4E : The Quantum Universe

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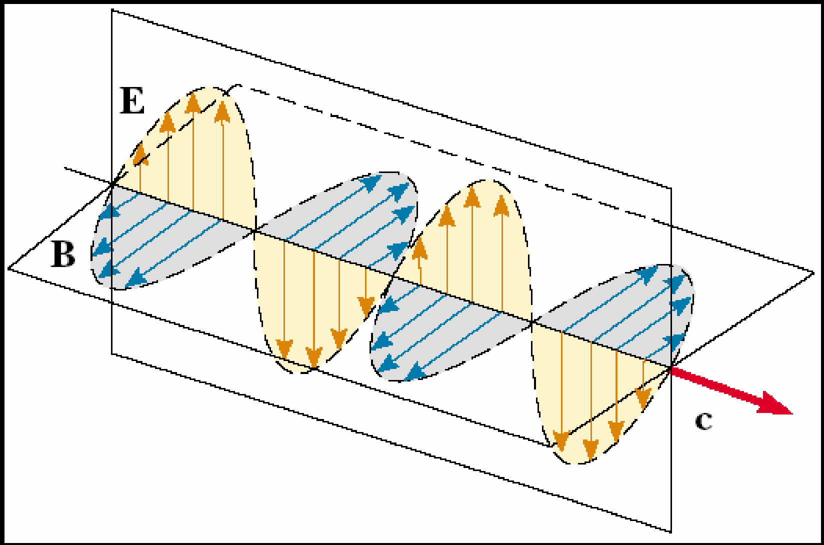
Properties of EM Waves: Maxwell's Equations



Energy Flow in EM Waves

Poynting Vector $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$

Power incident on an area A

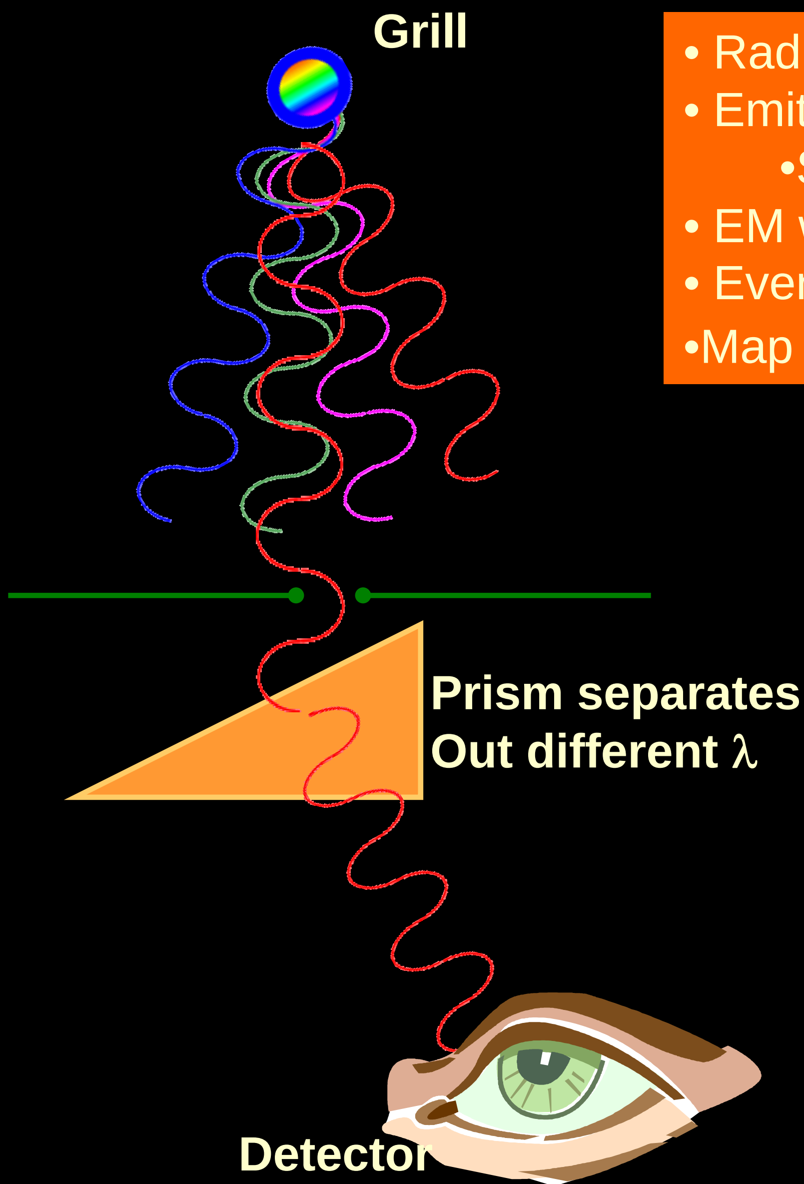
$$= \vec{S}.\vec{A} = \frac{1}{\mu_0}\left(AE_0 B_0 Sin^2(kx - \omega t)\right)$$

Intensity of Radiation $I = \frac{1}{2\mu_0 c} E_0^2$

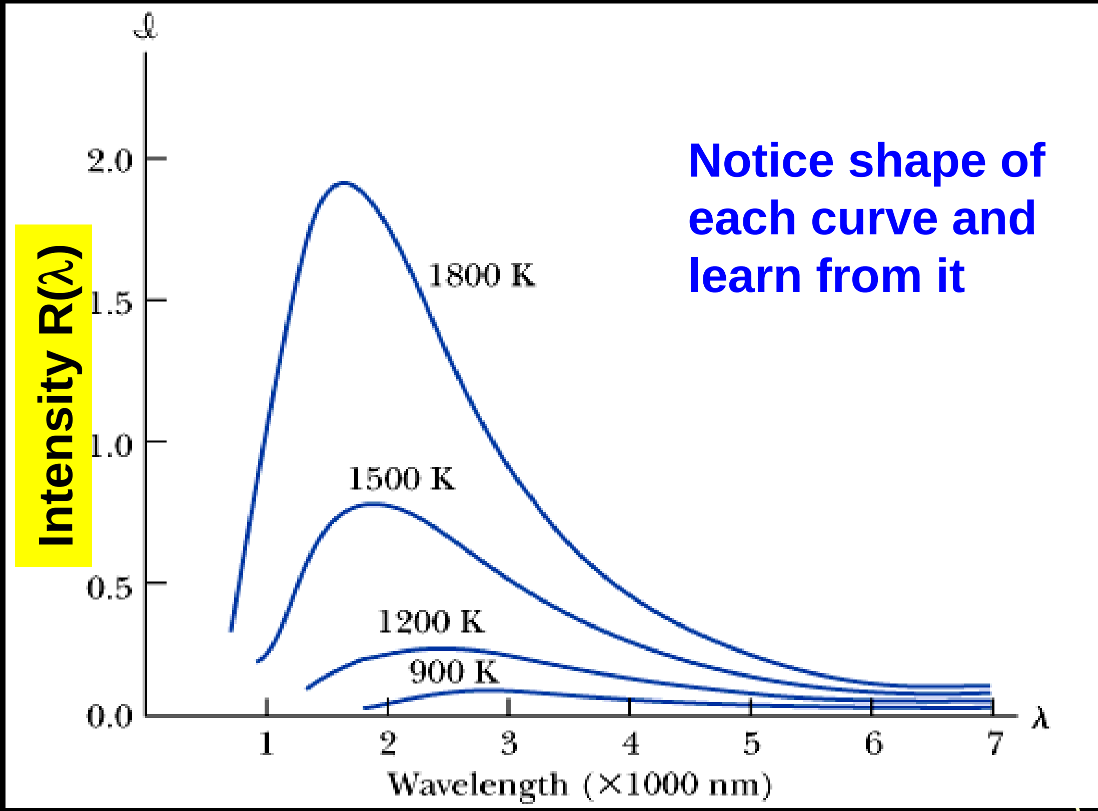
Larger the amplitude of Oscillation More intense is the radiation

Nature of Radiation: An Expt with BBQ Grill

Question : Distribution of Intensity of EM radiation Vs T & λ



- Radiator (BBQ grill) at some temp T
- Emits variety of wavelengths
  - Some with more intensity than others
- EM waves of diff. λ bend differently within prism
- Eventually recorded by a detector (eye)
- Map out emitted Power / area Vs λ



The Beginning of The End ! How BBQ Broke Physics

Classical Calculation

of standing waves between Wavelengths λ and λ+dλ are

$$N(\lambda)d\lambda = \frac{8\pi V}{\lambda^4} \bullet d\lambda \; ; \; V = \text{Volume of box} = L^3$$

Each standing wave contributes energy $E = kT$ to radiation in Box

Energy density $u(\lambda)$= [# of standing waves/volume]× Energy/Standing Wave

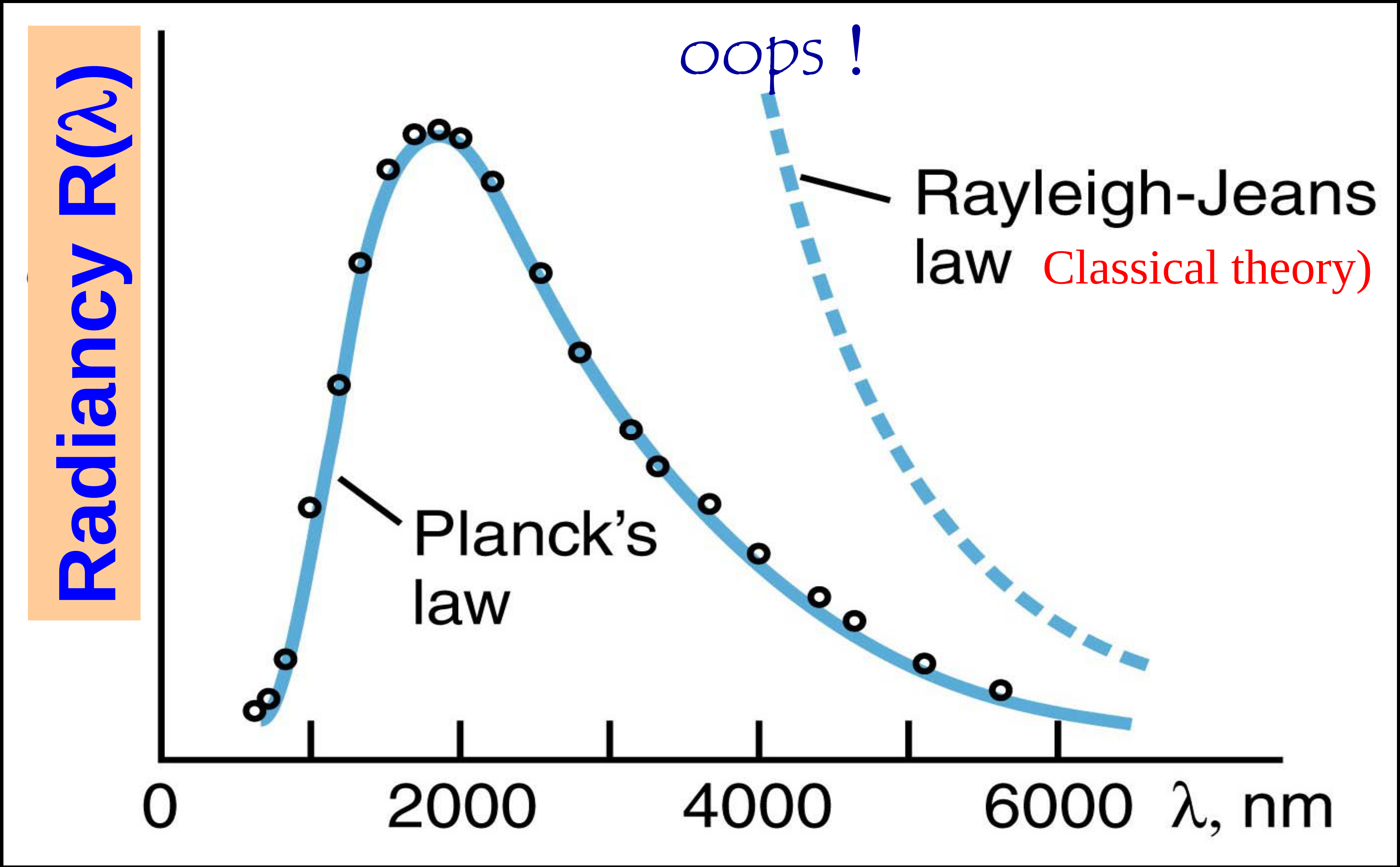
$$= \frac{8\pi V}{\lambda^4} \times \frac{1}{V} \times kT = \frac{8\pi}{\lambda^4} kT$$

$$\text{Radiancy } R(\lambda) = \frac{c}{4} u(\lambda) = \frac{c}{4}\frac{8\pi}{\lambda^4} kT = \frac{2\pi c}{\lambda^4} kT$$

Radiancy is Radiation intensity per unit λ interval: Lets plot it

Prediction : as λ → 0 (high frequency f), R(λ) → Infinity !
Oops !

Ultra Violet (Frequency) Catastrophe

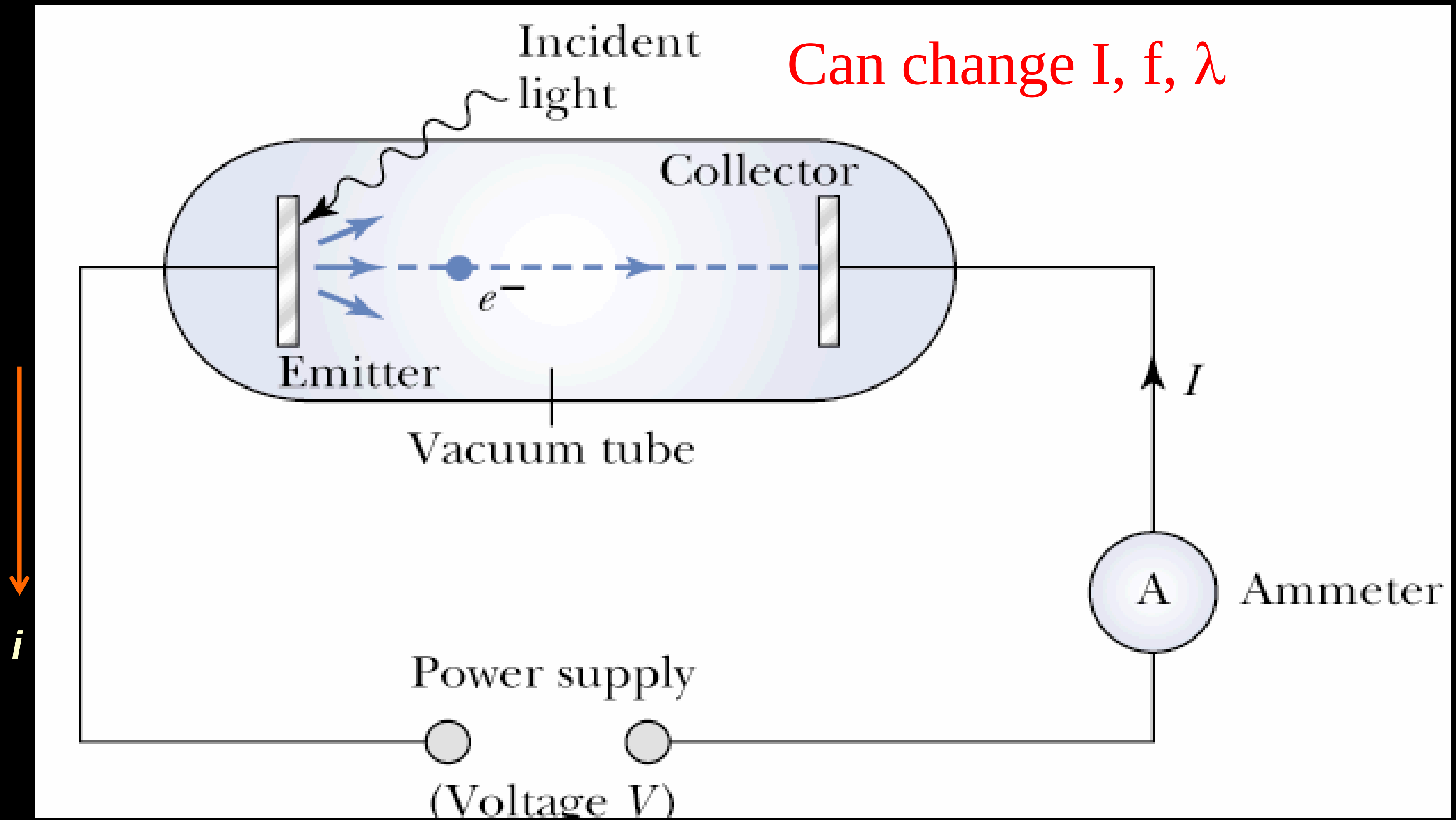


That was a Disaster !

(#1)

Disaster # 2 : Photo-Electric Effect

Light of intensity I, wavelength λ and frequency f incident on a photo-cathode

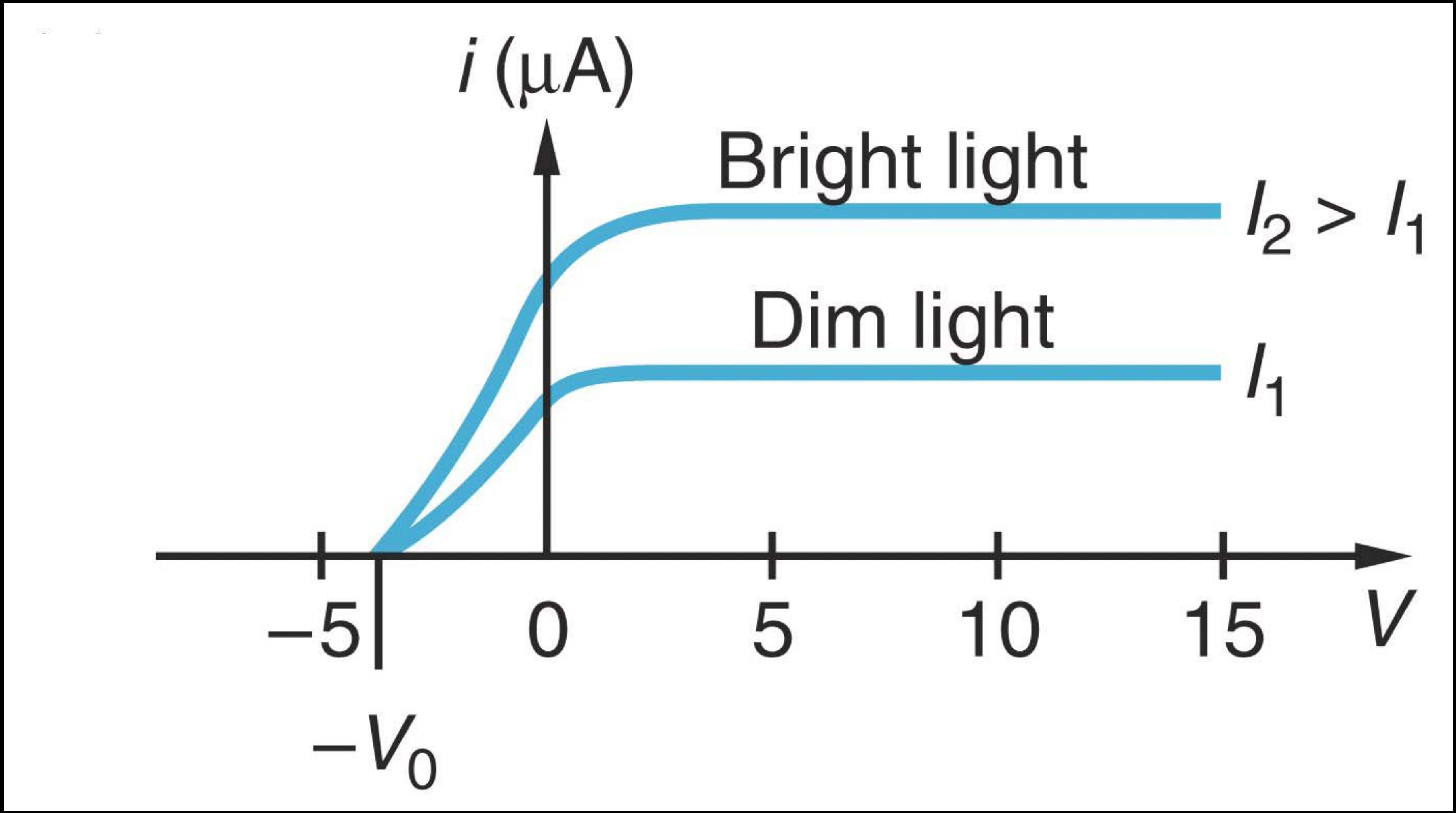


Measure characteristics of current in the circuit as a fn of I, f, λ

Photo Electric Effect: Measurable Properties

- Rate of electron emission from cathode
  - From current *i* seen in ammeter in the circuit. More photoelectrons → more current registered in ammeter
- Maximum kinetic energy of emitted electron
  - By applying retarding potential on electron moving left to tright towards Collector plate
    - **$K_{MAX} = eV_0$ (V_0 = Stopping voltage)**
    - **Stopping potential → no current flows**
- Photoelectric Effect on different types of photo-cathode metal surface
- Time between shining light and first sign of photo-current in the circuit

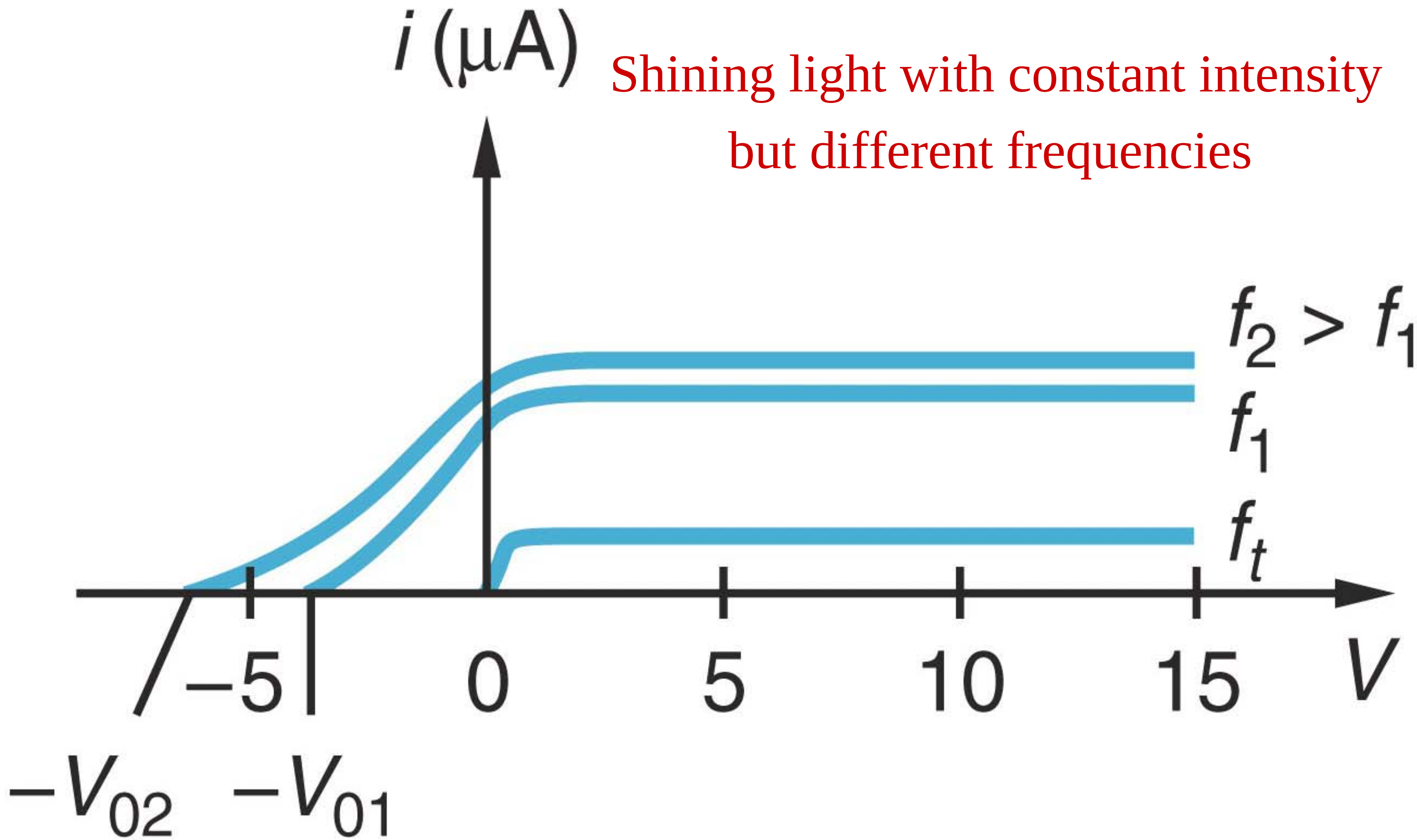
Observations:PhotoCurrent Vs Intensity of Incident Light



Observations: Photocurrent Vs frequency of incident light

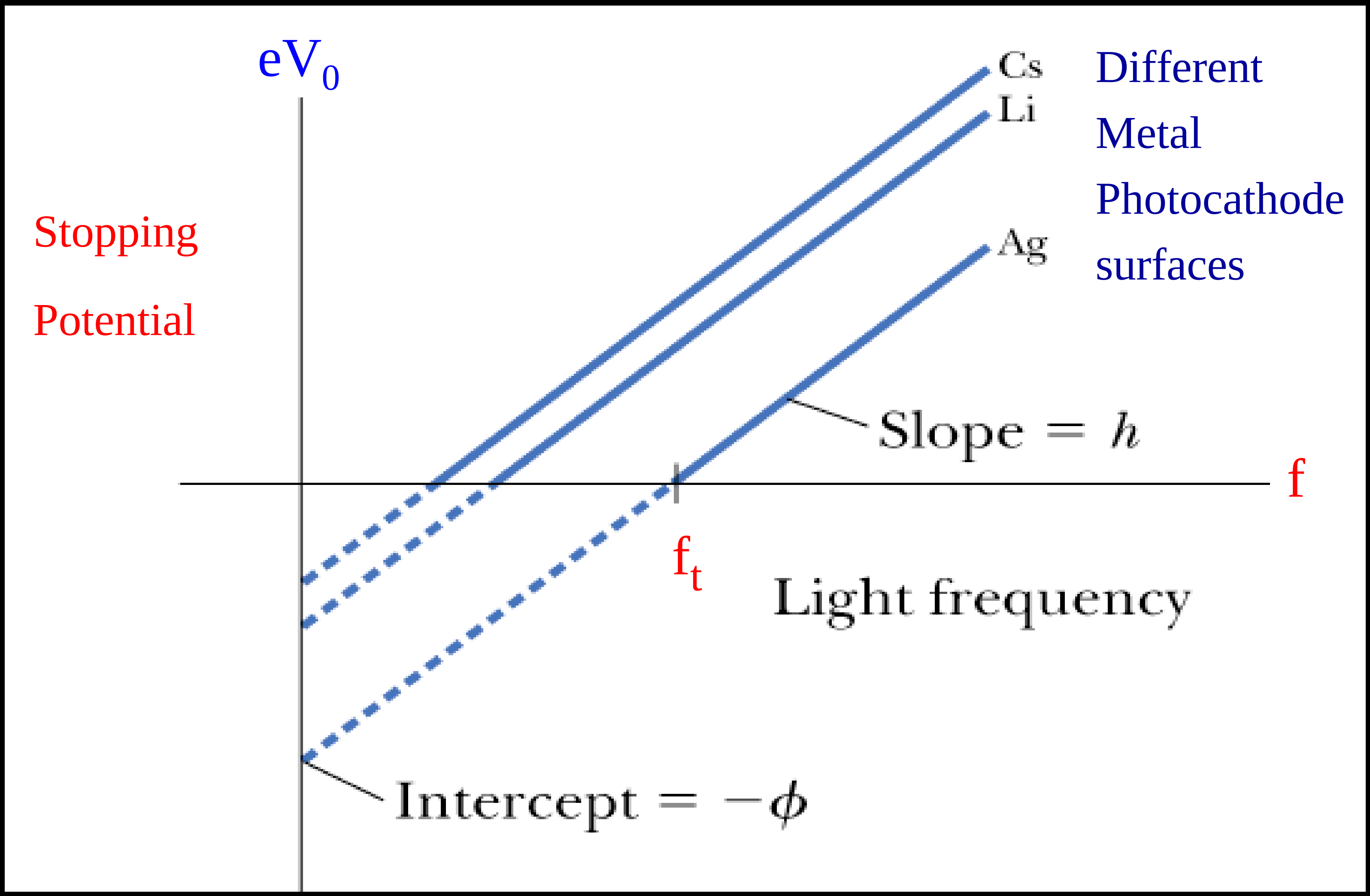
f

Shining light with constant intensity
but different frequencies



Stopping Voltage (V_0) Vs Incident Light Frequency (f)

Try different photocathode materials…..see what happens



Conclusions from the Experimental Observations

- Max Kinetic energy K_{MAX} independent of Intensity I for light of same frequency
- No photoelectric effect occurs if light frequency f is below a threshold no matter how high the intensity of light
- For a particular metal, light with $f > f_t$ causes photoelectric effect IRRESPECTIVE of light intensity.
  - f_t is characteristic of that metal
- Photoelectric effect is instantaneous !...not time delay

Can one Explain all this Classically !

Classical Explanation of Photo Electric Effect

- As light Intensity increased $\Rightarrow$ $\vec{E}$ field amplitude larger
  - E field and electrical force seen by the "charged subatomic oscillators" Larger
    - $\vec{F} = e\vec{E}$
    - More force acting on the subatomic charged oscillator
    - $\Rightarrow$ More (work done) ➔ more energy transferred to it
    - $\Rightarrow$ Charged particle "hooked to the atom" should leave the surface with more Kinetic Energy KE !! The intensity of light (EM Wave) shining rules !
- As long as light is intense enough , light of ANY frequency f should cause photoelectric effect
- Because the Energy in a Wave is uniformly distributed over the Spherical wavefront incident on cathode, should be a noticeable time lag ΔT between time is incident & the time a photo-electron is ejected : Energy absorption time
  - How much time for electron ejection ? Lets calculate it classically

Classical Physics: Time Lag in Photo-Electric Effect ?

- Electron absorbs energy incident on a surface area where the electron is confined ≅ size of atom in cathode metal
- Electron is "bound" by attractive Coulomb force in the atom, so it must absorb a minimum amount of radiation before its stripped off
- Example : Laser light Intensity I = 120W/m^2 on Na metal
  - Binding energy = 2.3 eV= "Work Function Φ "
  - Electron confined in Na atom, size ≅ 0.1nm; how long before ejection ?
  - Average Power Delivered $P_{AV} = \mathbf{I} \cdot \mathbf{A}$, $A = \pi r^2 \cong 3.1 \times 10^{-20}\ m^2$
  - If all energy absorbed then $\Delta E = P_{AV} \cdot \Delta T \Rightarrow \Delta T = \Delta E / P_{AV}$

$$\Delta T = \frac{(2.3eV)(1.6\times10^{-19}\ J/eV)}{(120W/m^2)(3.1\times10^{-20}\ m^2)} = 0.10\ S$$

  - Classical Physics predicts measurable delay even by the primitive clocks of 1900
  - But in experiment, the effect was observed to be instantaneous !!
  - Classical Physics fails in explaining all results

That was a Disaster !

(# 2)

Beginning of a search for a new hero or an explanation or both !

Max Planck & Birth of Quantum Physics

Back to Blackbody Radiation Discrepancy

Planck noted the Ultraviolet catastrophe at high frequency

"Cooked" calculation with new "ideas" so as bring:

$R(\lambda) \rightarrow 0$ as $\lambda \rightarrow 0$

$f \rightarrow \infty$

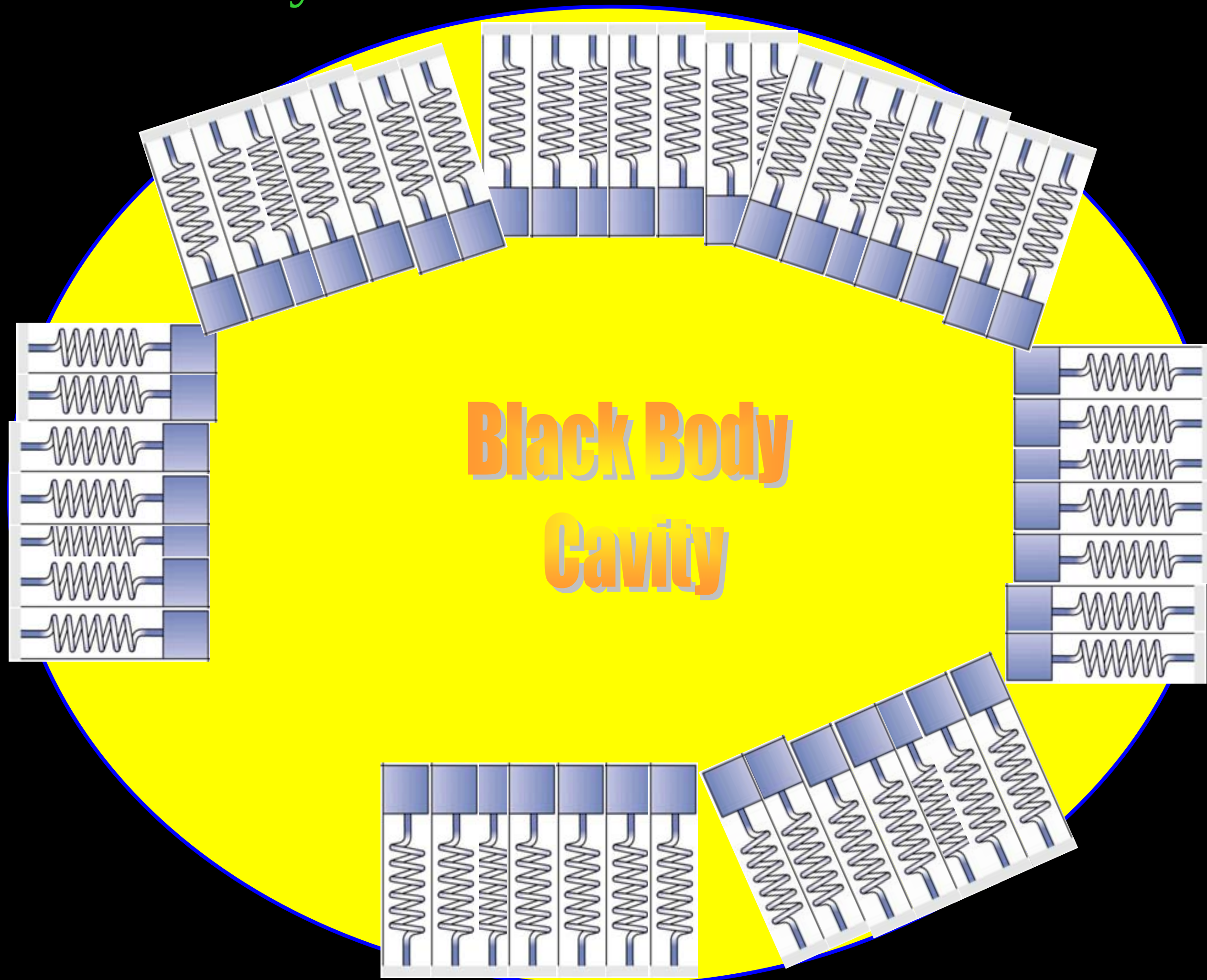
- Cavity radiation as equilibrium exchange of energy between EM radiation & "atomic" oscillators present on walls of cavity
- Oscillators can have any frequency f
- But the Energy exchange between radiation and oscillator NOT continuous, it is discrete …in packets of same amount
- $E = n\,hf$, with $n = 1, 2, 3, 4, \ldots \infty$

  h = constant he invented, a number he made up !

Planck's "Charged Oscillators" in a Black Body Cavity

Planck did not know about electrons, Nucleus etc:

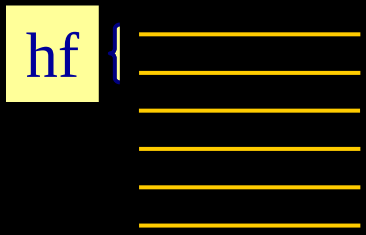
They had not been discovered then



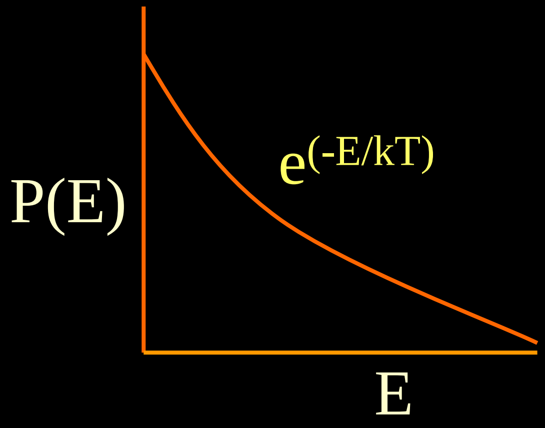
Planck, Quantization of Energy & BB Radiation

- Keep the rule of counting how many waves fit in a BB Volume
- BUT Radiation energy in cavity is quantized
- EM standing waves of frequency f have energy

  E = n hf (n = 1,2 ,3 …10 ….1000…)



- Probability Distribution: At an equilibrium temp T, possible energy of oscillators is distributed over a spectrum of states: $P(E) = e^{(-E/kT)}$
- Modes of Oscillation with :
  - Less energy: E=hf = favored
  - More energy: E=hf = disfavored



By this discrete statistics, large energy = high f modes of EM disfavored

Planck's Calculation: A preview to keep the story going

$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\left[\frac{hc}{\lambda}\left(\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\right)\right]$$

Odd looking form

When $\lambda \rightarrow$ large $\Rightarrow \dfrac{\text{hc}}{\lambda\text{kT}} \rightarrow$ small

Recall $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots.$

$$\Rightarrow e^{\frac{hc}{\lambda kT}} - 1 = (1 + \frac{hc}{\lambda kT} + \frac{1}{2}\left(\frac{hc}{\lambda kT}\right)^2 + \ldots.] - 1$$

$$= \frac{hc}{\lambda kT}$$

plugging this in R(λ) eq:

$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\frac{hc}{\lambda kT}$$

Graph & Compare
With BBQ data

Planck's Formula and Small λ

When λ is small (large f)

$$\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \cong \frac{1}{e^{\frac{hc}{\lambda kT}}} = e^{-\frac{hc}{\lambda kT}}$$

Substituting in R(λ) eqn:

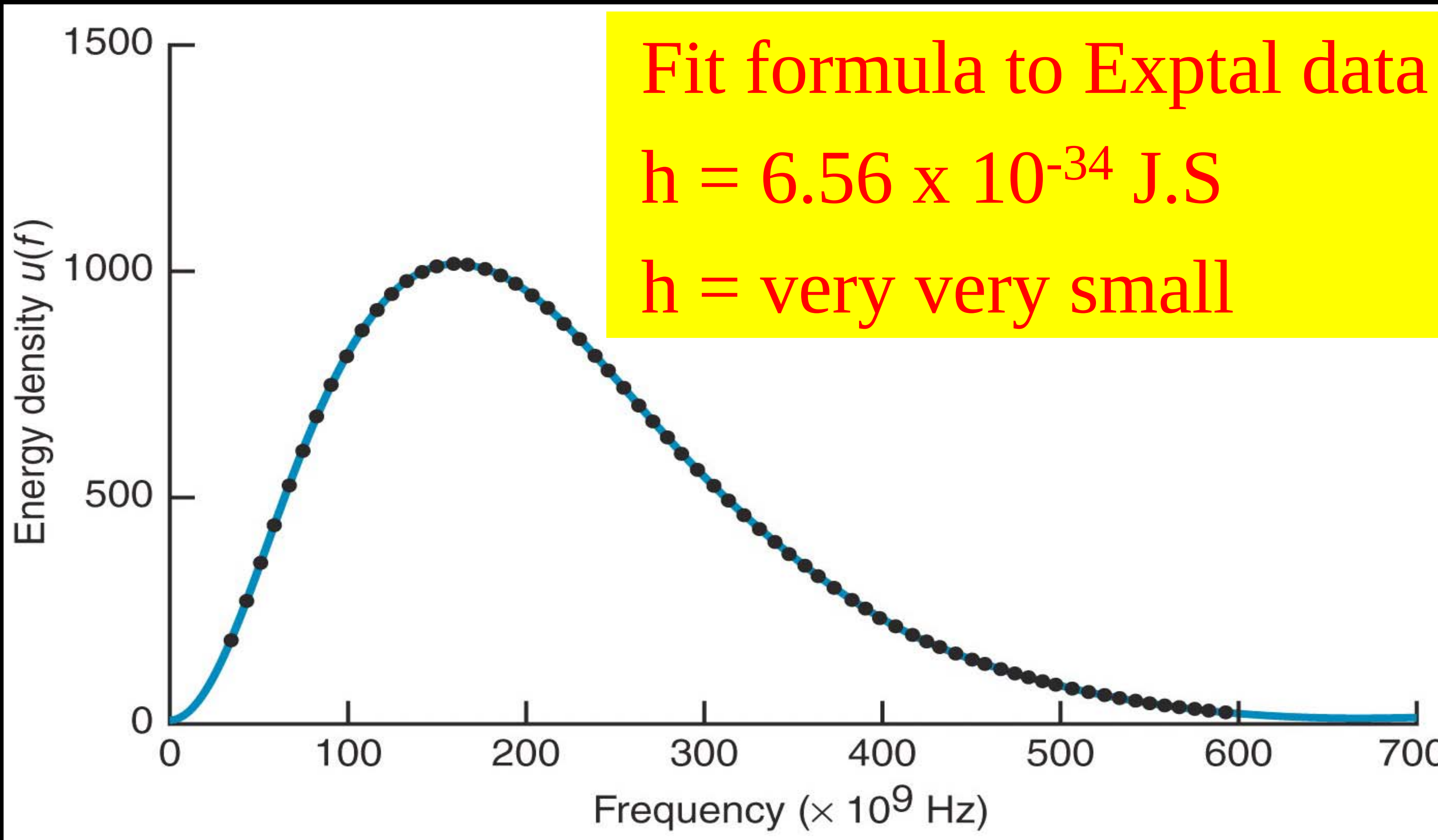
$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)e^{-\frac{hc}{\lambda kT}}$$

$$\text{As } \lambda \to 0,\ e^{-\frac{hc}{\lambda kT}} \to 0$$

$$\Rightarrow R(\lambda) \to 0$$

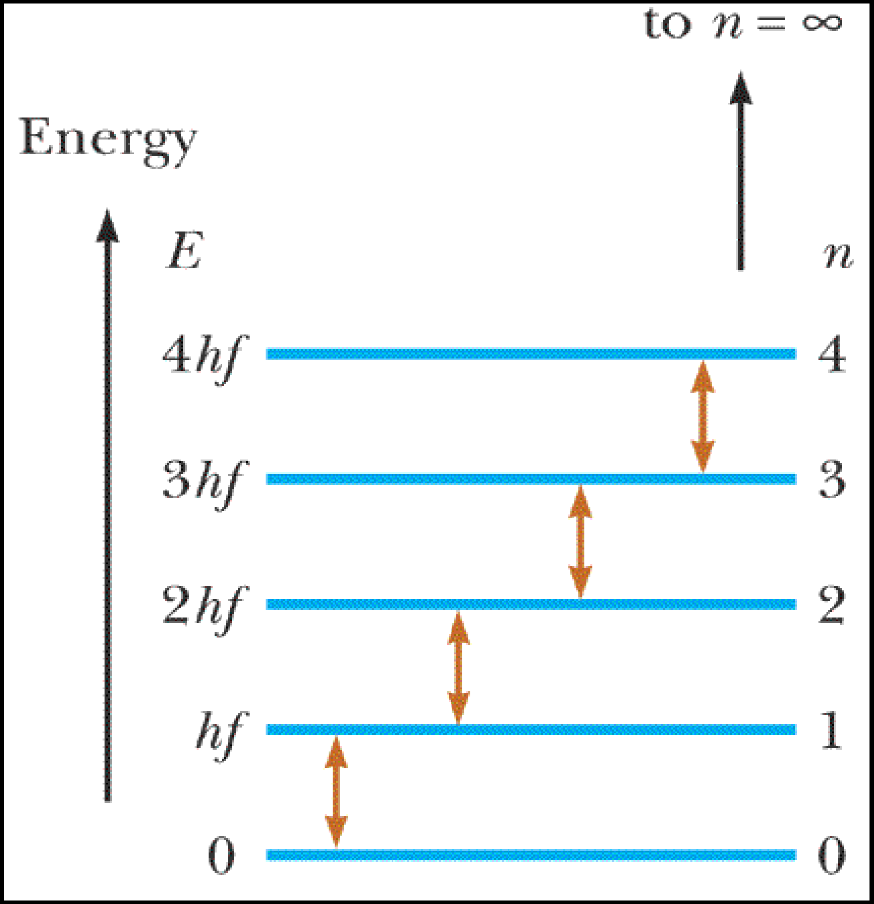
Just as seen in the experimental data !

Planck's Explanation of Black Body Radiation



Major Consequence of Planck's Postulate

Quantization of Energy !



Judging Planck's Postulate : Visionary or just a Wonk?