4E : The Quantum Universe

Lecture 29, May 24

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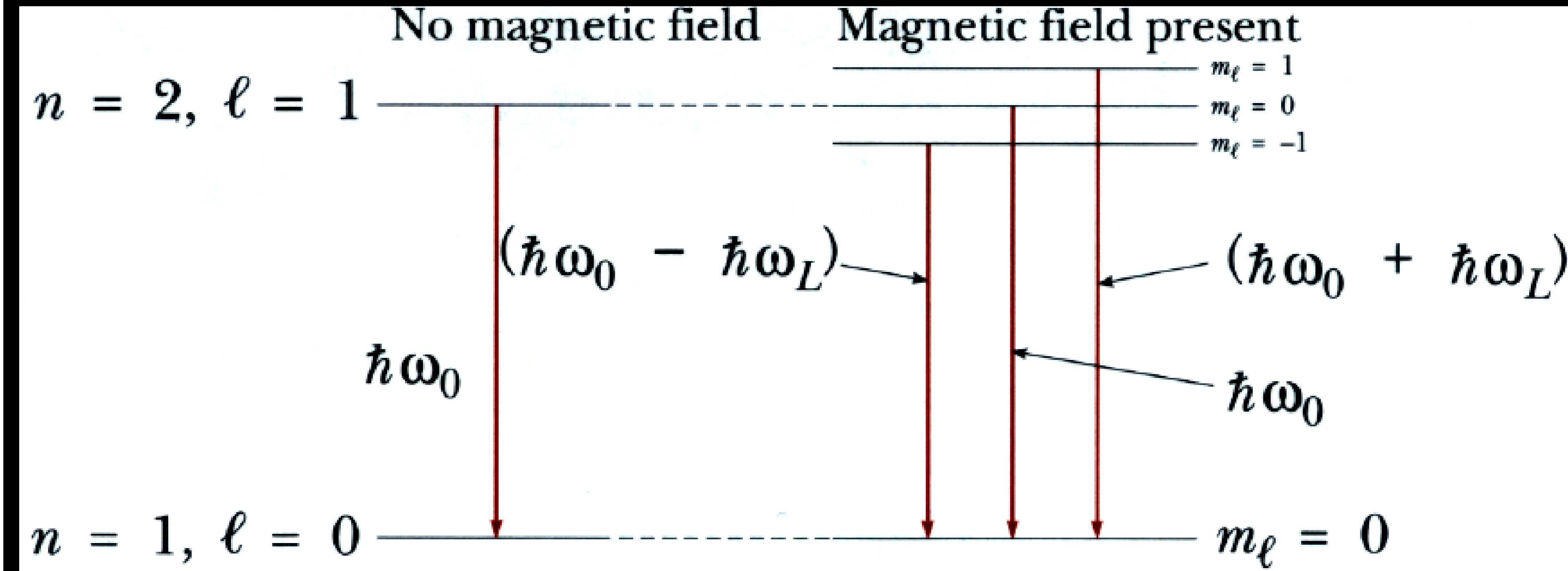
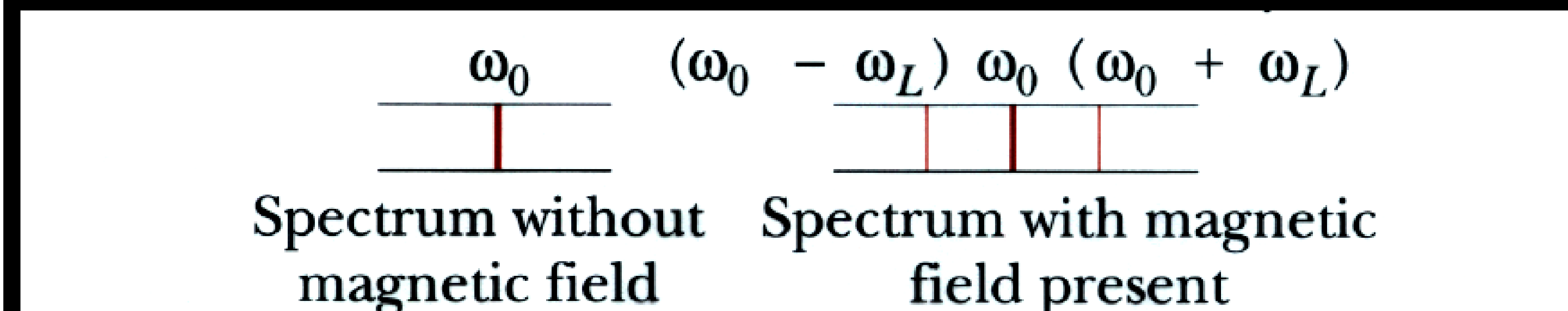
Zeeman Effect Due to Presence of External B field

Energy Degeneracy Is Broken





Electron has "Spin": An additional degree of freedom

Electron possesses additional "hidden" degree of freedom : "Spinning around itself" !

Spin Quantum # $s = \frac{1}{2}$ (either Up or Down)

How do we know this ? $\Rightarrow$ Stern-Gerlach expt

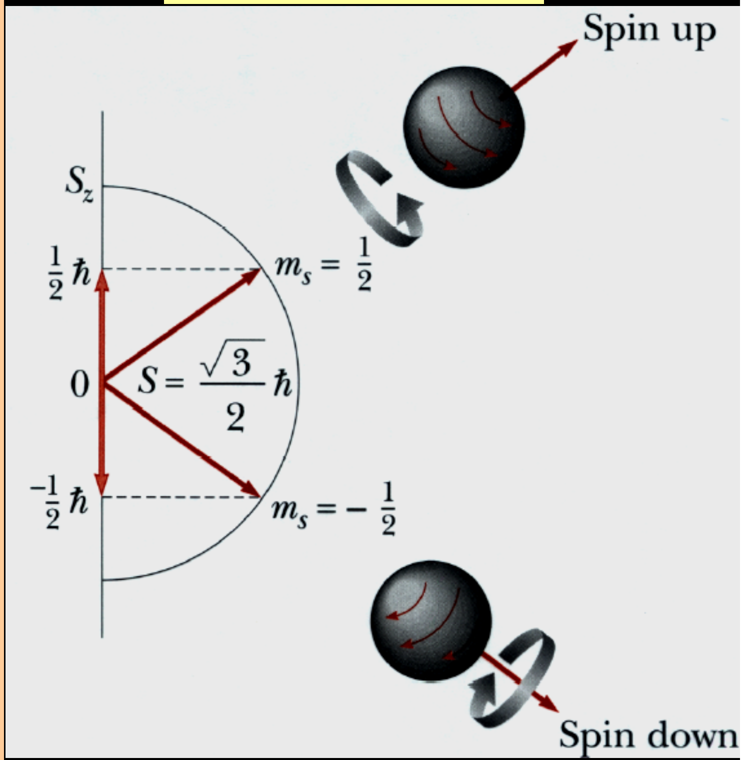
Spin Vector $\vec{S}$ (a form of angular momentum) is also Quantized

$$|\vec{S}| = \sqrt{s(s+1)}\ \hbar = \sqrt{\frac{3}{4}}\ \hbar$$

& $S_z = m_s\hbar;\ m_s = \pm\frac{1}{2}$

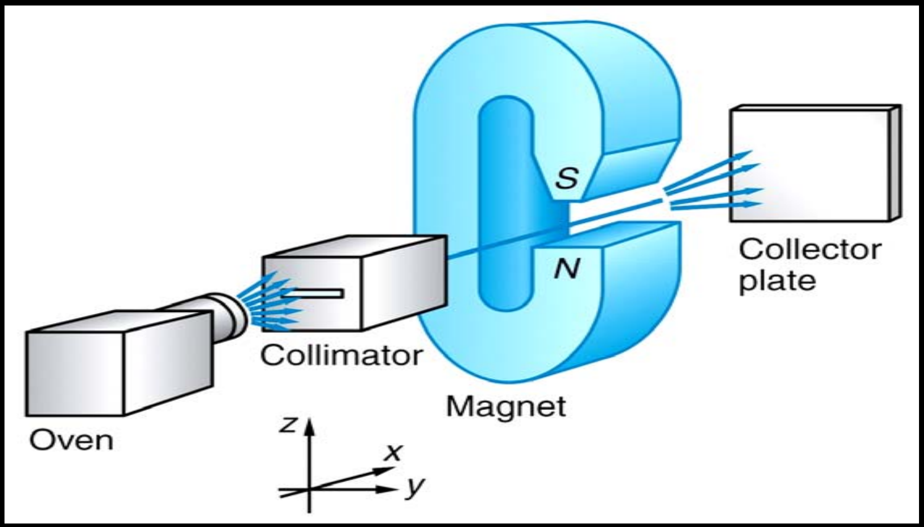
Spinning electron is an entitity defying any simple classical description. ....hidden D.O.F

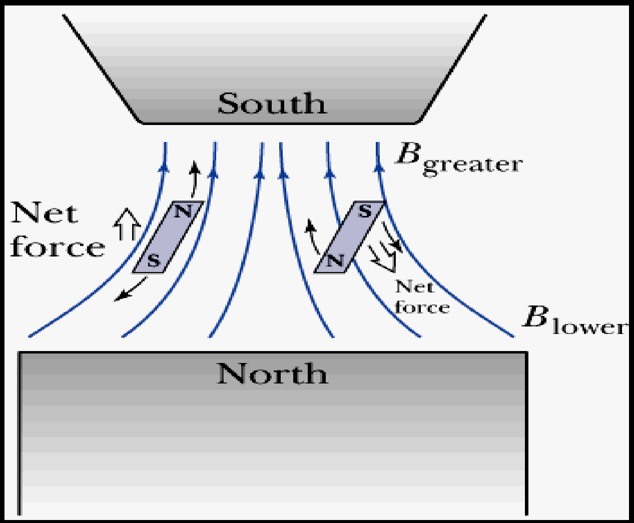
$$|\vec{S}| = \sqrt{s(s+1)}\ \hbar$$



Spin angular momentum S also exhibits Space quantization

Stern-Gerlach Expt ⇒ An additional degree of freedom: "Spin"





In an inhomogeneous field perpendicular to beam direction, magnetic moment μ experiences a force F_z whose direction depends on Z component of the net magnetic moment & inhomogeneity dB/dz. The force deflects magnetic moment up or down. Space Quantization means expect *(2l+1)* deflections. For l =0, expect *all* electrons to arrive on the screen at the center (no deflection)

$\vec{\mu}$ in inhomogenous $\vec{B}$ field, experiences force $\vec{F}$

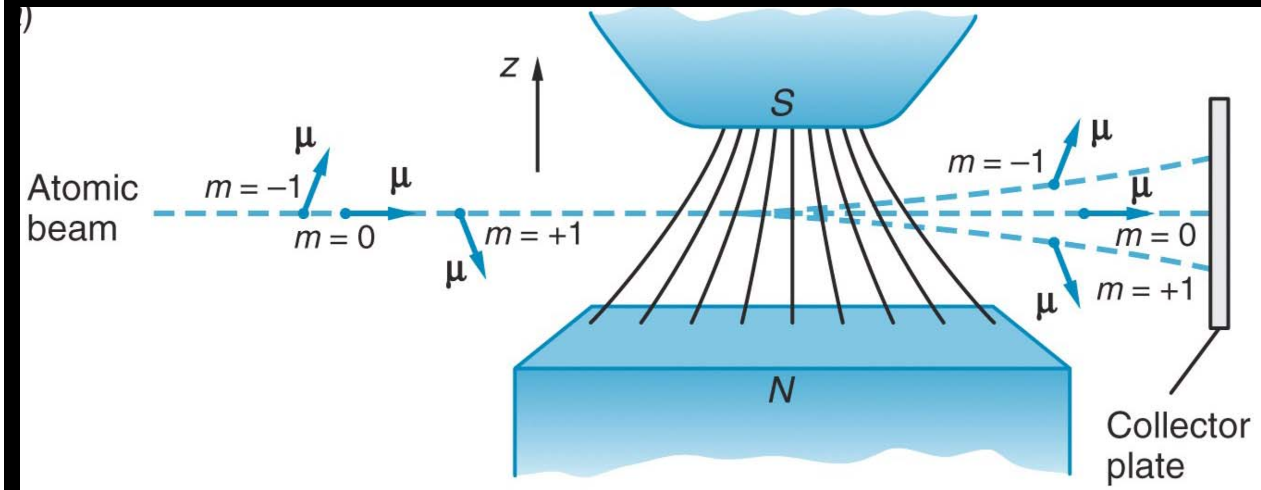
$$\vec{F} = -\nabla U_B = -\nabla(-\vec{\mu}.\vec{B})$$

When gradient only along z, $\frac{\partial B}{\partial z} \neq 0; \frac{\partial B}{\partial x} = \frac{\partial B}{\partial y} = 0$

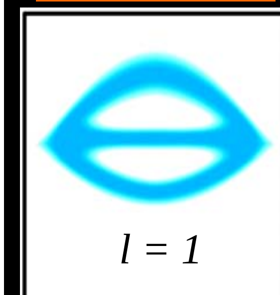
$$F_z = m\mu_B\left(\frac{\partial B}{\partial z}\right) \text{ moves particle up or down}$$

(in addition to torque causing magnetic moment $\vec{\mu}$ to precess about B field direction

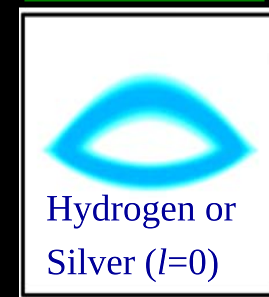
An Additional degree of freedom: "Spin" for lack of a better name !



Expected



Observed



!

This was a big surprise for Stern-Gerlach ! They had accidentally Discovered a new degree of freedom for electron : "spin" which Can take only two orientations for angular momentum S : up or down Leads to a new quantum number *s=1/2*. As a result:

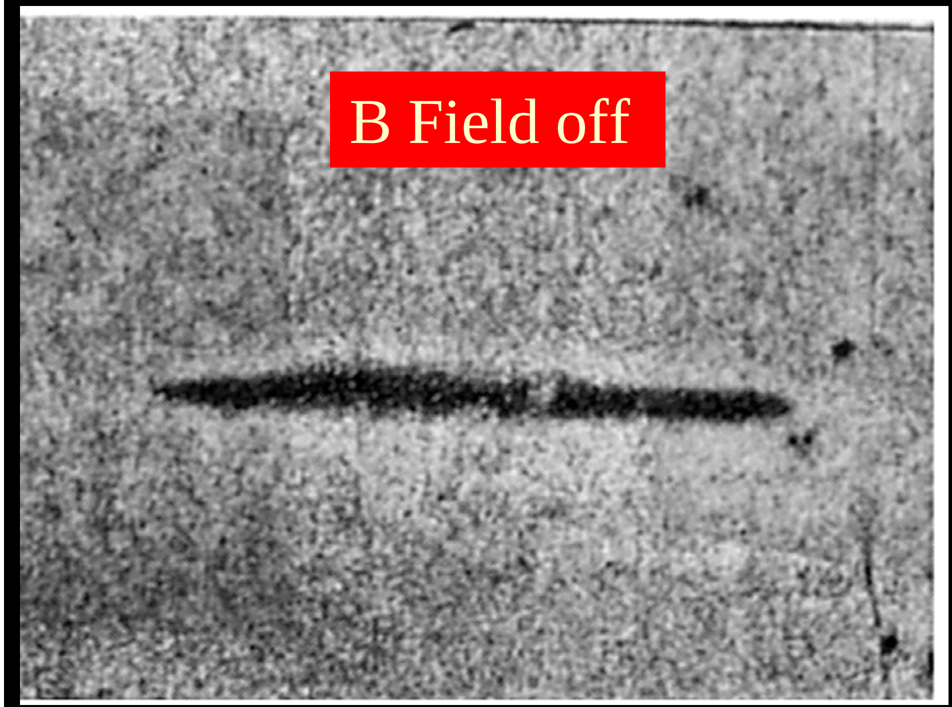
Z Component of Spin Angular Momentum $S_z = m_s \hbar$

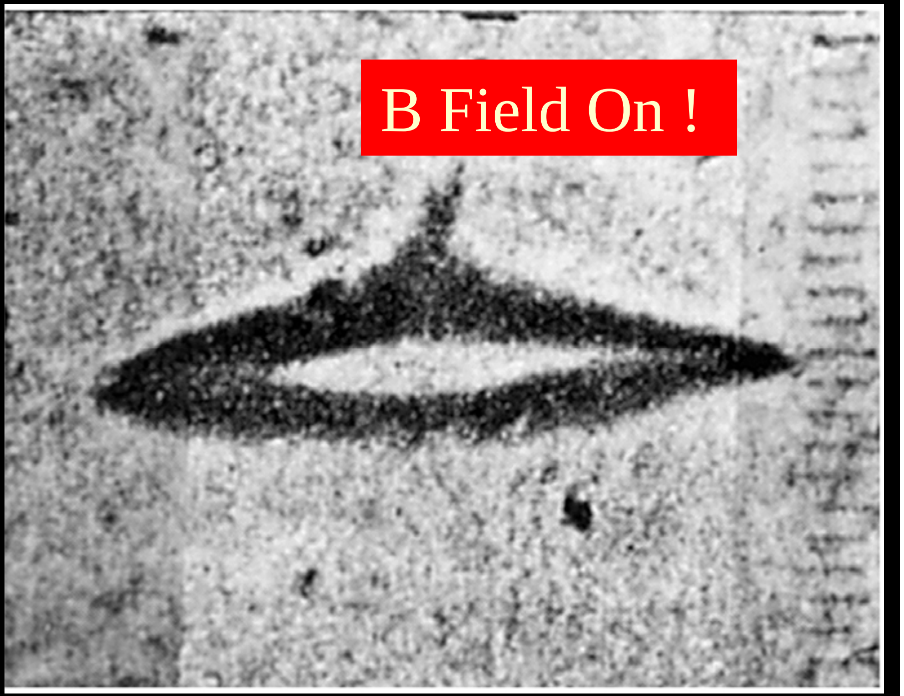
The magnitude $|S| = \sqrt{s(s+1)}\ \hbar$ is FIXED, never changes !

Allowed orientations are $s(s+1) = 2$

$\vec{S} \Rightarrow \vec{\mu}_S$; The corresponding Spin Magnetic Moment

What Stern&Gerlach Saw in l=0 Silver Atoms

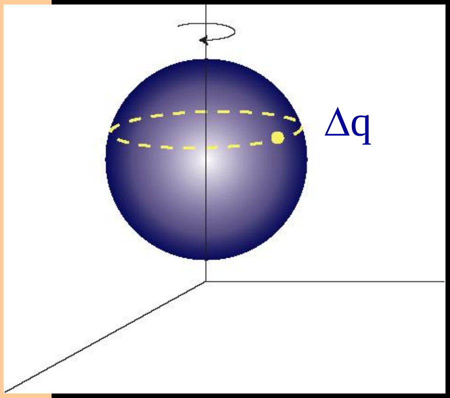




Picture changes instantaneously as the external
Field is switched off or on….discovery !

Four (not 3) Numbers Describe Hydrogen Atom → n, l, m_l, m_s

"Spinning" charge gives rise to a dipole moment $\vec{\mu}_s$:



Imagine (semi-clasically, incorrectly!) electron as sphere: charge q, radius r

Total charge uniformly distributed: $q=\sum_i \Delta q_i$;

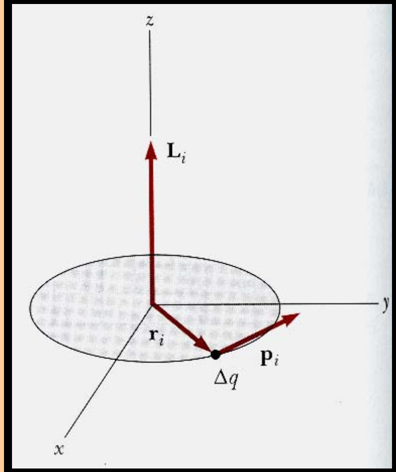
as electron spins, each "chargelet" rotates ⇒ current ⇒ dipole moment $\vec{\mu}_{s_i}$

$$\vec{\mu}_s = \left(\frac{q}{2m_e}\right)\sum_i \vec{\mu}_{s_i} = g\left(\frac{q}{2m_e}\right)\vec{S}; g = 2$$

In a Magnetic Field $\vec{B}$ ⇒ magnetic energy due to spin $U_S = \vec{\mu}_s . \vec{B}$

Net Angular Momentum in H Atom $\vec{J} = \vec{L} + \vec{S}$

Net Magnetic Moment of H atom: $\vec{\mu} = \vec{\mu}_0 + \vec{\mu}_s = \left(\frac{-e}{2m_e}\right)(\vec{L} + g\vec{S})$



Notice that since g=2, net dipole moment vector $\vec{\mu}$ is not || to $\vec{J}$

(There are many such "ubiquitous" quantum numbers for elementary particles!)

Magnetic Energy in an External B Field

Contributions from Orbital and Spin motions. Defining Z axis to be the orientation of the B field:

$$\text{U}=-\vec{\mu}.\vec{B}=\frac{e}{2m}B\left\{L_z+gS_z\right\}=\frac{e\hbar}{2m}B\left\{m_l+gm_s\right\}$$

Example: Zeeman spectrum in B=1T produced by Hydrogen initially in n=2 state after taking spin into account: n=2 $\Rightarrow \text{E}_2=-13.6eV/2^2=-3.40eV$

Since $\text{m}_\text{l}=0,\pm1,$ orbital contribution to Magnetic energy $\text{U}_0=m_l\hbar\omega_L$

This splits energy levels to $\text{E}=\text{E}_2\pm\hbar\omega_L$; for $\text{m}_\text{l}=\pm1$ states

These states get further split in pairs due to spin magnetic moment

Since g=2 and $\text{m}_\text{s}=\pm\frac{1}{2}$; spin energy is again Zeeman energy=$\hbar\omega_L$

As a result electrons in this shell have one of the following energies

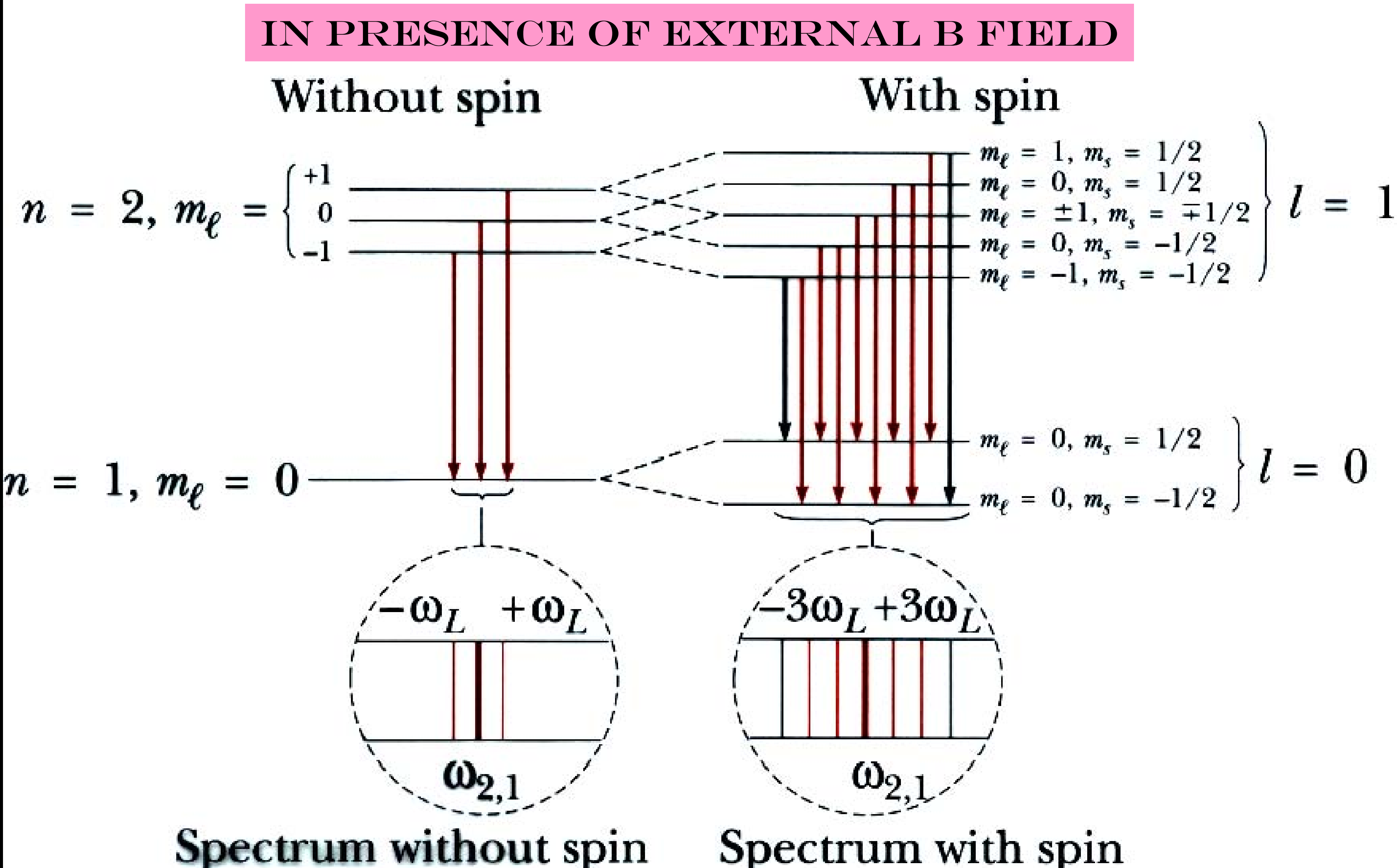
$\boxed{\text{E}_2}$ $\boxed{\text{E}_2\pm\hbar\omega_L}$ $\boxed{\text{E}_2\pm2\hbar\omega_L}$

This leads to a variety of allowed ($\Delta(\text{m}_\text{l}+\text{m}_\text{s})=0,\pm1$) energy transitions with different intensities (Principal and satellites) which are visible when B field is large (ignore LS coupling

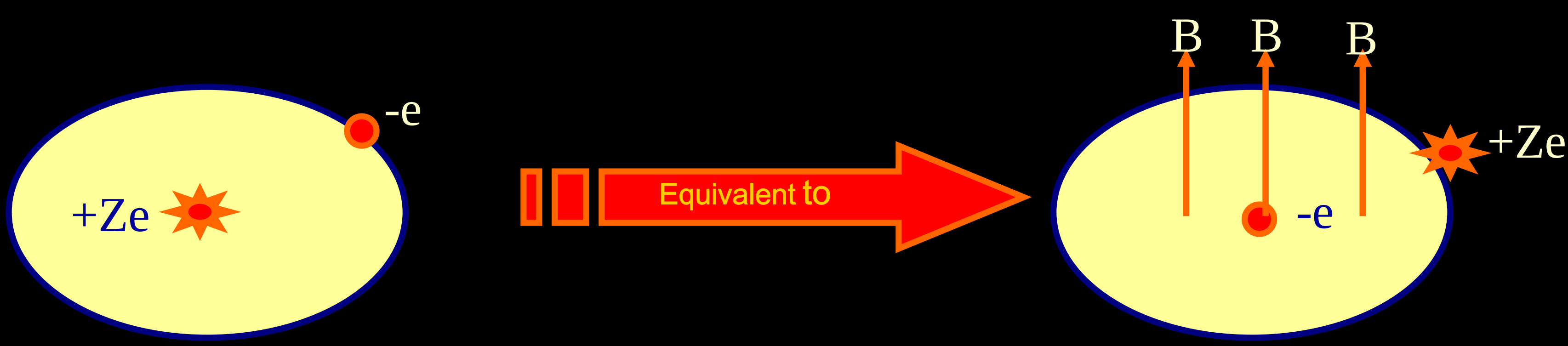
See energy level diagram on next page

Doubling of Energy Levels Due to Spin Quantum Number

Under Intense B field, each {n , m_l } energy level splits into two depending on spin up or down



Spin-Orbit Interaction: L and S Momenta are Linked Magnetically



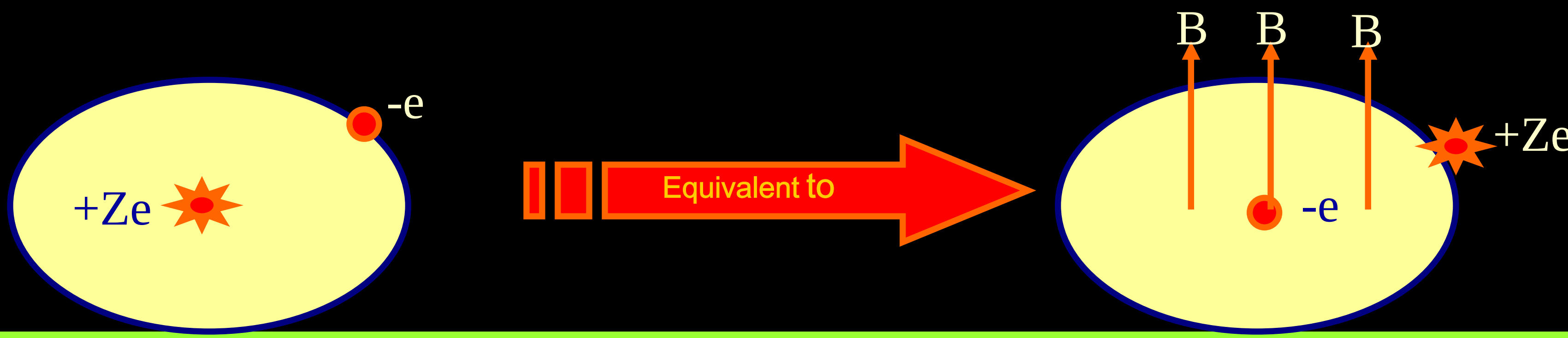
Electron revolving around Nucleus finds itself in a "internal" B field because in its frame of reference, the nucleus is orbiting around it

This B field, due to orbital motion, interacts with electron's spin dipole moment $\vec{\mu}_s$

$U_m = -\vec{\mu}.\vec{B} \Rightarrow$ Energy larger when $\vec{S} \parallel \vec{B}$, smaller when anti-parallel

$\Rightarrow$ States with same (n, l, m_l) but diff. spins $\Rightarrow$ energy level splitting/doubling due to $\vec{S}$

Spin-Orbit Interaction: Angular Momenta are Linked Magnetically



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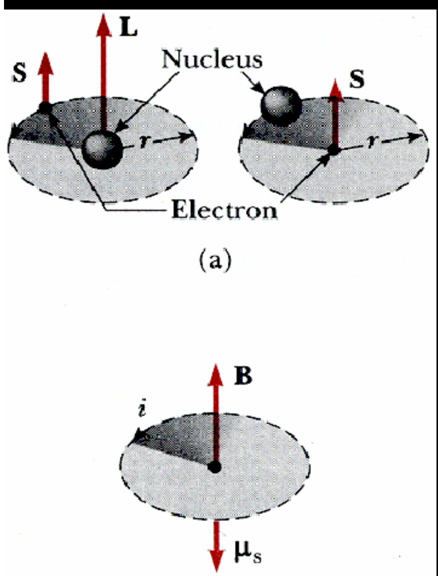
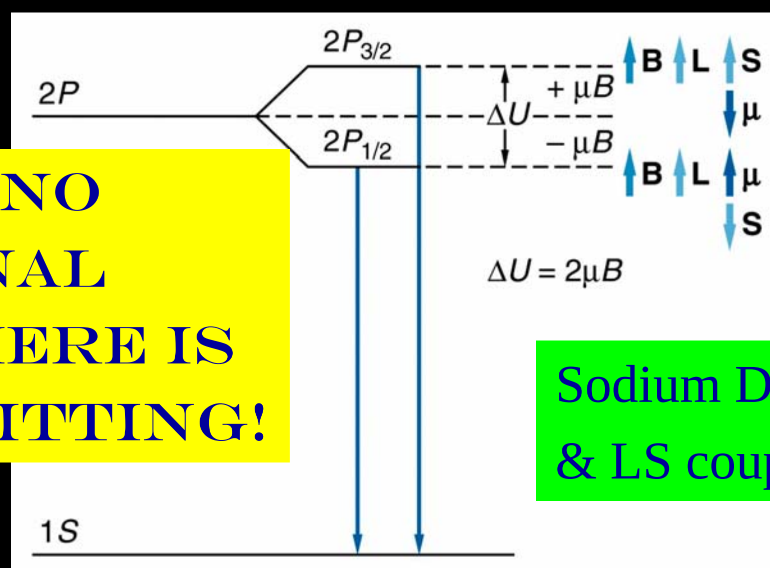
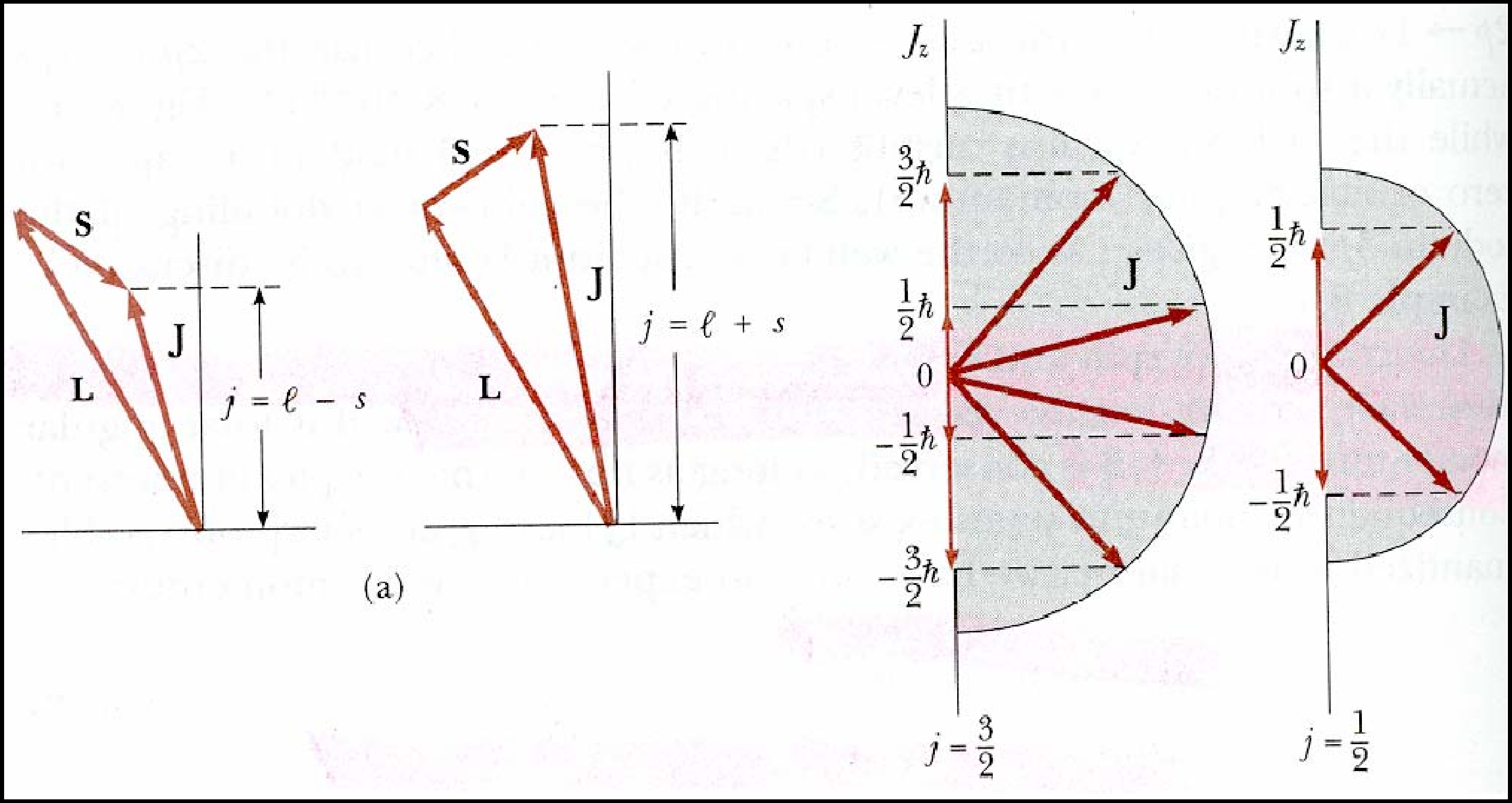


UNDER NO EXTERNAL B FIELD THERE IS STILL A SPLITTING!



Sodium Doublet & LS coupling

Vector Model For Total Angular Momentum J



Vector Model For Total Angular Momentum J

Coupling of Orbital & Spin magnetic moments $\Rightarrow$

Neither Orbital nor Spin angular Momentum are conserved seperately!

$\vec{J} = \vec{L} + \vec{S}$ is conserved so long as there are no external torques present

Rules for Total Angular Momentum Quantization :

$|J| = \sqrt{j(j+1)}\,\hbar$ with $j = |l+s|,\ l+s-1,\ l+s-2 \ldots\ldots, \ldots\ldots, |l-s|$

$J_z = m_j \hbar$ with $m_j = j, j-1, j-2 \ldots\ldots, -j$

Example: state with $(l = 1, s = \frac{1}{2})$

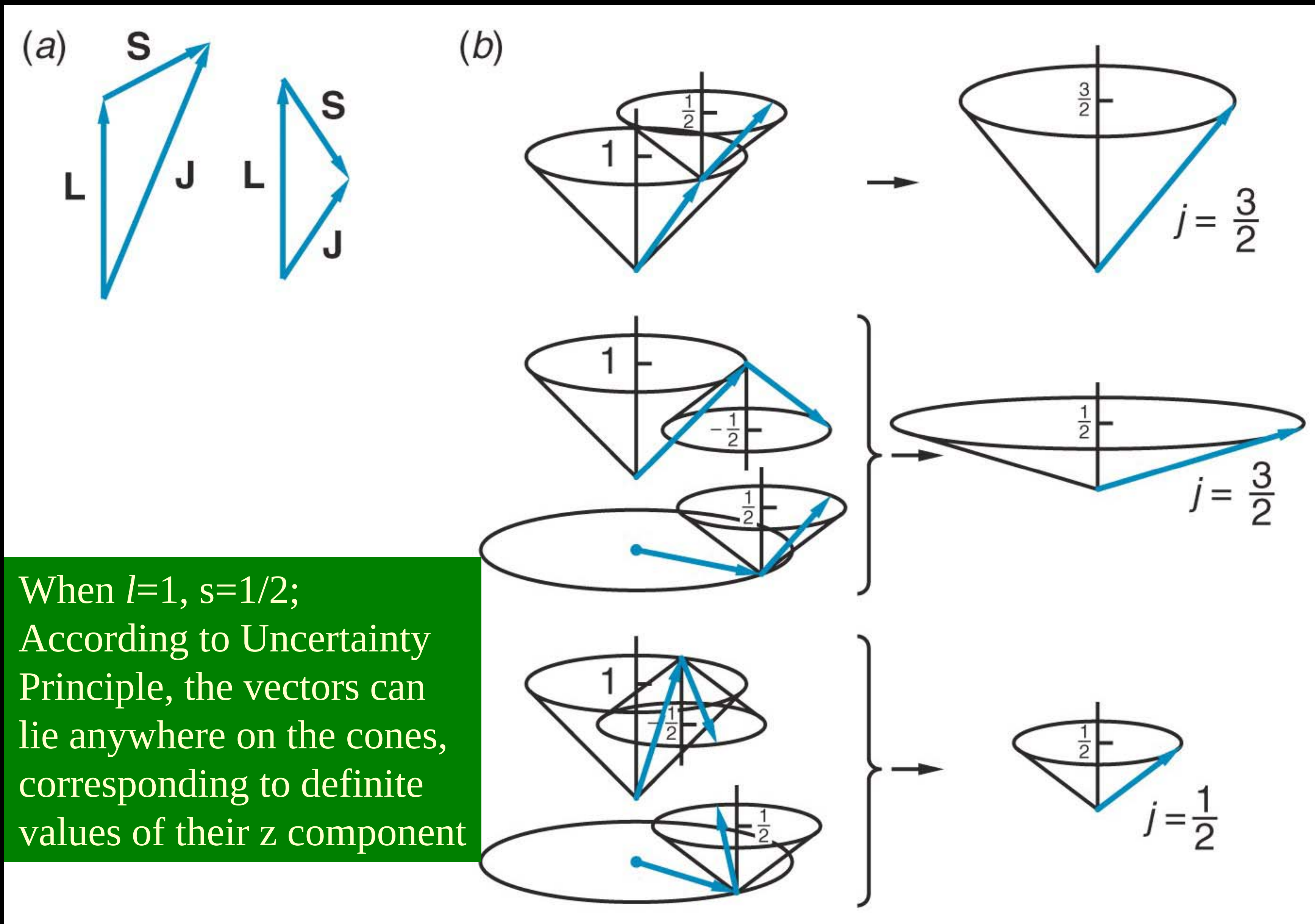
$j = 3/2 \Rightarrow m_j = -3/2, -1/2, 1/2, 3/2$

$j = 1/2 \Rightarrow m_j = \pm 1/2$

In general m_j takes $(2j+1)$ values

$\Rightarrow$ Even # of orientations

Addition of Orbital and Spin Angular Momenta



When l=1, s=1/2;
According to Uncertainty Principle, the vectors can lie anywhere on the cones, corresponding to definite values of their z component

Complete Description of Hydrogen Atom

Full description

of the Hydrogen atom:

$\{n, l, m_l, m_s\}$

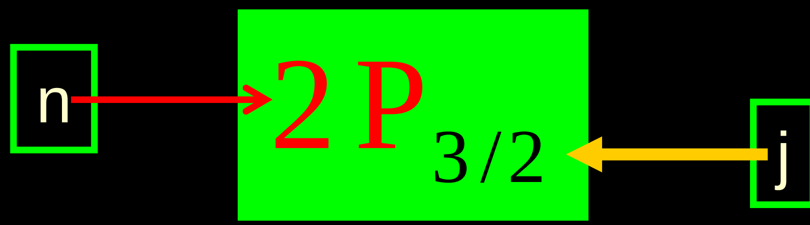
$\Downarrow$

LS Coupling

$\Downarrow$

$\{n, l, j, m_s\}$

corresponding to 4 D.O.F.



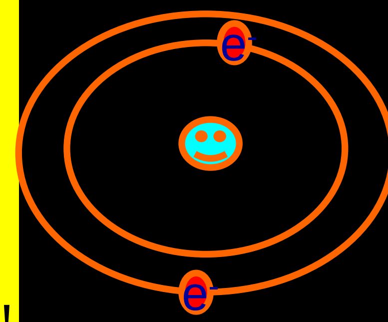
How to describe multi-electrons atoms like He, Li etc?

How to order the Periodic table?

- Four guiding principles:
  - Indistinguishable particle & Pauli Exclusion Principle
  - Independent particle model (ignore inter-electron repulsion)
  - Minimum Energy Principle for atom
  - Hund’s “rule” for order of filling vacant orbitals in an atom

Multi-Electron Atoms : >1 electron in orbit around Nucleus

In Hydrogen Atom $\psi(r,\theta,\phi) = R(r).\Theta(\theta).\Phi(\phi) \quad \equiv \{n, l, j, m_j\}$



In n-electron atom, to simplify, ignore electron-electron interactions

complete wavefunction, in "independent"particle approx" :

$$\psi(1,2,3,..n) = \psi(1).\psi(2).\psi(3)...\psi(n) \quad ???$$

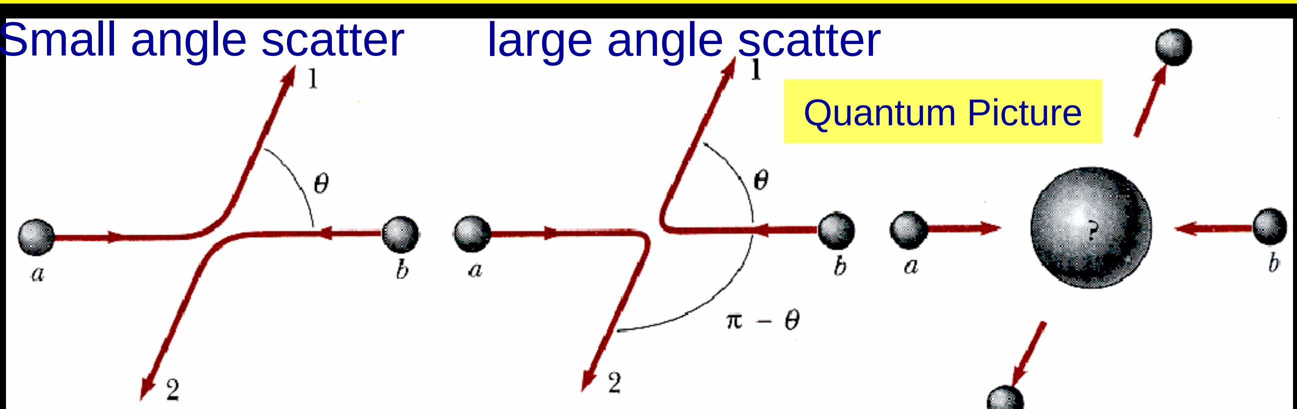
Complication $\rightarrow$ Electrons are identical particles, labeling meaningless!

Question: How many electrons can have same set of quantum #s?

Answer: No two electrons in an atom can have SAME set of quantum#s

(if not, all electrons would occupy 1s state (least energy)... no structure!!

Example of Indistinguishability: electron-electron scattering



If we cant follow electron path, don't know between which of the two scattering events actually happened