4E : The Quantum Universe

Lecture 4

Vivek Sharma

modphys@hepmail.ucsd.edu

Max Planck & Birth of Quantum Physics

Back to Blackbody Radiation Discrepancy

Planck noted the Ultraviolet catastrophe at high frequency

"Cooked" calculation with new "ideas" so as bring:

$R(\lambda) \rightarrow 0$ as $\lambda \rightarrow 0$

$f \rightarrow \infty$

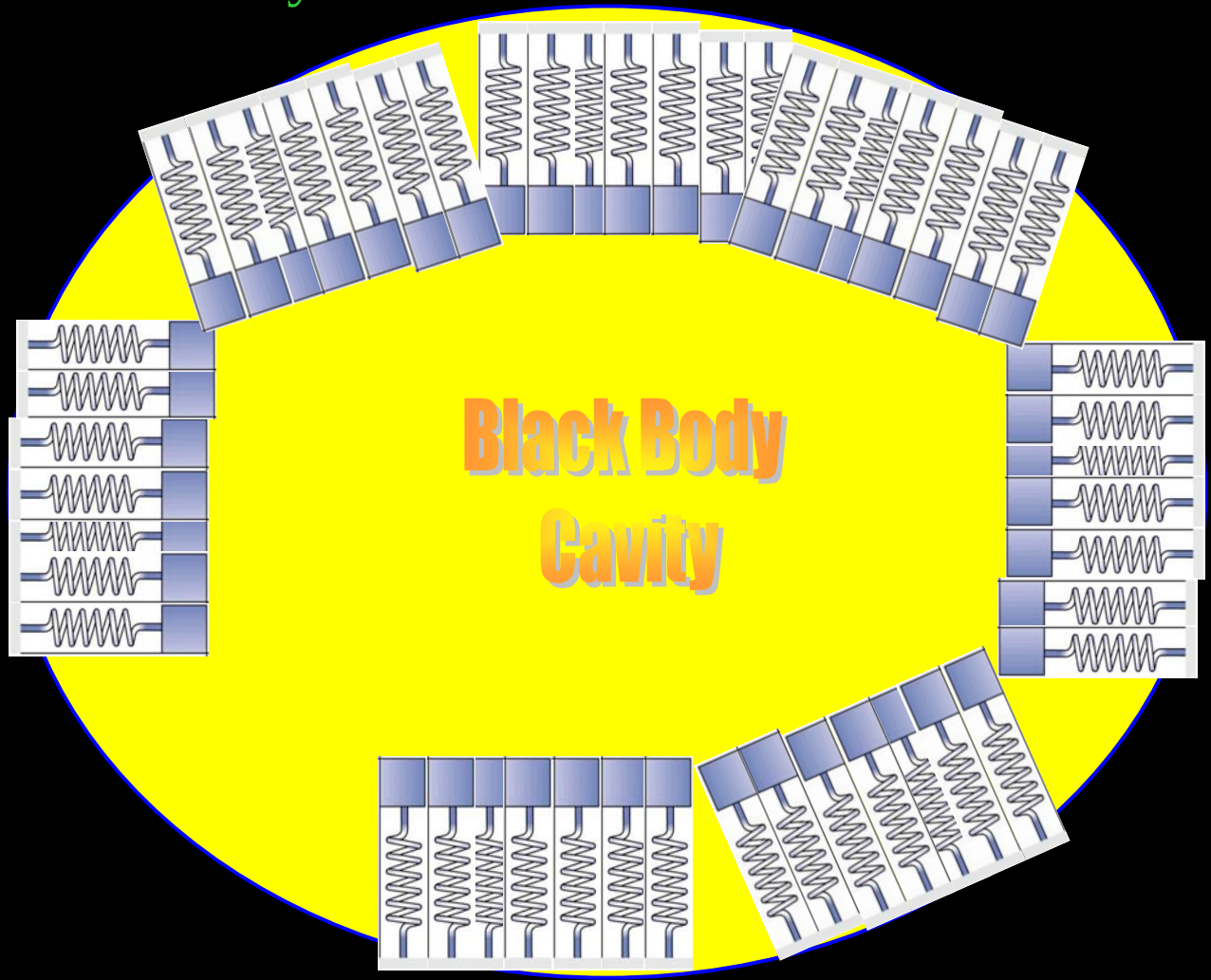
- Cavity radiation as equilibrium exchange of energy between EM radiation & "atomic" oscillators present on walls of cavity
- Oscillators can have any frequency f
- But the Energy exchange between radiation and oscillator NOT continuous, it is discrete …in packets of same amount
- $E = n\,hf$, with $n = 1,2,3,4,\ldots \infty$

  h = constant he invented, a number he made up !

Planck's "Charged Oscillators" in a Black Body Cavity

Planck did not know about electrons, Nucleus etc:

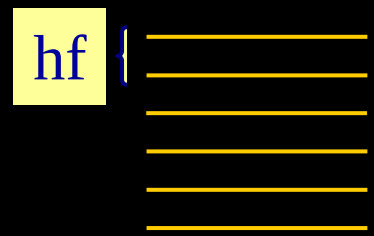
They had not been discovered then



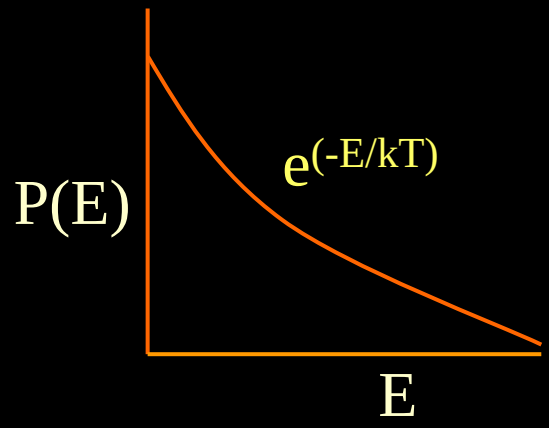
Planck, Quantization of Energy & BlackBody Radiation

- Keep the rule of counting how many waves fit in a BB Volume
- Radiation Energy in cavity is quantized
- EM standing waves of frequency f have energy

  E = n hf (n = 1,2 ,3 …10 ….1000…)



- Probability Distribution: At an equilibrium temp T, possible energy of oscillators is distributed over a spectrum of states: $P(E) = e^{(-E/kT)}$
- Modes of Oscillation with :
  - Less energy E=hf = favored
  - More energy E=hf = disfavored



By this discrete statistics, large energy = high f modes of EM disfavored

Planck's Calculation: A preview to keep the story going

$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\left[\frac{hc}{\lambda}\left(\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\right)\right]$$

Odd looking form

When $\lambda \rightarrow$ large $\Rightarrow \dfrac{\text{hc}}{\lambda \text{kT}} \rightarrow$ small

Recall $\boxed{e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...}$

$$\Rightarrow e^{\frac{hc}{\lambda kT}} - 1 = (1 + \frac{hc}{\lambda kT} + \frac{1}{2}\left(\frac{hc}{\lambda kT}\right)^2 + ...] - 1$$

$$= \frac{hc}{\lambda kT}$$ plugging this in R(λ) eq:

$$\boxed{R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\frac{hc}{\lambda kT}}$$

Graph & Compare With BBQ data

Planck's Formula and Small λ

When λ is small (large f)

$$\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \cong \frac{1}{e^{\frac{hc}{\lambda kT}}} = e^{-\frac{hc}{\lambda kT}}$$

Substituting in R(λ) eqn:

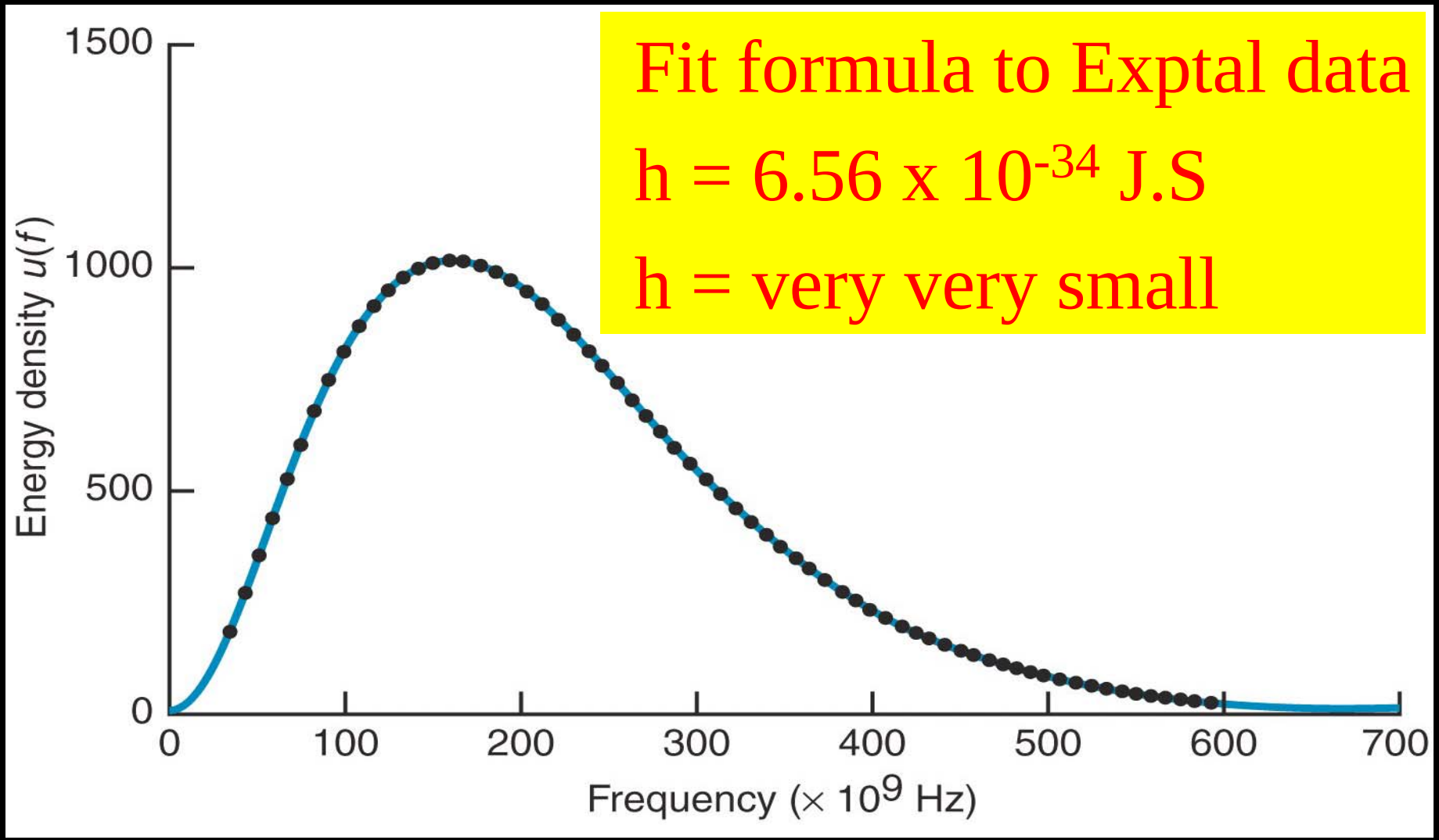
$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right) e^{-\frac{hc}{\lambda kT}}$$

As $\lambda \to 0$, $e^{-\frac{hc}{\lambda kT}} \to 0$

$$\Rightarrow R(\lambda) \to 0$$

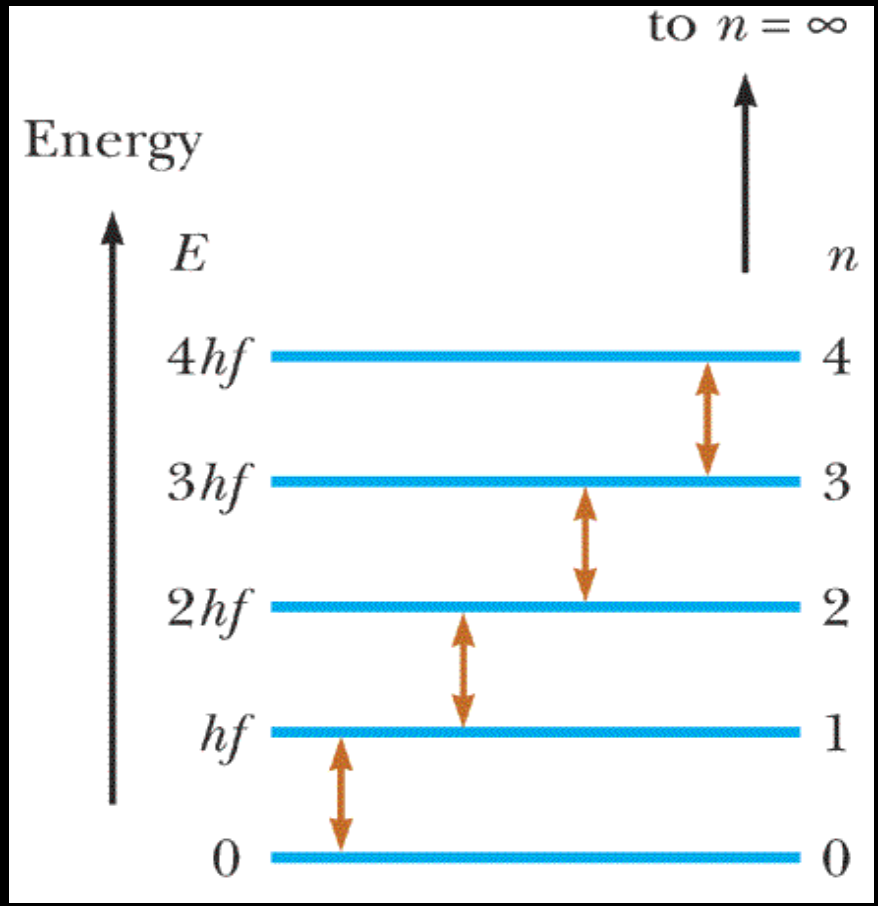
Just as seen in the experimental data !

Planck's Explanation of Black Body Radiation

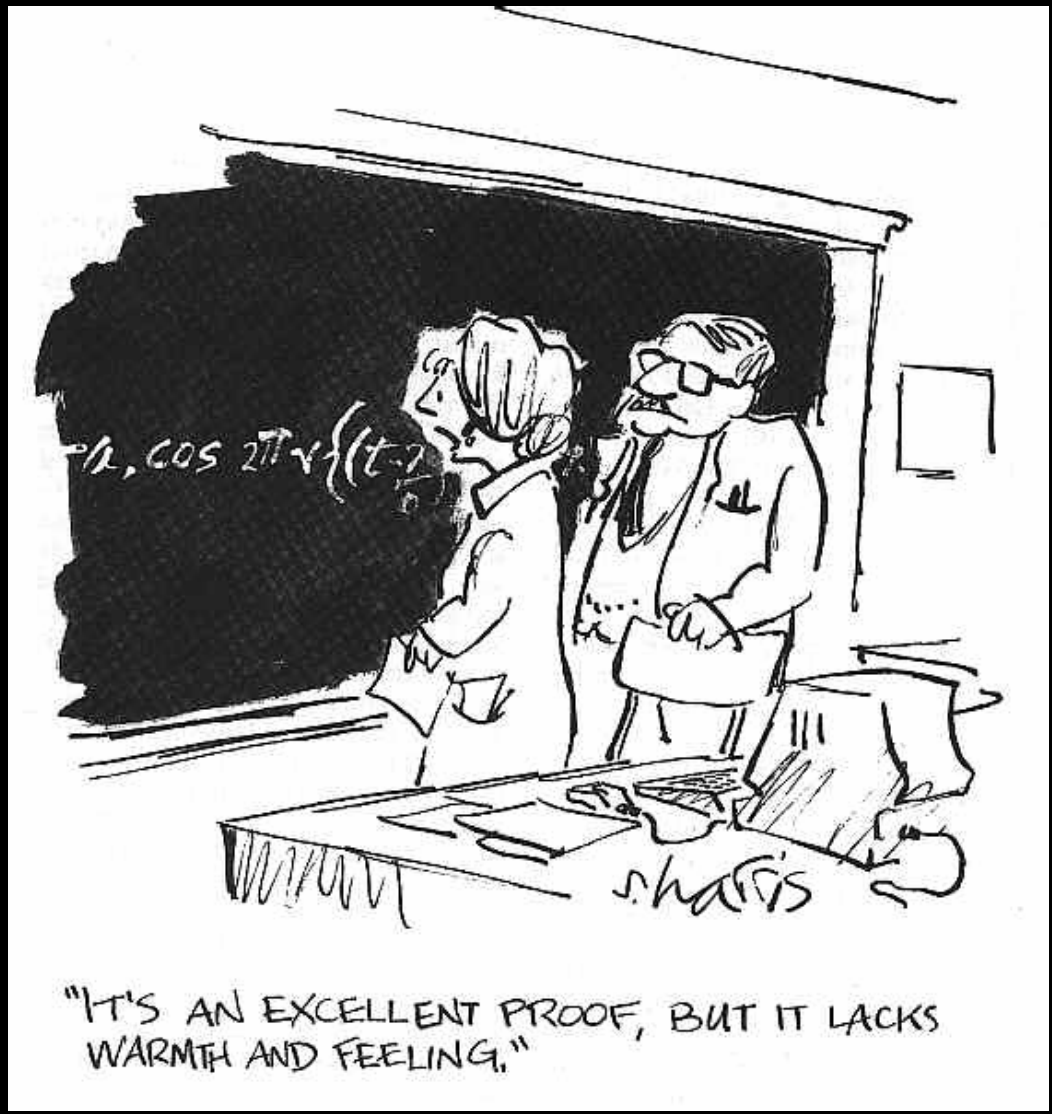


Major Consequence of Planck's Energy Postulate

Quantization of Energy !



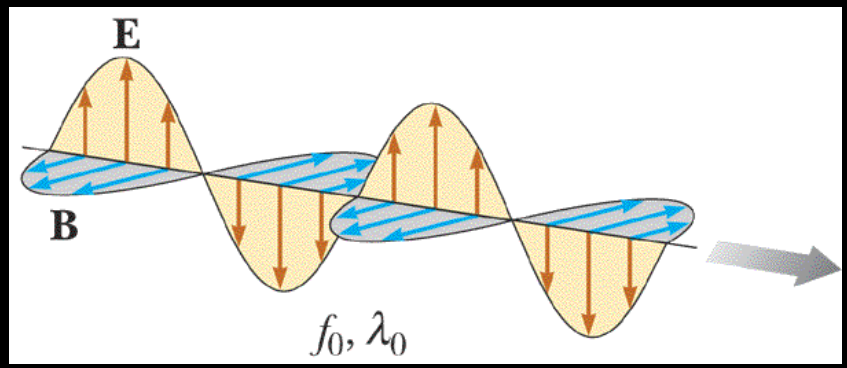
Judging Planck's Postulate : Visionary or just a Wonk?



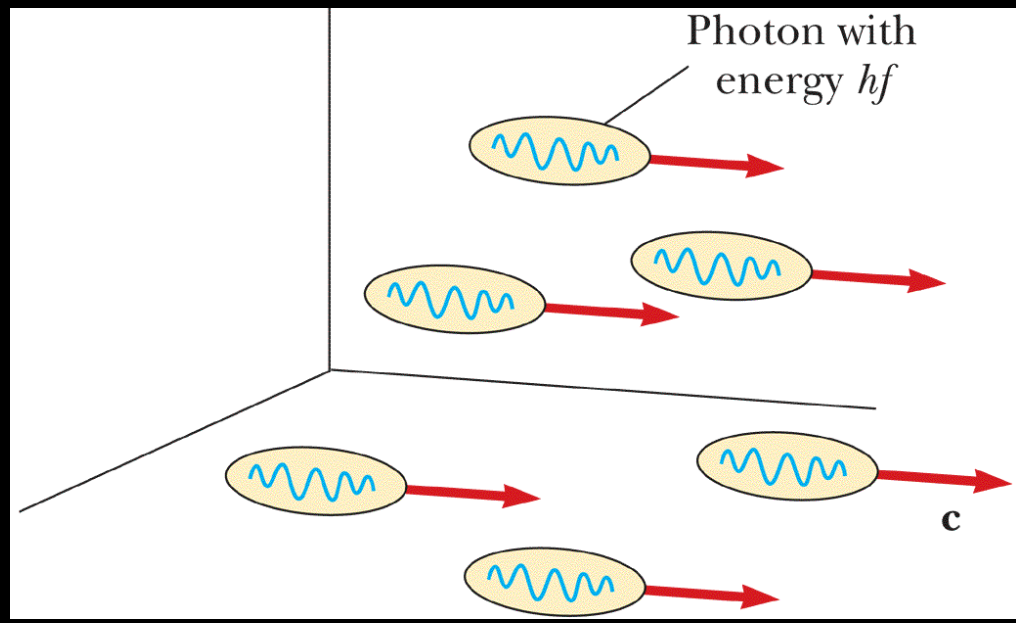
Einstein Provided the "warmth & feeling" to Planck's Wonky idea

Einstein's Explanation of Photo Electric Effect

What Maxwell Saw of EM Waves



What Einstein Saw of EM Waves



Light as bullets of "photons"
Energy concentrated in photons
Energy exchanged instantly
Energy of EM Wave E= hf

Einstein's Explanation of Photoelectric Effect

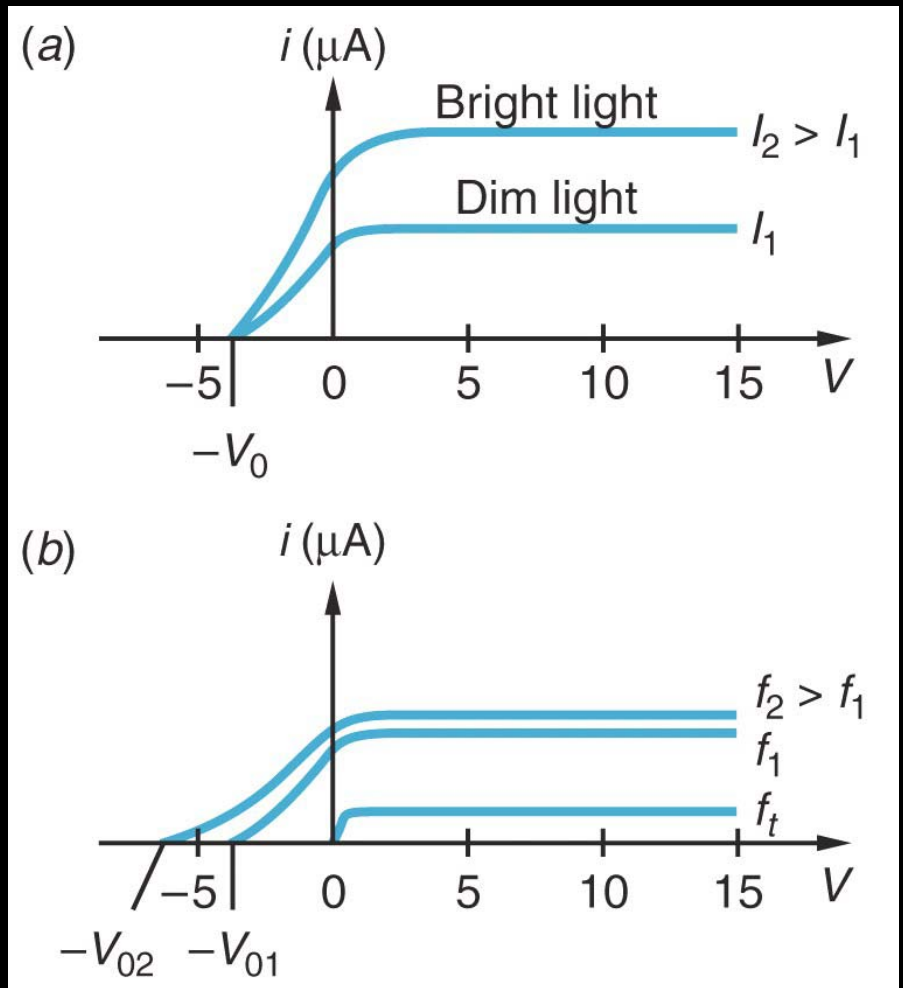
- Energy associated with EM waves not uniformly distributed over wavefront, rather is contained in packets of energy $\Rightarrow$ PHOTON
- $E = hf = hc/\lambda$ [but is it the same h as in Planck's th.?]
- Light shining on metal emitter/cathode is a *stream of photons* of energy E which depends on frequency f
- Photons knock off electron from metal instantaneously
  - Transfer all energy to electron
  - Energy gets used up to pay for Work Function Φ. Remaining energy shows up as KE of electron $KE = hf - \Phi$
- Cutoff Frequency $hf_0 = \Phi$ (pops an electron, KE = 0)
- Larger intensity I $\rightarrow$ more photons incident
- Low frequency light f $\rightarrow$ not energetic enough to overcome work function of electron in atom

Einstein's Interpretation of Photoelectric Effect (1905)

$$E = hf = \varphi + KE_{electron}$$

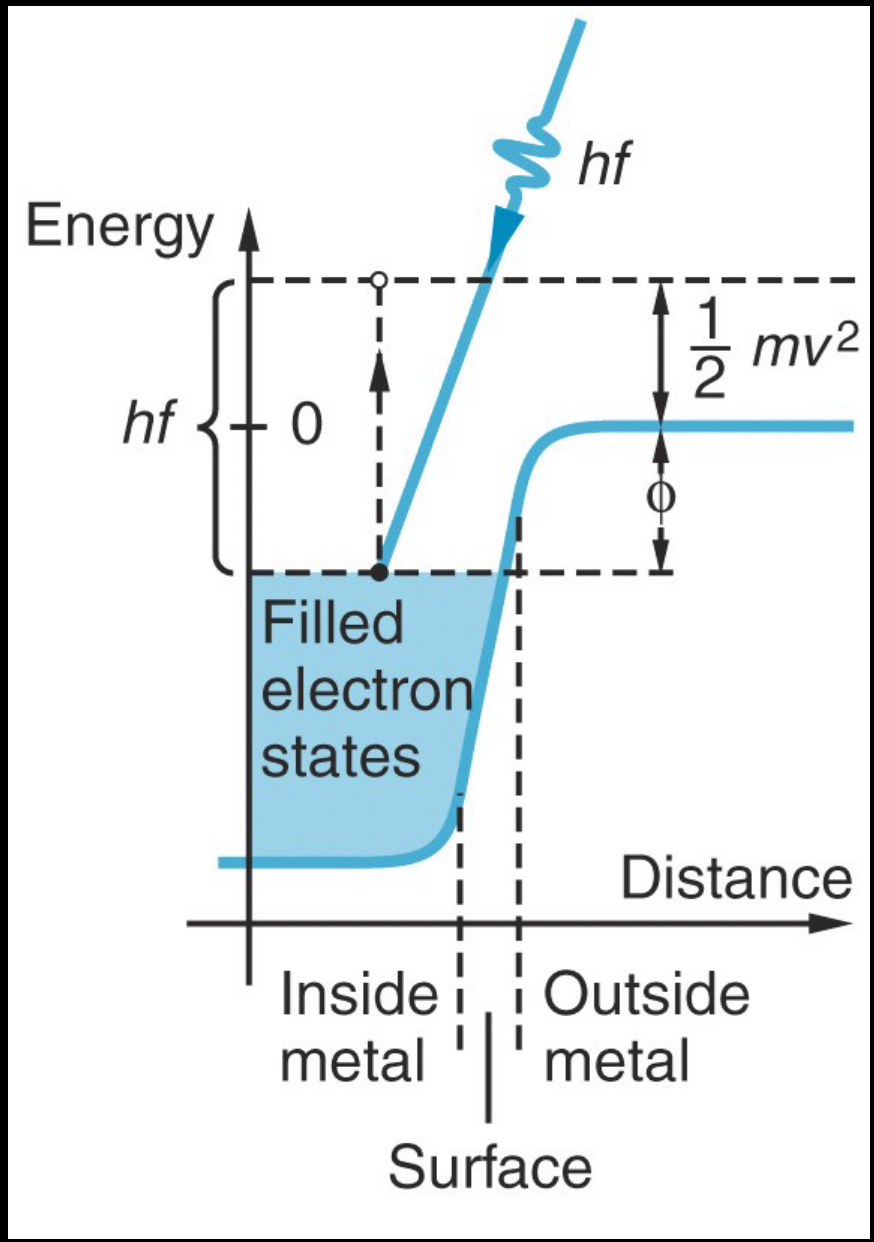
$$eV_0 = KE = hf - \varphi$$

Now interpret the experimental data
Under the "single bullet" theory



Makes Sense !

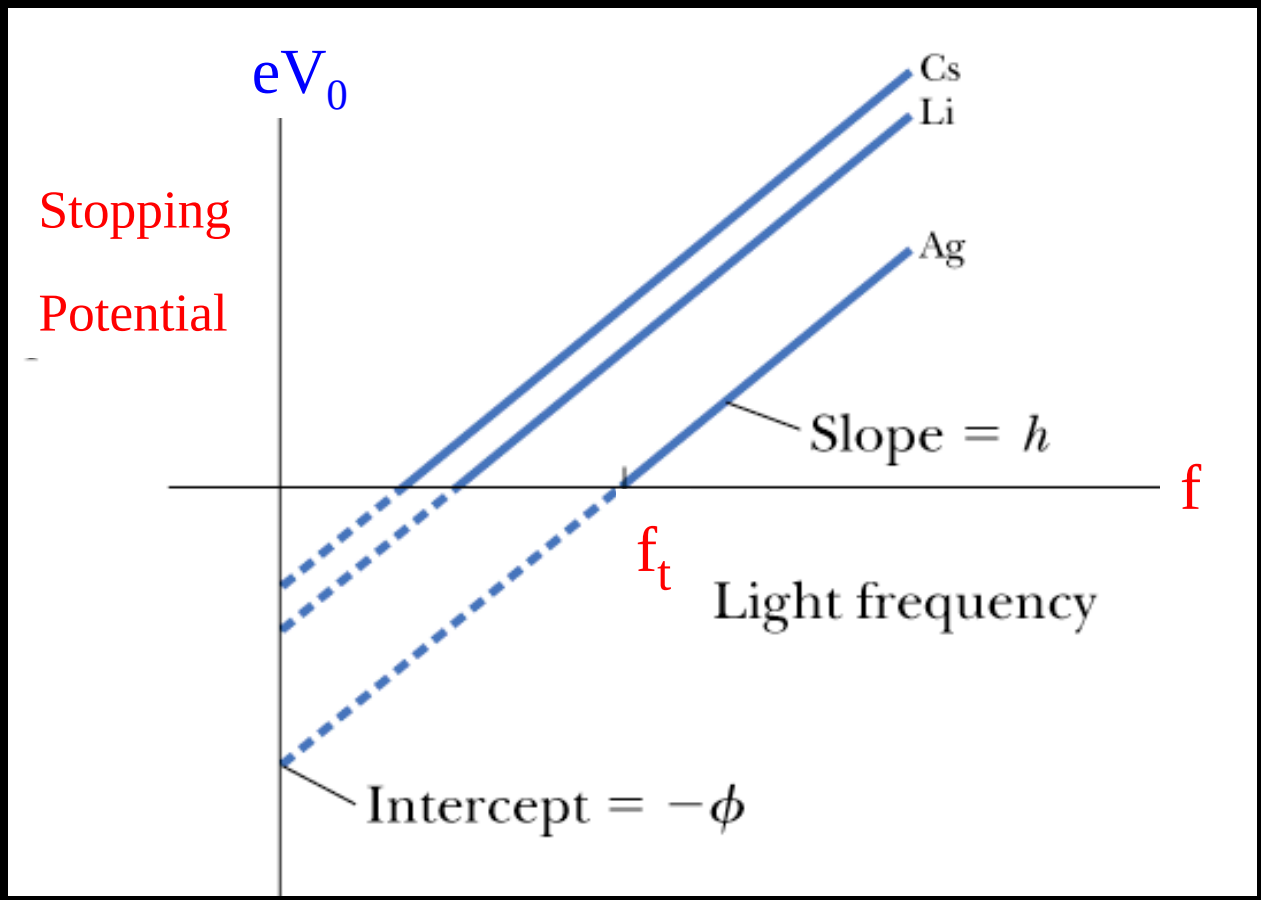
Modern View of Photoelectric Effect



Is "h" same in Photoelectric Effect as BB Radiation?

Slope h = 6.626×10^{-34} JS

Einstein → Nobel Prize!



NOBEL PRIZE FOR PLANCK !

Work Function (Binding Energy) In Metals

TABLE 3-1 Photoelectric work functions

Element	ϕ (eV)
Na	2.28
C	4.81
Cd	4.07
Al	4.08
Ag	4.73
Pt	6.35
Mg	3.68
Ni	5.01
Se	5.11
Pb	4.14

Reinterpreting Photoelectric Effect With Light as Photons

Photoelectric Effect on An Iron Surface

Light of Intensity I = 1.0 μW/cm^2 incident on 1.0cm^2 surface of Fe

Assume Fe reflects 96% of light

further *only* 3% *of incident light* is Violet region (λ = 250nm)

barely above threshold frequency for Photoelectric effect

(a) Intensity available for Ph. El effect I = 3% $\times$ 4% $\times$ (1.0 μW/cm^2)

(b) how many photo-electrons emitted per second ?

$$\#\text{ of photoelectrons} = \frac{\text{Power}}{h f} = \frac{3\% \times 4\% \times (1.0\ \mu\text{W/cm}^2)\,\lambda}{hc}$$

$$= \frac{(250\times10^{-9}\,m)(1.2\times10^{-9}\,J/s)}{(6.6\times10^{-34}\,J\cdot s)(3.0\times10^{8}\,m/s)}$$

$$1.5\times10^{9}$$

(c) Current in Ammeter : $i = (1.6\times10^{-19}C)(1.5\times10^{9}) = 2.4\times10^{-10}\,A$

(d) Work Function $\Phi = hf_0 = (4.14\times10^{-15}eV\cdot s)(1.1\times10^{15}\,s^{-1})$

$= 4.5$ eV

Facts about Light Quantum

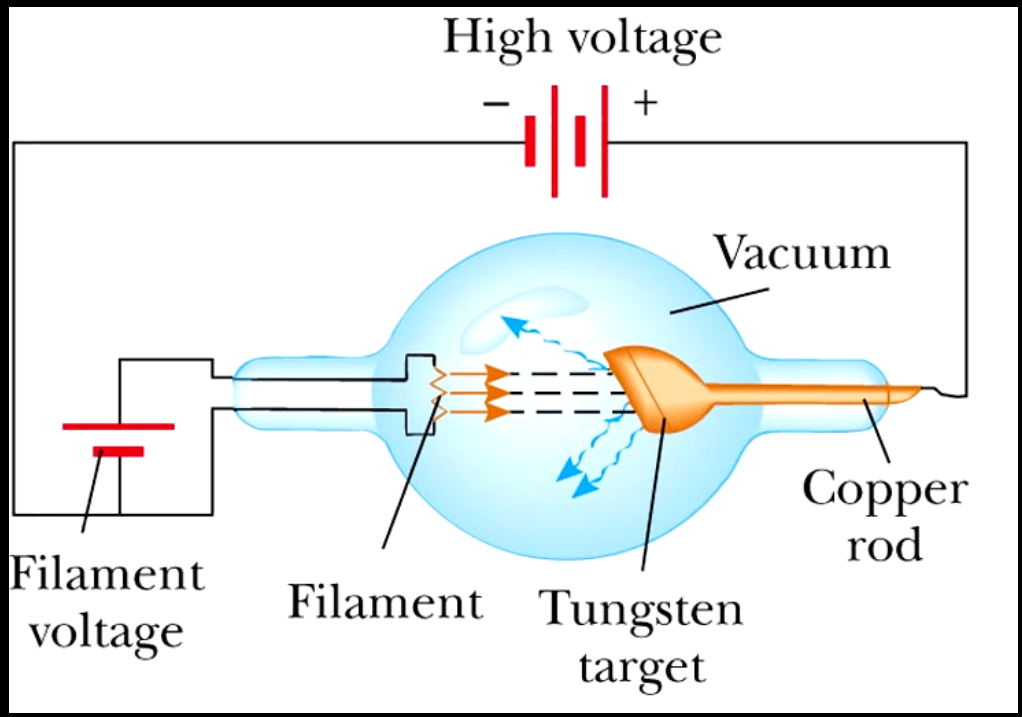
- The human eye is a sensitive photon detector at visible wavelengths: Need >5 photons of ≅ 550nm to register on your optical sensor
- The Photographic process :
  - Energy to Dissociate an AgBr molecule = 0.6eV
- Photosynthesis Process : 9 sunlight photon per reaction cycle of converting CO_2 and water to carbohydrate & O_2
  - chlorophyll absorbs best at $\lambda \cong$ 650-700 nm
- Designing Space Shuttle “skin” : Why Platinum is a good thing
- designing Solar cells : picking your metal cathode

Photon & Relativity: Wave or a Particle ?

- Photon associated with EM waves, travel with speed =c
- For light (m =0) : Relativity says $E^2 = (pc)^2 + (mc^2)^2$
- $\Rightarrow E = pc$
- But Planck tells us : $E = hf = h(c/\lambda)$
- Put them together : $hc/\lambda = pc$
  - $\Rightarrow \quad p = h/\lambda$
  - Momentum of the photon (light) is inversely proportional to λ
- But we associate λ with waves & p with particles ….what is going on??
  - Quantum Physics !

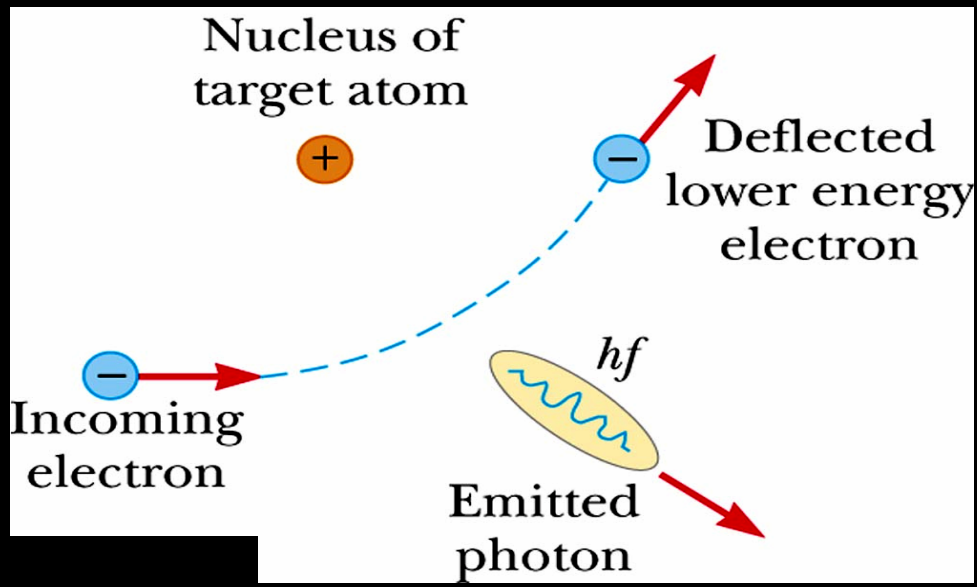
X Rays "Bremsstrahlung": The Braking Radiation

- EM radiation, produced by bombarding a metal target with energetic electrons.
- Produced in general by ALL decelerating charged particles
- X rays : very short $\lambda \cong 60\text{-}100$ pm (10^{-12}m), large frequency f
- Very penetrating because very energetic E = hf !!



Useful for probing structure of sub-atomic Particles (and your teeth !)

X Ray Production Mechanism

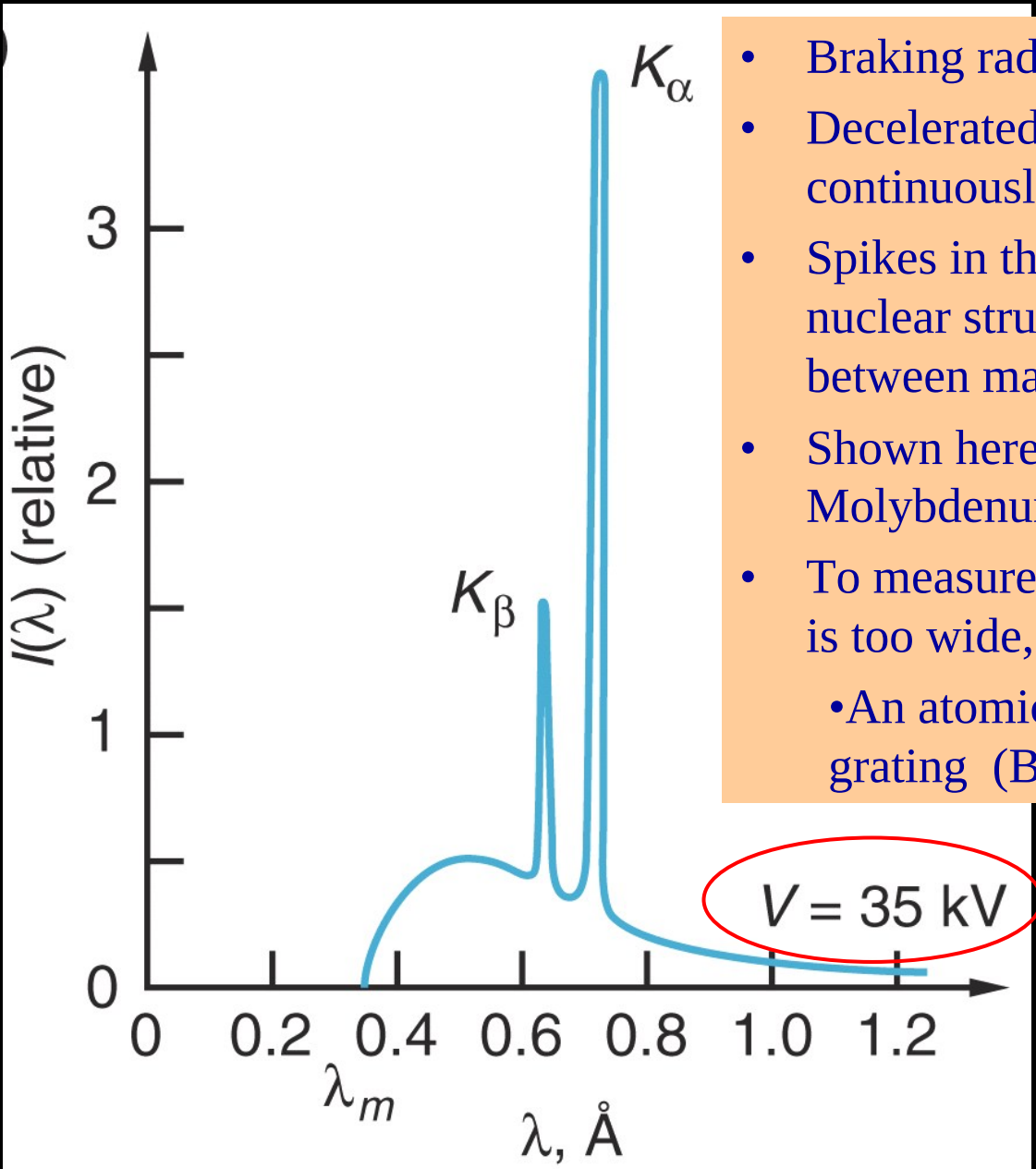


When electron passes near a positively charged target nucleus contained in target material, its deflected from its path because of Coulomb attraction, experiences acceleration.

E&M ➔ that any charged particle will emit radiation when accelerated.
This EM radiation "appears" as photons. Since photo carries energy and momentum, the electron must lose same amount. If all of electron's energy is lost in just one single collision then:

$$e\,\Delta V = \mathrm{hf}_{\max} = \frac{hc}{\lambda_{\min}} \text{ or } \boxed{\lambda_{\min} = \frac{hc}{e\,\Delta V}}$$

X Ray Spectrum in Molybdenum (Mo)



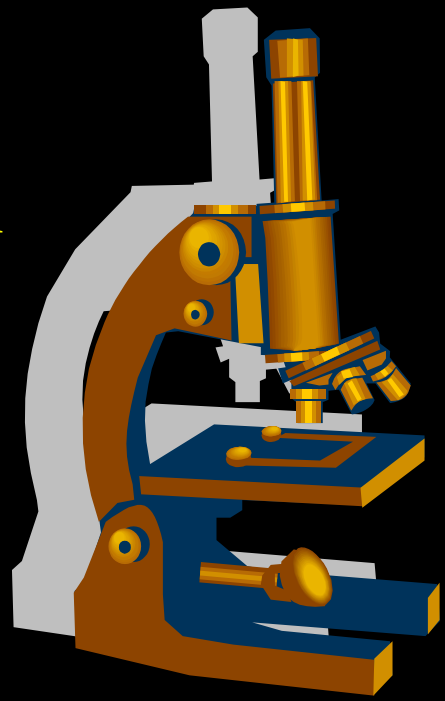
- Braking radiation predicted by Maxwell's eqn
- Decelerated charged particle will radiate continuously
- Spikes in the spectrum are characteristic of the nuclear structure of target material and varies between materials
- Shown here are the α and β lines for Molybdenum (Mo)
- To measure the wavelength, diffraction grating is too wide, need smaller slits
  - An atomic crystal lattice as diffraction grating (Bragg)

X rays As Subatomic Probes

X rays are EM waves of low wavelength, high frequency (and energy) and demonstrate characteristic features of a wave

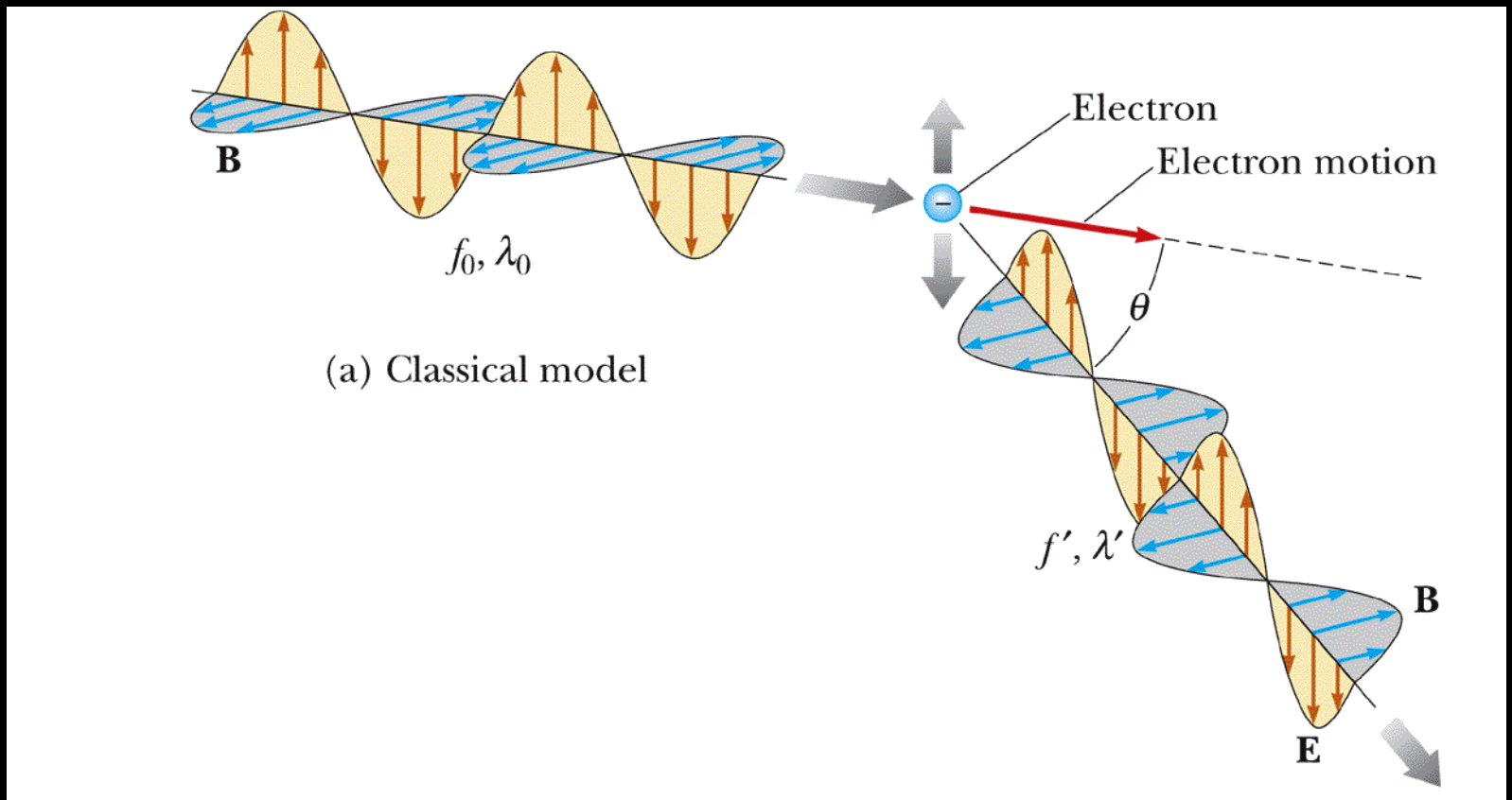
- Interference &Diffraction

- To probe into a structure size ΔX you need a light source with wavelength much smaller than the features of the object being probed
  - Good Resolution → $\lambda_{SOURCE} << \Delta X$
  - X rays allows one probe at atomic size (10^{-10})m



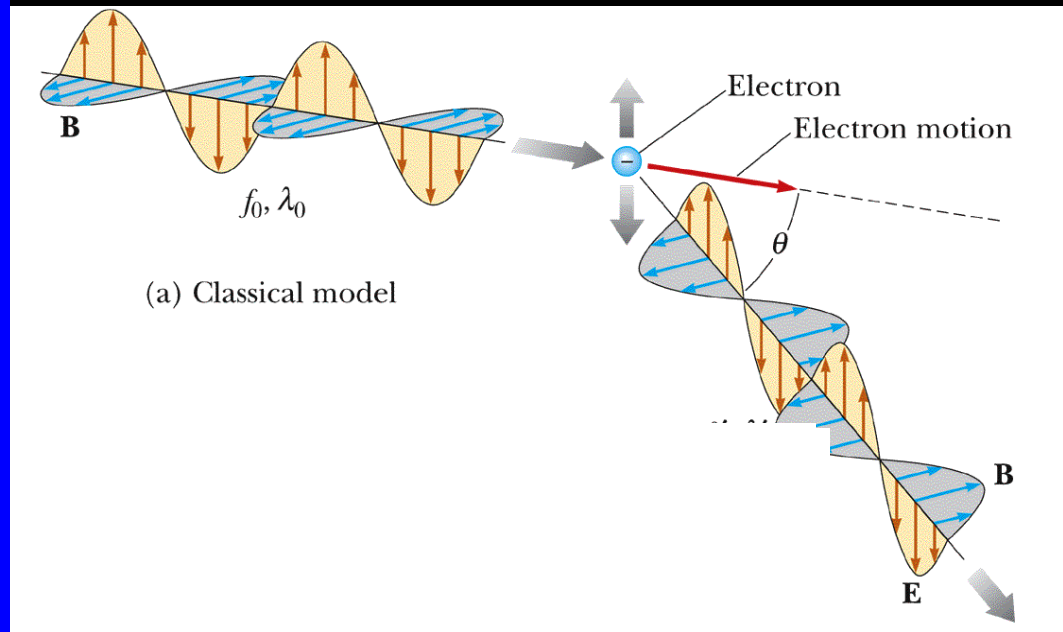
Compton Scattering : Quantum Pool !

- Arthur Compton (USA) proves that X-rays (EM Waves) have particle like properties (acts like photons)
  - Showed that classical theory failed to explain the scattering effect of X rays on to free (not bound, barely bound electrons)
- Experiment : shine X ray on to a surface with “almost” free electrons
  - Watch the scattering of light off electron : measure time + λ of scattered X-ray

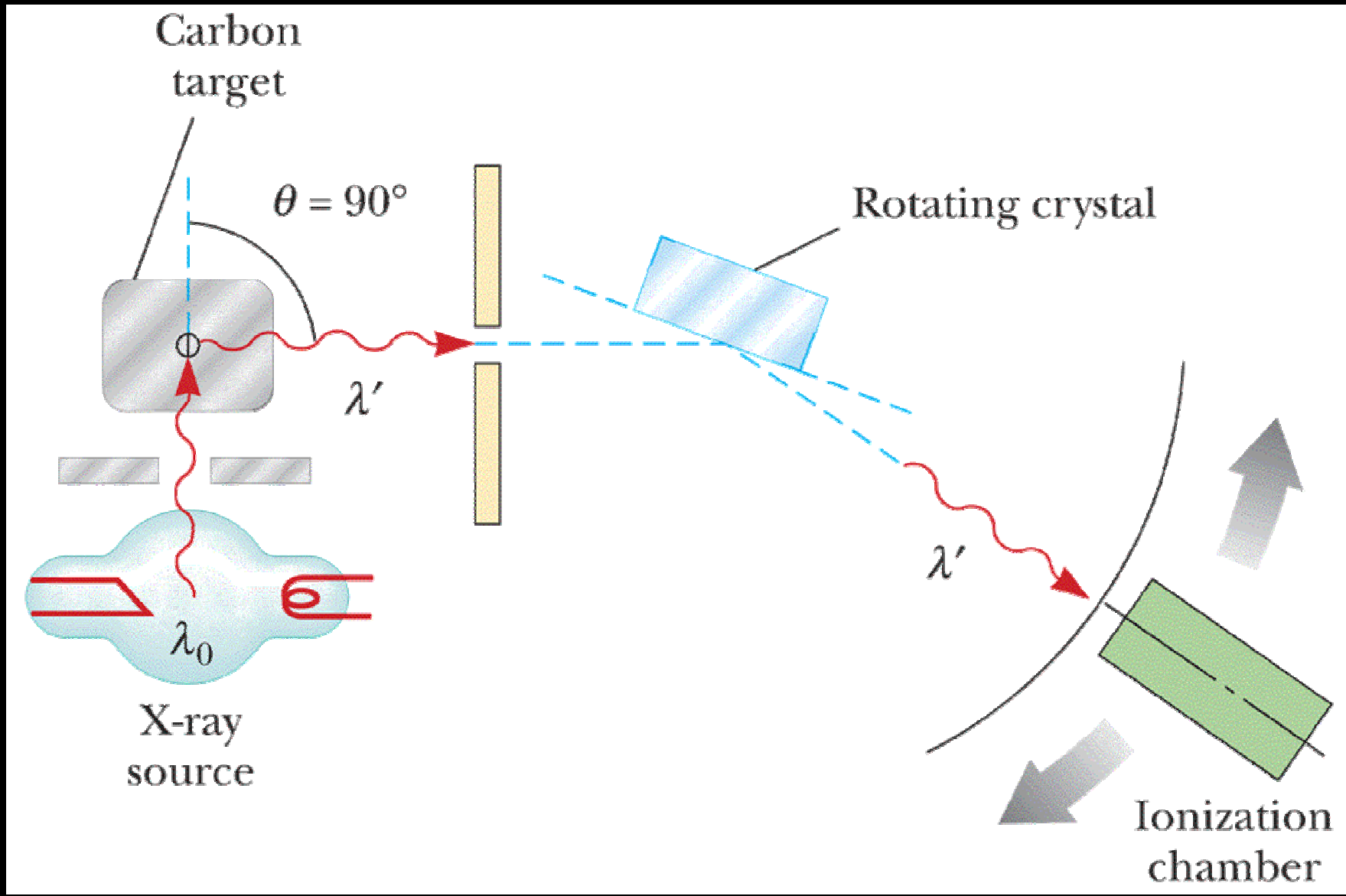


Compton Effect: what should Happen Classically?

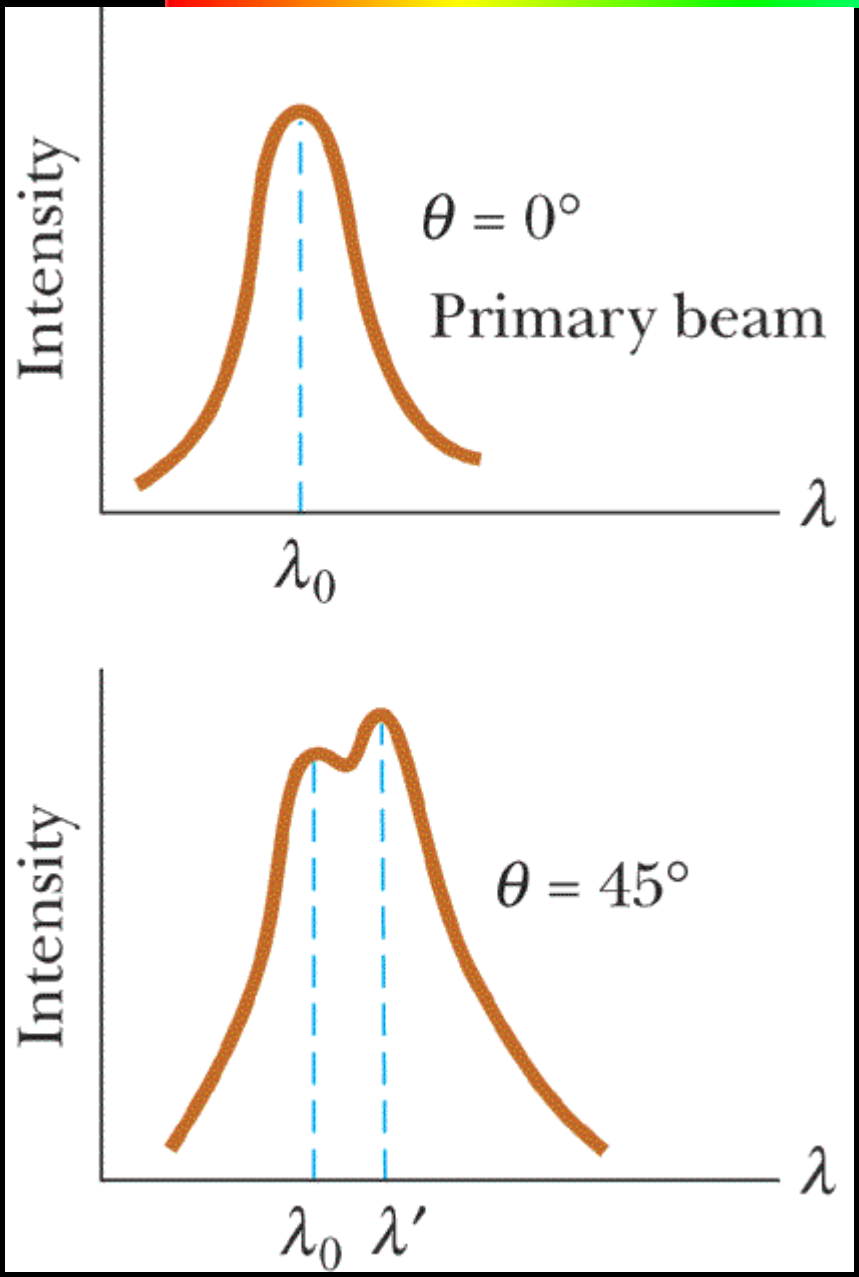
- Plane wave [f,λ] incident on a surface with loosely bound electrons → interaction of E field of EM wave with electron: **F** = e**E**
- Electron oscillates with $f = f_{incident}$
- Eventually radiates spherical waves with $f_{radiated} = f_{incident}$
  - At all scattering angles, Δf & Δλ must be zero
- Time delay while the electron gets a "tan" : soaks in radiation

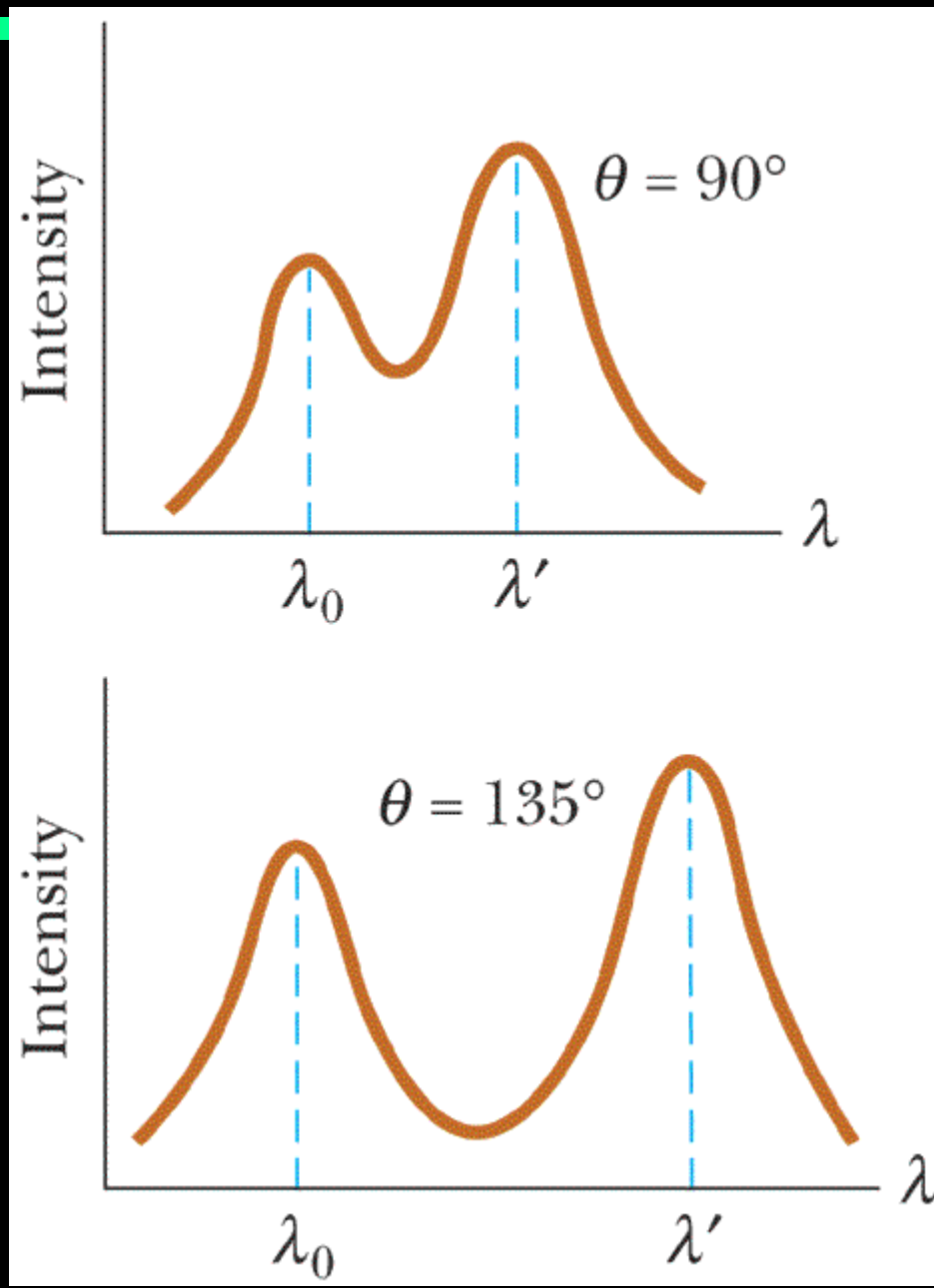


Compton Scattering : Experimental Setup & Results

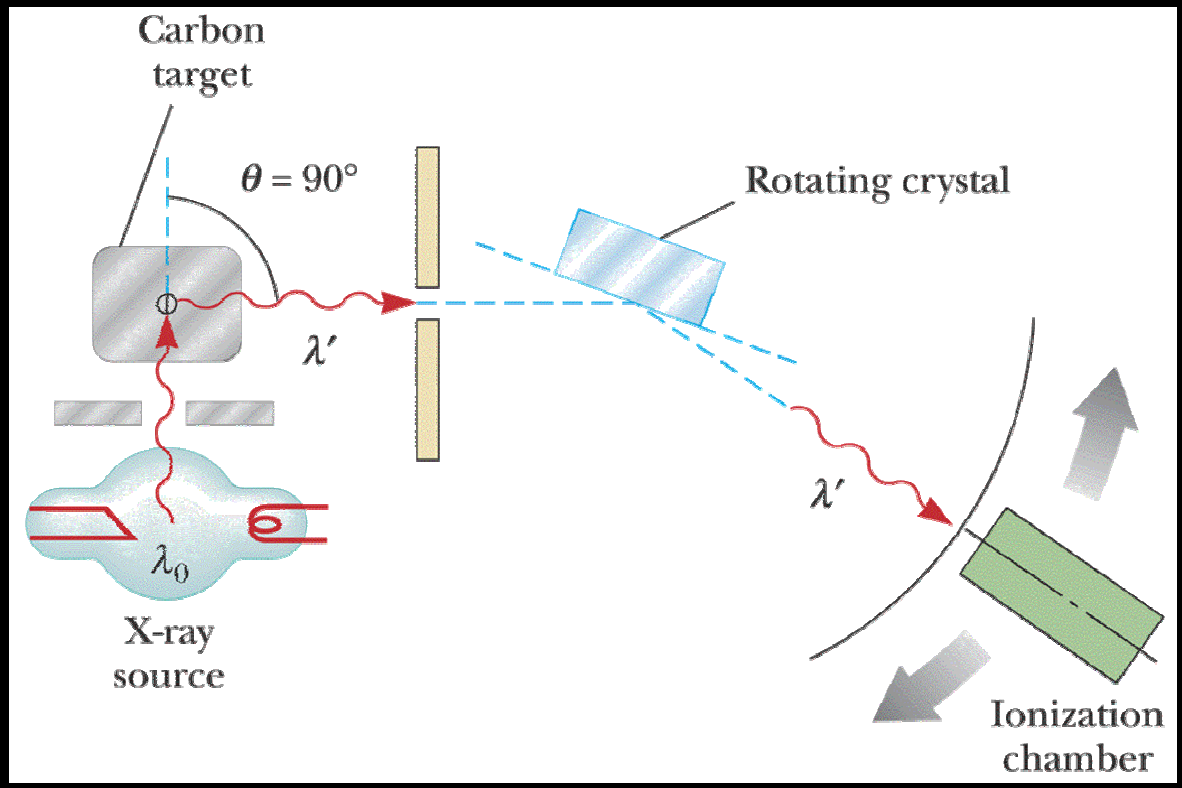


Compton Scattering : Observations





Compton Scattering : Summary of Observations

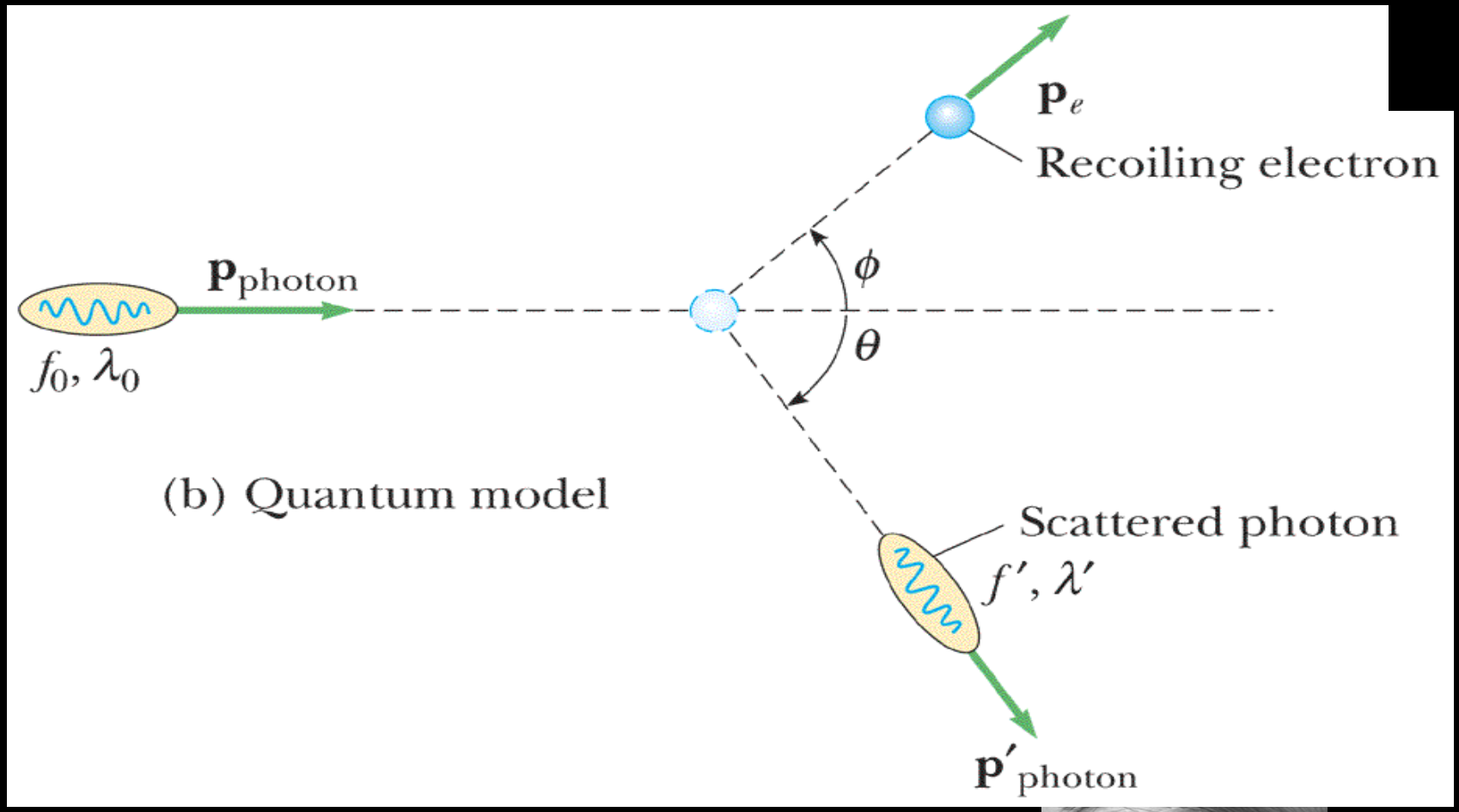


$$\Delta\lambda = (\lambda' - \lambda) \propto (1 - \cos\theta)\ !$$

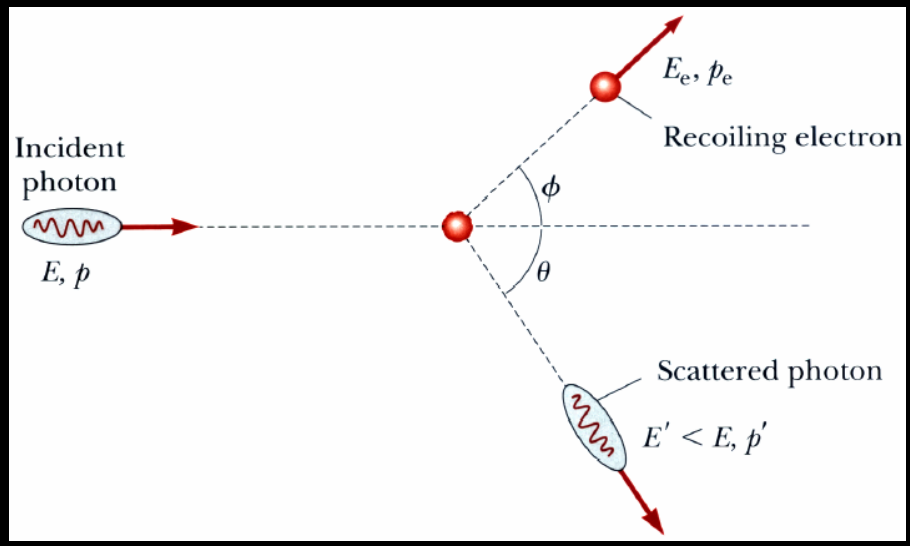
Not isotropy in distribution of scattered radiation

How does one explain this startling anisotropy?

Compton Effect : Quantum (Relativistic) Pool



Compton Scattering: The Quantum Picture



Energy Conservation:

$E + m_e c^2 = E' + E_e$

Momentum Conserv:

$p = p'\cos\theta + p_e \cos\phi$

$0 = p'\sin\theta - p_e \sin\phi$

Use these to eliminate electron deflection angle (not measured)

$\Rightarrow$

$p_e \cos\phi = p - p'\cos\theta$

$p_e \sin\phi = p'\sin\theta$

Square and add $\Rightarrow$

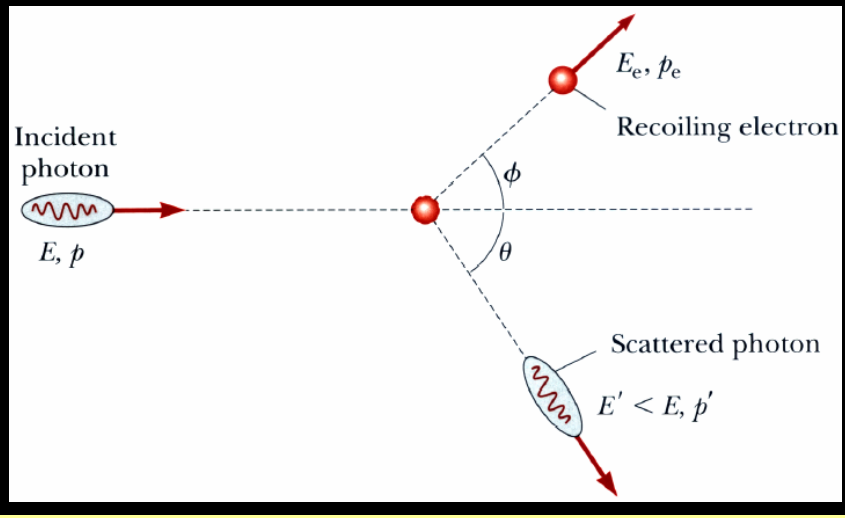
$p_e^2 = p^2 - 2pp'\cos\theta + p'^2$

Eliminate p_e & E_e using

$E_e^2 = p_e^2 c^2 + m_e^2 c^4$ &

$E_e = (E - E') + m_e c^2$

Compton Scattering: The Quantum Picture



$$\left((E-E')+m_e c^2\right)^2 = \left[\, p^2 - 2pp'\cos\theta + p'^2 \,\right] + (m_e c^2)^2$$

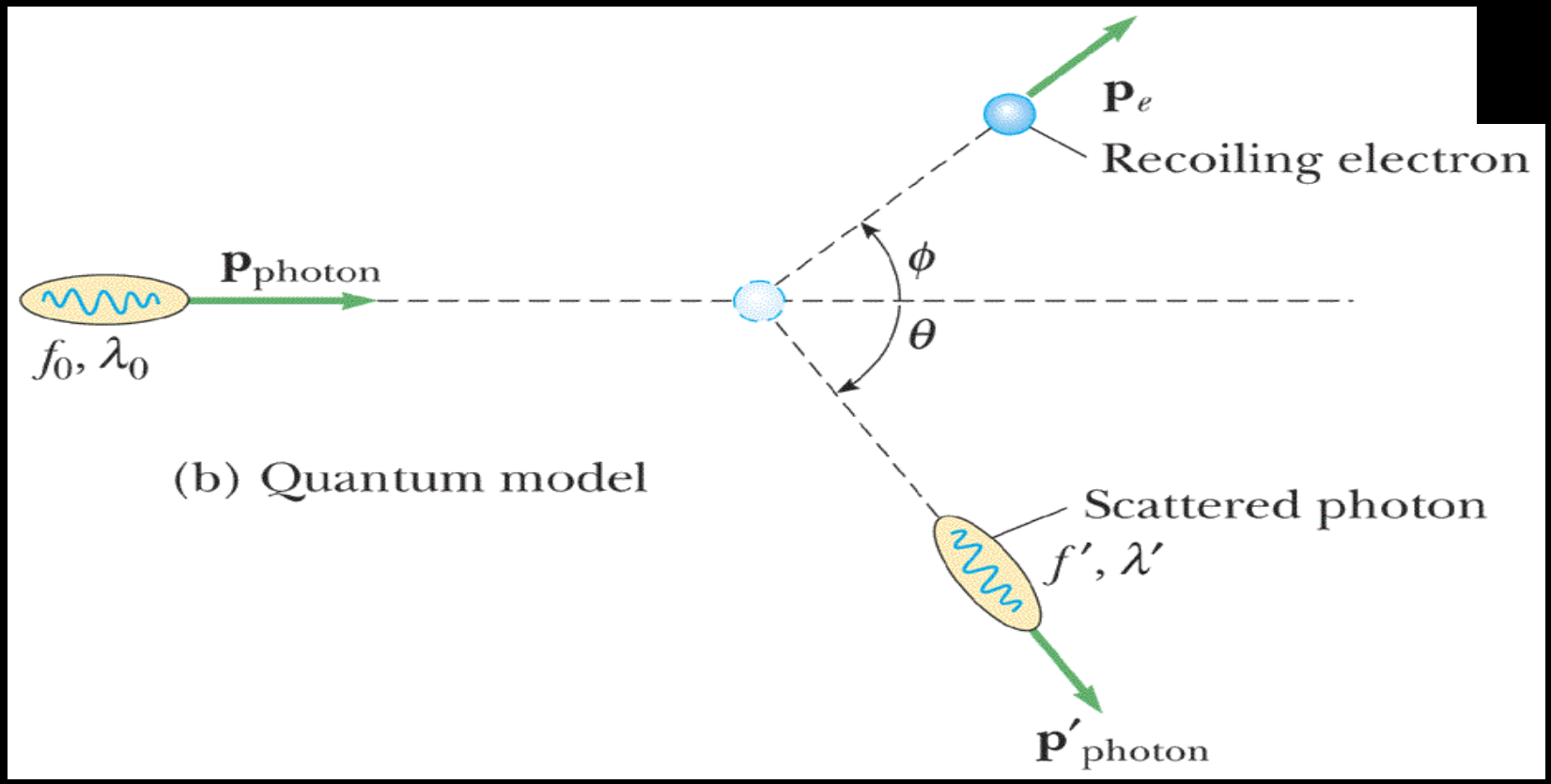
For light $p = \frac{E}{c} \Rightarrow$

$$E^2 + E'^2 - 2EE' + 2(E-E')mc^2 = \left[\frac{E^2}{c^2} - 2\frac{EE'}{c^2}\cos\theta + \frac{E'^2}{c^2}\right]c^2$$

$$\Rightarrow -EE' + (E-E')mc^2 = -EE'\cos\theta$$

$$\Rightarrow \quad \frac{E-E'}{EE'} = -\frac{1}{m_e c^2}(1-\cos\theta) \Rightarrow \boxed{(\lambda' - \lambda) = \left(\frac{h}{m_e c}\right)(1-\cos\theta)}$$

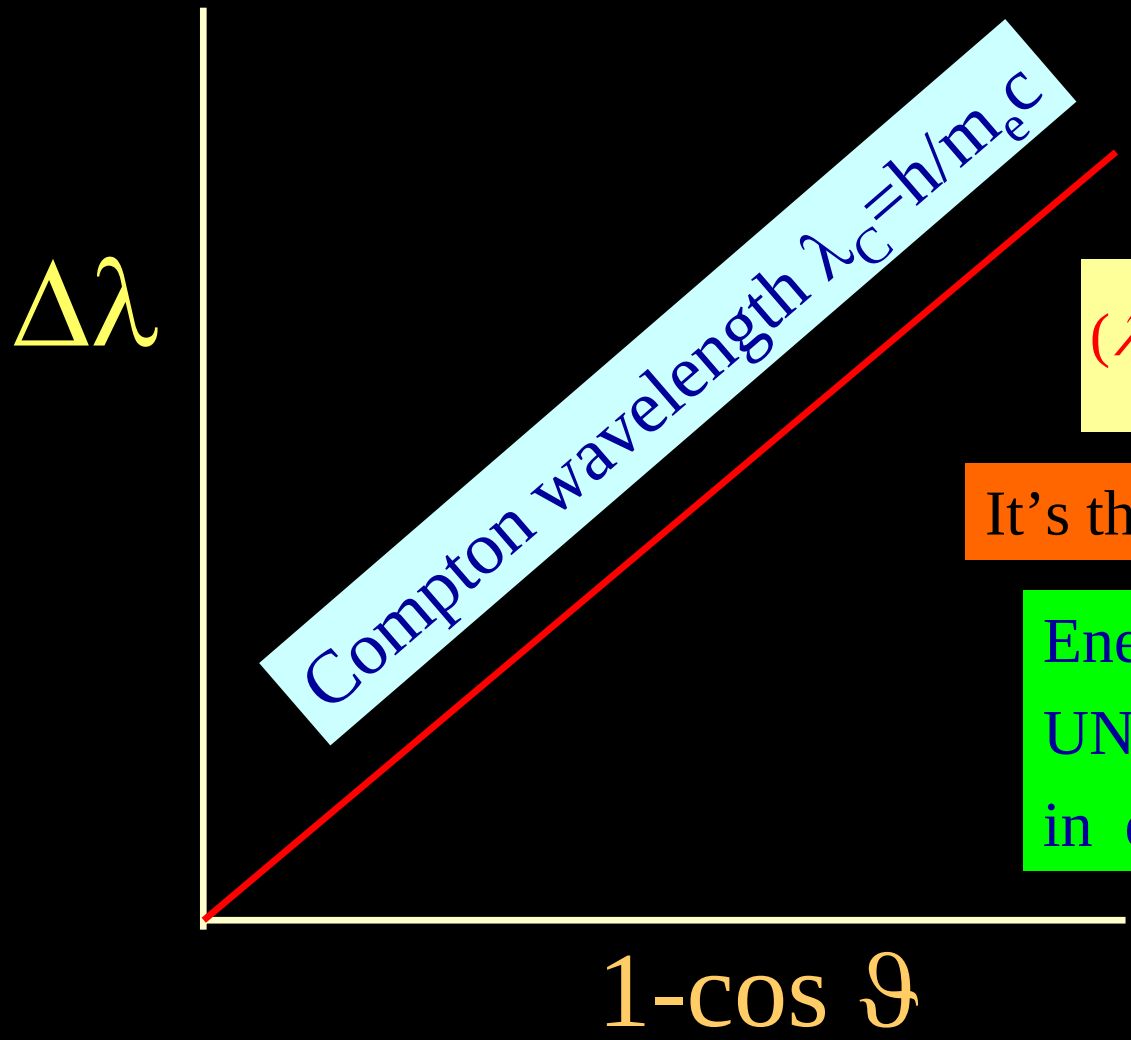
Rules of Quantum Pool between Photon and Electron



$$(\lambda' - \lambda) = \left(\frac{h}{m_e c}\right)(1 - \cos\theta)$$

Checking for "h" in Compton Scattering

From scattered photon λ, plot Δλ, calculate slope and measure "h"



$$(\lambda' - \lambda) = (\frac{h}{m_e c})(1 - \cos\theta)$$

It's the same value for h again !

Energy Quantization is a UNIVERSAL characteristic in energy transactions !