1 M_2}{r}$$

Changes every thing: $E, \ r, \ f$ etc

"Importance of constants in your life"