Properties of EM Waves: Maxwell’s Equations

Energy Flow in EM Waves

Poynting Vector \( \mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}) \)

Power incident on an area \( A \)

\[ = \mathbf{S} \cdot \mathbf{A} = \frac{1}{\mu_0} \left( A E_0 B_0 \sin^2 (kx - \omega t) \right) \]

Intensity of Radiation \( I = \frac{1}{2\mu_0 c} \frac{E_0^2}{\mu_0} \)

Larger the amplitude of Oscillation
More intense is the radiation
Nature of Radiation: An Expt with BBQ Grill

Question: Distribution of Intensity of EM radiation Vs T & $\lambda$

- Radiator (BBQ grill) at some temp T
- Emits variety of wavelengths
  - Some with more intensity than others
- EM waves of diff. $\lambda$ bend differently within prism
- Eventually recorded by a detector (eye)
- Map out emitted Power / area Vs $\lambda$

Notice shape of each curve and learn from it.
The Beginning of The End! How BBQ Broke Physics

Classical Calculation

# of standing waves between Wavelengths $\lambda$ and $\lambda + d\lambda$ are

$$N(\lambda)d\lambda = \frac{8\pi V}{\lambda^4} \cdot d\lambda; \ V = \text{Volume of box} = L^3$$

Each standing wave contributes energy $E = kT$ to radiation in Box

Energy density $u(\lambda) = \left[\frac{\text{# of standing waves}}{\text{volume}}\right] \times \text{Energy/Standing Wave}$

$$= \frac{8\pi V}{\lambda^4} \times \frac{1}{V} \times kT = \frac{8\pi}{\lambda^4} \ kT$$

Radiancy $R(\lambda) = \frac{c}{4} u(\lambda) = \frac{c}{4} \frac{8\pi}{\lambda^4} \ kT = \frac{2\pi c}{\lambda^4} \ kT$

Radiancy is Radiation intensity per unit $\lambda$ interval: Let's plot it

Prediction: as $\lambda \rightarrow 0$ (high frequency $f$), $R(\lambda) \rightarrow \text{Infinity!}$

Oops!
Ultra Violet (Frequency) Catastrophe

Radiancy $R(\lambda)$

- Planck's law
- Rayleigh-Jeans law (Classical theory)

$\text{oops!}$
That was a Disaster!

(#1)
Disaster # 2 : Photo-Electric Effect

Light of intensity I, wavelength \( \lambda \) and frequency \( f \) incident on a photo-cathode

Can change I, f, \( \lambda \)

Measure characteristics of current in the circuit as a fn of I, f, \( \lambda \)
Photo Electric Effect: Measurable Properties

• Rate of electron emission from cathode
  – From current \( i \) seen in ammeter in the circuit. More photoelectrons \( \rightarrow \) more current registered in ammeter

• Maximum kinetic energy of emitted electron
  – By applying retarding potential on electron moving left to right towards Collector plate
    
    \[
    K_{\text{MAX}} = eV_0 \quad (V_0 = \text{Stopping voltage})
    \]
    
    • Stopping potential \( \rightarrow \) no current flows

• Photoelectric Effect on different types of photo-cathode metal surface

• Time **between** shining light and first sign of photo-current in the circuit
Observations: PhotoCurrent Vs Intensity of Incident Light

The graph shows the relationship between the photo current ($i$) in microamperes ($\mu A$) and the voltage ($V$) across the device. The intensity of the incident light is represented by $I_1$ for dim light and $I_2$ for bright light, with $I_2 > I_1$. The voltage is labeled as $-V_0$. The graph indicates that the current increases as the intensity of the light increases.
Observations: Photocurrent Vs frequency of incident light

Shining light with constant intensity but different frequencies
Try different photocathode materials…..see what happens
Conclusions from the Experimental Observations

• Max Kinetic energy $K_{\text{MAX}}$ independent of Intensity $I$ for light of same frequency

• No photoelectric effect occurs if light frequency $f$ is below a threshold no matter how high the intensity of light

• For a particular metal, light with $f > f_t$ causes photoelectric effect IRRESPECTIVE of light intensity.
  − $f_t$ is characteristic of that metal

• Photoelectric effect is instantaneous !...not time delay

Can one Explain all this Classically !
Classical Explanation of Photo Electric Effect

- As light intensity increased $\Rightarrow \vec{E}$ field amplitude larger
  - $E$ field and electrical force seen by the “charged subatomic oscillators” larger
    - $\vec{F} = e\vec{E}$
  - More force acting on the subatomic charged oscillator
  - $\Rightarrow$ More (work done) $\Rightarrow$ more energy transferred to it
  - $\Rightarrow$ Charged particle “hooked to the atom” should leave the surface with more Kinetic Energy $KE$ !! The intensity of light (EM Wave) shining rules !

- As long as light is **intense enough**, light of ANY frequency $f$ should cause photoelectric effect

- Because the Energy in a Wave is uniformly distributed over the Spherical wavefront incident on cathode, should be a **noticeable time lag** $\Delta T$ between time is incident & the time a photo-electron is ejected: Energy absorption time
  - How much time for electron ejection? Lets calculate it classically
Classical Physics: Time Lag in Photo-Electric Effect?

- Electron absorbs energy incident on a surface area where the electron is confined ≈ size of atom in cathode metal.
- Electron is “bound” by attractive Coulomb force in the atom, so it must absorb a minimum amount of radiation before it’s stripped off.
- Example: Laser light Intensity $I = 120\, \text{W/m}^2$ on Na metal
  - Binding energy = $2.3\, \text{eV} = \text{“Work Function } \Phi \text{”}$
  - Electron confined in Na atom, size ≈ $0.1\, \text{nm}$; how long before ejection?
  - Average Power Delivered $P_{AV} = I \cdot A$, $A = \pi r^2 \approx 3.1 \times 10^{-20}\, \text{m}^2$
  - If all energy absorbed then $\Delta E = P_{AV} \cdot \Delta T \Rightarrow \Delta T = \Delta E / P_{AV}$

$$\Delta T = \frac{(2.3\, \text{eV})(1.6 \times 10^{-19}\, \text{J/eV})}{(120\, \text{W/m}^2)(3.1 \times 10^{-20}\, \text{m}^2)} = 0.10\, \text{s}$$

- Classical Physics predicts measurable delay even by the primitive clocks of 1900.
- But in experiment, the effect was observed to be instantaneous!!
- Classical Physics fails in explaining all results.
That was a Disaster!

(# 2)

Beginning of a search for a new hero or an explanation or both!
Max Planck & Birth of Quantum Physics

Back to Blackbody Radiation Discrepancy

Planck noted the Ultraviolet catastrophe at high frequency

“Cooked” calculation with new “ideas” so as bring:

\[ R(\lambda) \to 0 \text{ as } \lambda \to 0 \]
\[ f \to \infty \]

- Cavity radiation as equilibrium exchange of energy between EM radiation & “atomic” oscillators present on walls of cavity
- Oscillators can have any frequency \( f \)
- But the Energy exchange between radiation and oscillator NOT continuous, it is discrete …in packets of same amount
- \[ E = n \hbar f \], with \( n = 1, 2, 3, 4, \ldots \infty \)
  \( \hbar = \text{constant he invented, a number he made up!} \)
Planck’s “Charged Oscillators” in a Black Body Cavity

Planck did not know about electrons, Nucleus etc:
They had not been discovered then
Planck, Quantization of Energy & BB Radiation

- Keep the rule of counting how many waves fit in a BB Volume
- BUT Radiation energy in cavity is quantized
- EM standing waves of frequency $f$ have energy
  $$E = n \, h f \quad (n = 1, 2, 3 \ldots 10 \ldots 1000\ldots)$$
- Probability Distribution: At an equilibrium temp $T$, possible energy of oscillators is distributed over a spectrum of states: $P(E) = e^{-E/kT}$
- Modes of Oscillation with:
  - Less energy: $E = hf$ = favored
  - More energy: $E = hf$ = disfavored

By this discrete statistics, large energy = high $f$ modes of EM disfavored
Planck’s Calculation: A preview to keep the story going

\[ R(\lambda) = \left( \frac{c}{4} \right) \left( \frac{8\pi}{\lambda^4} \right) \left[ \frac{hc}{\lambda} \left( \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \right) \right] \]

Odd looking form

When \( \lambda \rightarrow \) large \( \Rightarrow \frac{hc}{\lambda kT} \rightarrow \) small

Recall

\[ e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \]

\[ \Rightarrow e^{\frac{hc}{\lambda kT}} - 1 = \left(1 + \frac{hc}{\lambda kT} + \frac{1}{2} \left( \frac{hc}{\lambda kT} \right)^2 + \ldots \right) - 1 \]

\[ = \frac{hc}{\lambda kT} \quad \text{plugging this in } R(\lambda) \text{ eq:} \]

\[ R(\lambda) = \left( \frac{c}{4} \right) \left( \frac{8\pi}{\lambda^4} \right) \frac{hc}{\lambda kT} \]

Graph & Compare With BBQ data
Planck’s Formula and Small $\lambda$

When $\lambda$ is small (large f)

$$\frac{1}{\frac{hc}{\lambda \kappa T}} \approx \frac{1}{\frac{hc}{hc}} = e^{-\frac{hc}{\lambda \kappa T}}$$

$$e^{\frac{hc}{\lambda \kappa T}} - 1 \quad e^{\frac{hc}{\lambda \kappa T}}$$

Substituting in $R(\lambda)$ eqn:

$$R(\lambda) = \left( \frac{c}{4} \right) \left( \frac{8\pi}{\lambda^4} \right) e^{-\frac{hc}{\lambda \kappa T}}$$

As $\lambda \to 0$, $e^{-\frac{hc}{\lambda \kappa T}} \to 0$

$$\Rightarrow R(\lambda) \to 0$$

Just as seen in the experimental data!
Planck’s Explanation of Black Body Radiation

Fit formula to Exptal data

\[ h = 6.56 \times 10^{-34} \text{ J.S} \]

\[ h = \text{very very small} \]
Major Consequence of Planck’s Postulate

Quantization of Energy!

Diagram:
- Energy levels: 0, hf, 2hf, 3hf, 4hf, up to \( n = \infty \)
- Arrows indicate transitions between energy levels
"IT'S AN EXCELLENT PROOF, BUT IT LACKS WARMTH AND FEELING."