4E : The Quantum Universe

Lecture 28, May 19

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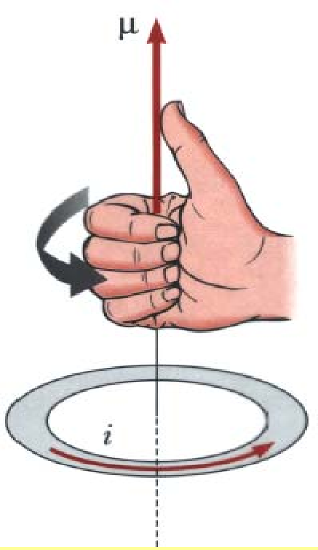
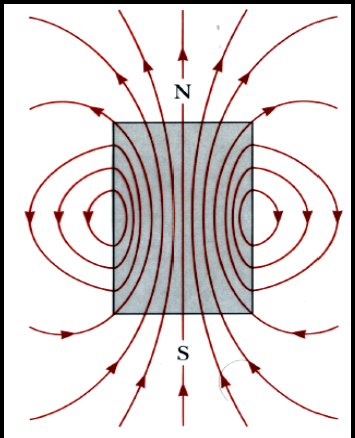
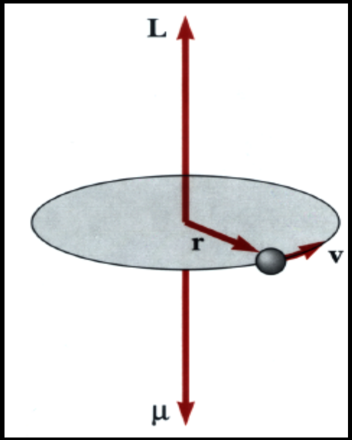
The "Magnetism" of an Orbiting Electron

Precessing electron → Current in loop → Magnetic Dipole moment μ







Electron in motion around nucleus ⇒ circulating charge ⇒ curent i

$$i = \frac{-e}{T} = \frac{-e}{\frac{2\pi r}{v}} = \frac{-ep}{2\pi m r};$$ Area of current loop $A = \pi r^2$

Magnetic Moment $|\mu| = iA = \left(\frac{-e}{2m}\right) rp$; $\vec{\mu} = \left(\frac{-e}{2m}\right)\vec{r} \times \vec{p} = \left(\frac{-e}{2m}\right)\vec{L}$

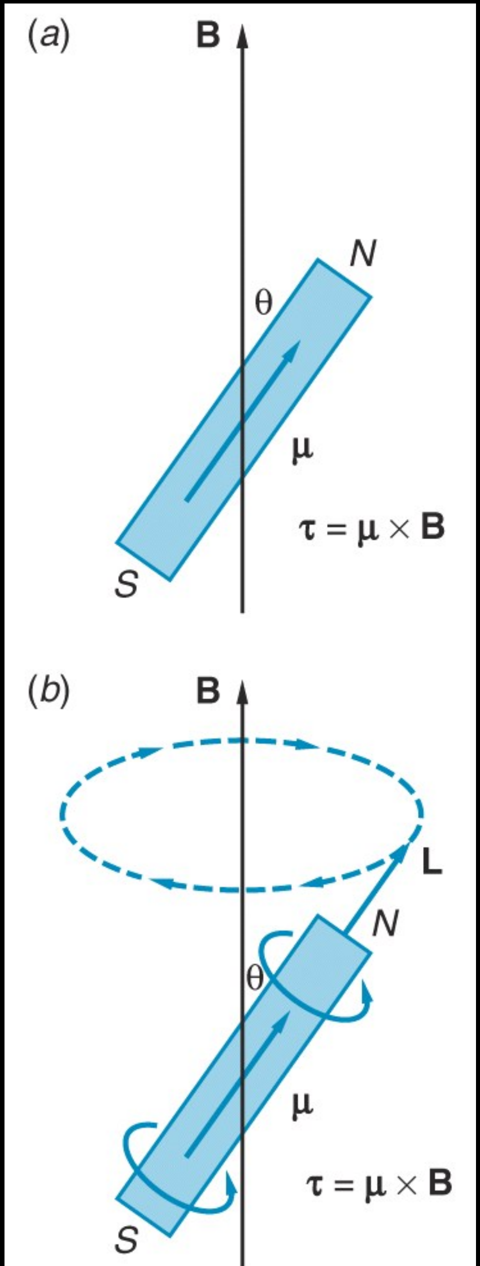
Like the $\vec{L}$, magnetic moment $\vec{\mu}$ also precesses about "z" axis

z component, $\mu_z = \left(\frac{-e}{2m}\right) L_z = \left(\frac{-e\hbar}{2m}\right) m_l = -\mu_B m_l = quantized$!

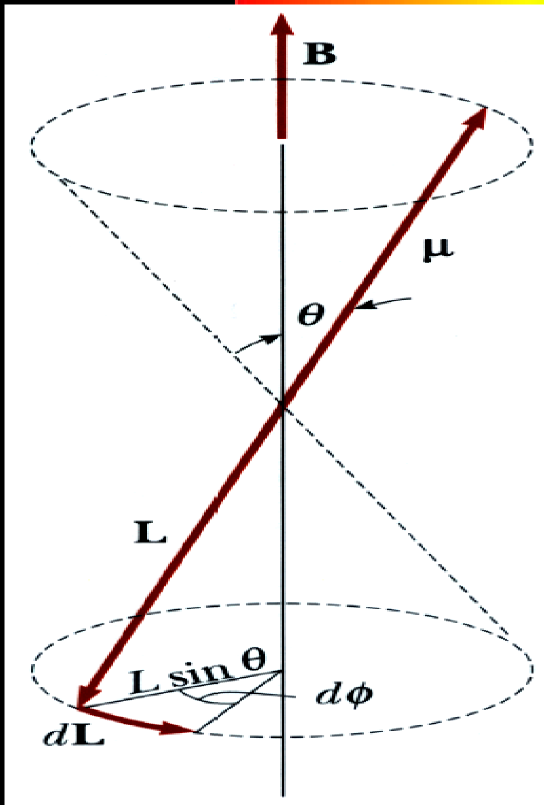
Bar Magnet Model of Magnetic Moment

In external B field, magnet experiences a torque which tends to align it with the field direction

If the magnet is spinning, torque causes magnet to precess around the ext. B field with a constant frequency: Larmor frequency



"Lifting" Degeneracy : Magnetic Moment in External B Field



Apply an External $\vec{B}$ field on a Hydrogen atom (viewed as a dipole)

Consider $\vec{B} \parallel \vec{Z}$ axis (could be any other direction too)

The dipole moment of the Hydrogen atom (due to electron orbit)

experiences a Torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ which does work to align $\vec{\mu} \parallel \vec{B}$

but this can not be (same Uncertainty principle argument)

$\Rightarrow$ So, Instead, $\vec{\mu}$ precesses (dances) around $\vec{B}$... like a spinning top

The Azimuthal angle ϕ changes with time : calculate frequency

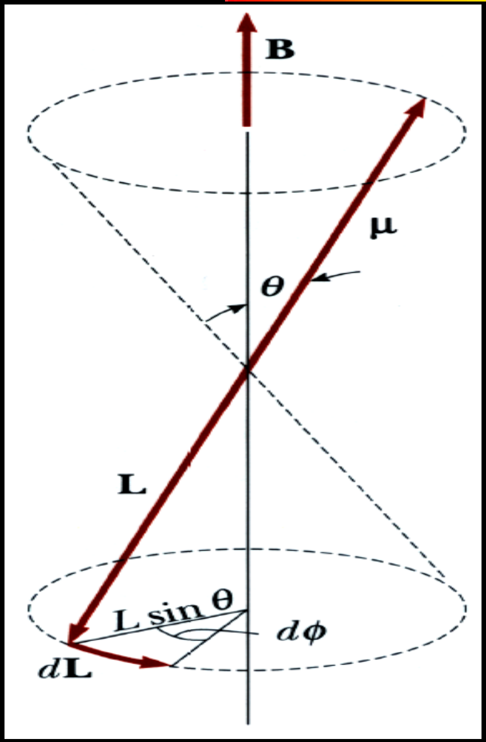
Look at Geometry: |projection along x-y plane : $|dL| = L\sin\theta . d\phi$

$\Rightarrow d\phi = \dfrac{|dL|}{L\sin\theta}$; Change in Ang Mom. $|dL| = |\tau|\, dt = \left|\dfrac{q}{2m} LB\sin\theta\right| dt$

$$\Rightarrow \boxed{\omega_L = \frac{d\phi}{dt} = \frac{1}{L\sin\theta}\frac{|dL|}{dt} = \frac{1}{L\sin\theta}\frac{q}{2m} LB\sin\theta = \frac{qB}{2m_e}}$$ Larmor Freq

ω_L depends on B, the applied external magnetic field

"Lifting" Degeneracy : Magnetic Moment in External B Field



WORK done to reorient $\vec{\mu}$ against $\vec{B}$ field: $\boxed{dW=\tau d\theta=-\mu B\sin\theta d\theta}$

$dW = d(\mu B\cos\theta)$: This work is stored as orientational Pot. Energy U

dW= -dU

Define Magnetic Potential Energy $U=-\vec{\mu}.\vec{B} = -\mu\cos\theta.B = -\mu_z B$

Change in Potential Energy $U = \frac{e\hbar}{2m_e} m_l B = \boxed{\hbar\omega_L m_l}$

Zeeman Effect in Hydrogen Atom

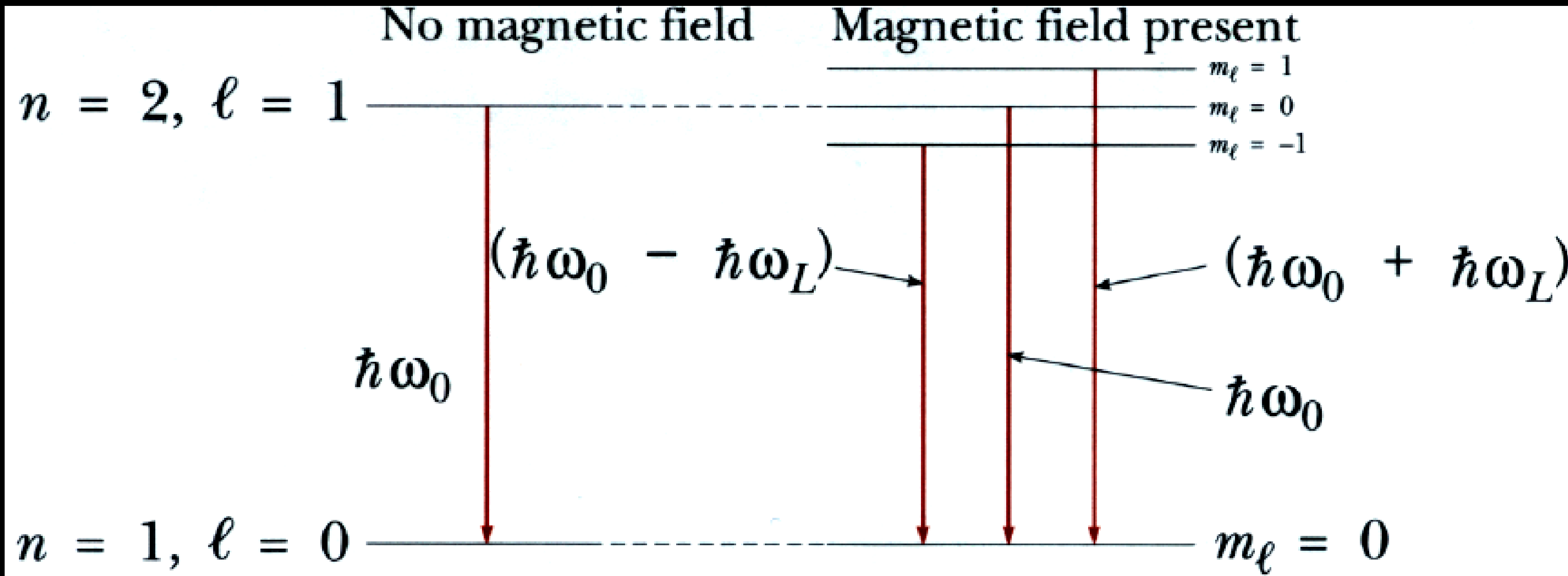
In presence of External B Field, Total energy of H atom changes to

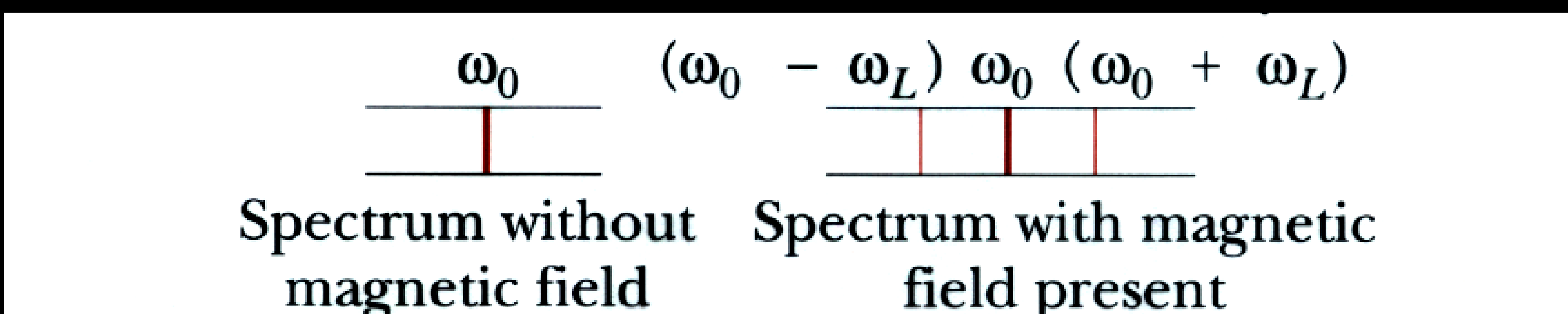
$$\boxed{E=E_0 + \hbar\omega_L m_l}$$

So the Ext. B field can break the E degeneracy "organically" inherent in the H atom. The Energy now depends not just on *n* but also m_l

Zeeman Effect Due to Presence of External B field

Energy Degeneracy Is Broken





Electron has "Spin": An additional degree of freedom

Electron possesses additional "hidden" degree of freedom : "Spinning around itself" !

Spin Quantum # $\boxed{s = \frac{1}{2}}$ (either Up or Down)

How do we know this ? $\Rightarrow$ Stern-Gerlach expt

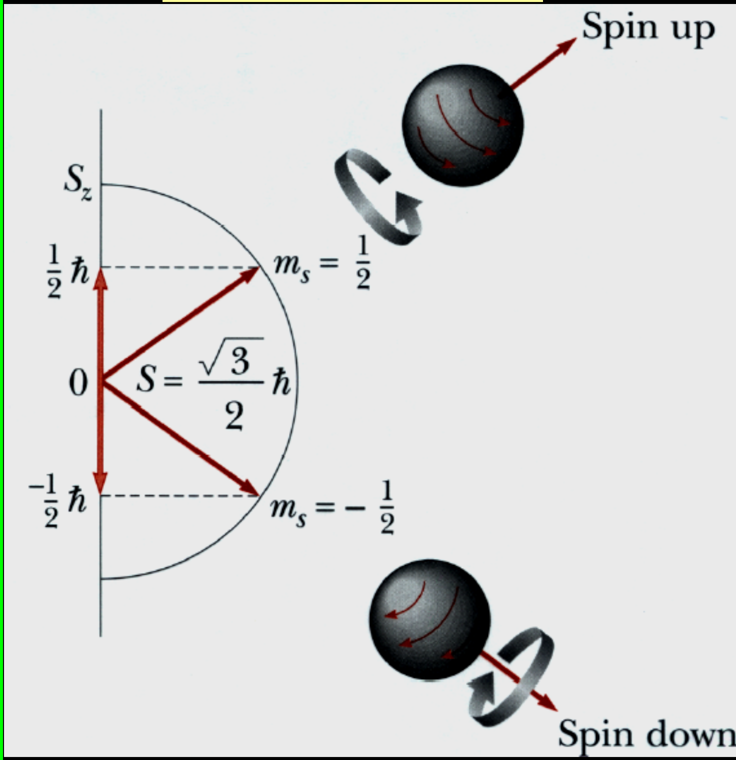
Spin Vector $\vec{S}$ (a form of angular momentum) is also Quantized

$$\boxed{|\vec{S}| = \sqrt{s(s+1)}\ \hbar = \sqrt{\frac{3}{2}}\ \hbar}$$

& $\boxed{S_z = m_s\hbar;\ m_s = \pm\frac{1}{2}}$

Spinning electron is an entitity defying any simple classical description. Dont try to visualize it (e.g see HW problem 7)...hidden D.O.F

$$|\vec{S}| = \sqrt{s(s+1)}\ \hbar$$



Stern-Gerlach Expt ⇒ An additional degree of freedom: "Spin" for lack of a better name

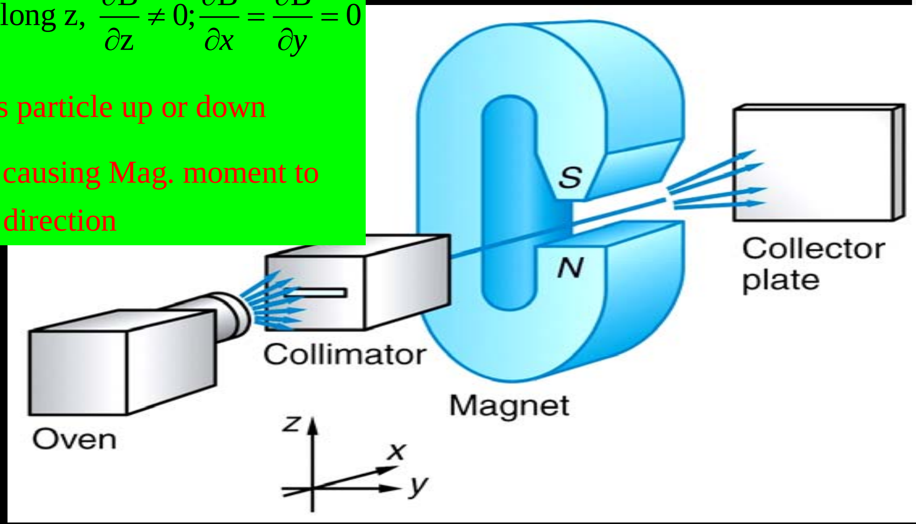
$\vec{\mu}$ in inhomogenous $\vec{B}$ field, experiences force $\vec{F}$

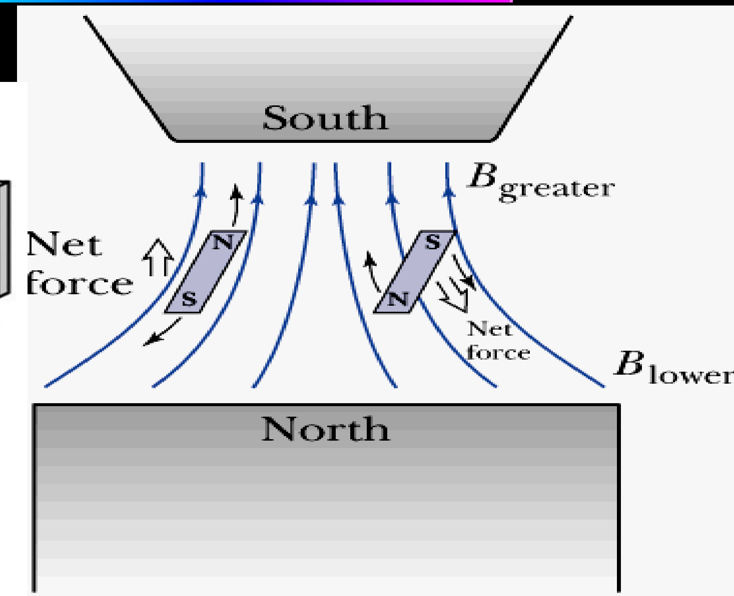
$\vec{F} = -\nabla U_B = -\nabla(-\vec{\mu}.\vec{B})$

When gradient only along z, $\frac{\partial B}{\partial z} \neq 0; \frac{\partial B}{\partial x} = \frac{\partial B}{\partial y} = 0$

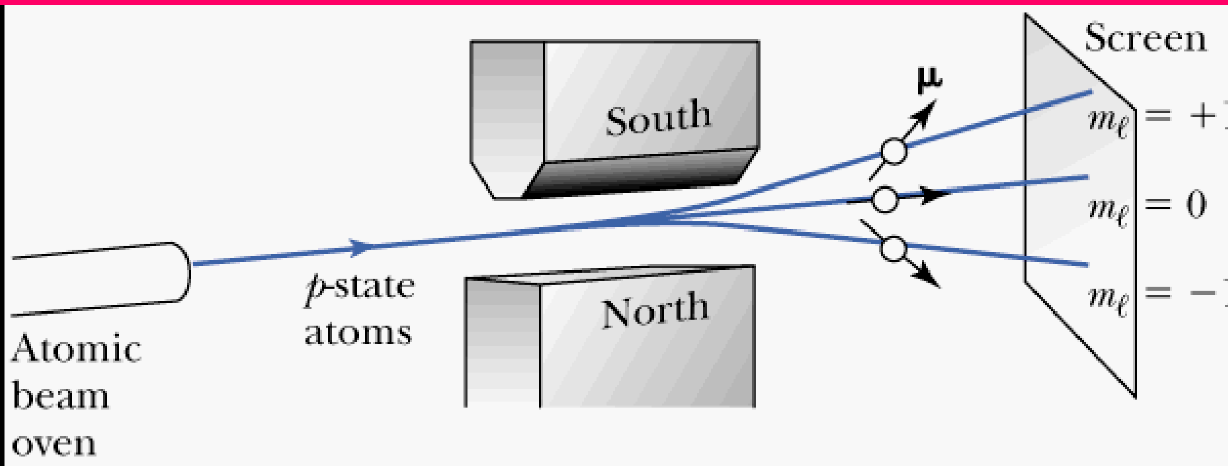
$F_z = m\mu_B(\frac{\partial B}{\partial z})$ moves particle up or down

(in addition to torque causing Mag. moment to precess about B field direction





In an inhomogeneous field, magnetic moment μ experiences a force F_z whose direction depends on component of the net magnetic moment & inhomogeneity dB/dz. Quantization means expect *(2l+1)* deflections. For l=0, expect all electrons to arrive on the screen at the center (no deflection)



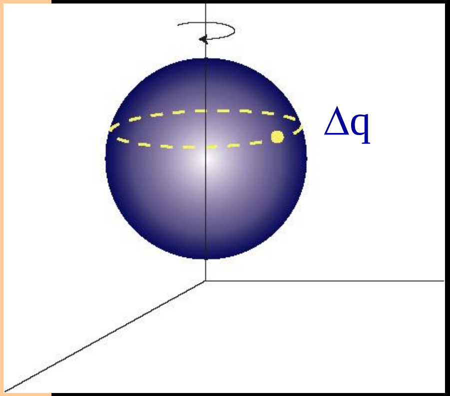
Silver

Hydrogen (l=0)

!

Four (not 3) Numbers Describe Hydrogen Atom → n, l, m_l, m_s

"Spinning" charge gives rise to a dipole moment $\vec{\mu}_s$:



Imagine (semi-clasically, incorrectly!) electron as sphere: charge q, radius r

Total charge uniformly distributed: $q=\sum_i \Delta q_i$;

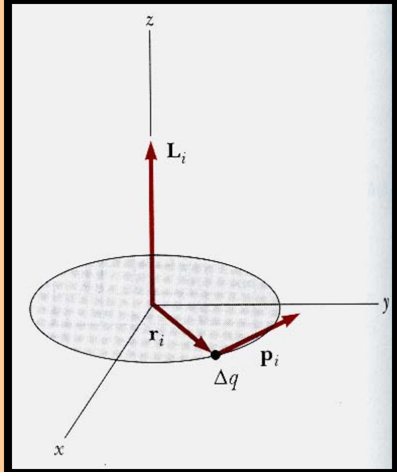
as electron spins, each "chargelet" rotates ⇒ current ⇒ dipole moment $\vec{\mu}_{s_i}$

$$\vec{\mu}_s = \left(\frac{q}{2m_e}\right)\sum_i \vec{\mu}_{s_i} = g\left(\frac{q}{2m_e}\right)\vec{S}$$

In a Magnetic Field $\vec{B}$ ⇒ magnetic energy due to spin $U_S = \vec{\mu}_s . \vec{B}$

Net Angular Momentum in H Atom $\vec{J} = \vec{L} + \vec{S}$

Net Magnetic Moment of H atom: $\vec{\mu} = \vec{\mu}_0 + \vec{\mu}_s = \left(\frac{-e}{2m_e}\right)(\vec{L} + g\vec{S})$

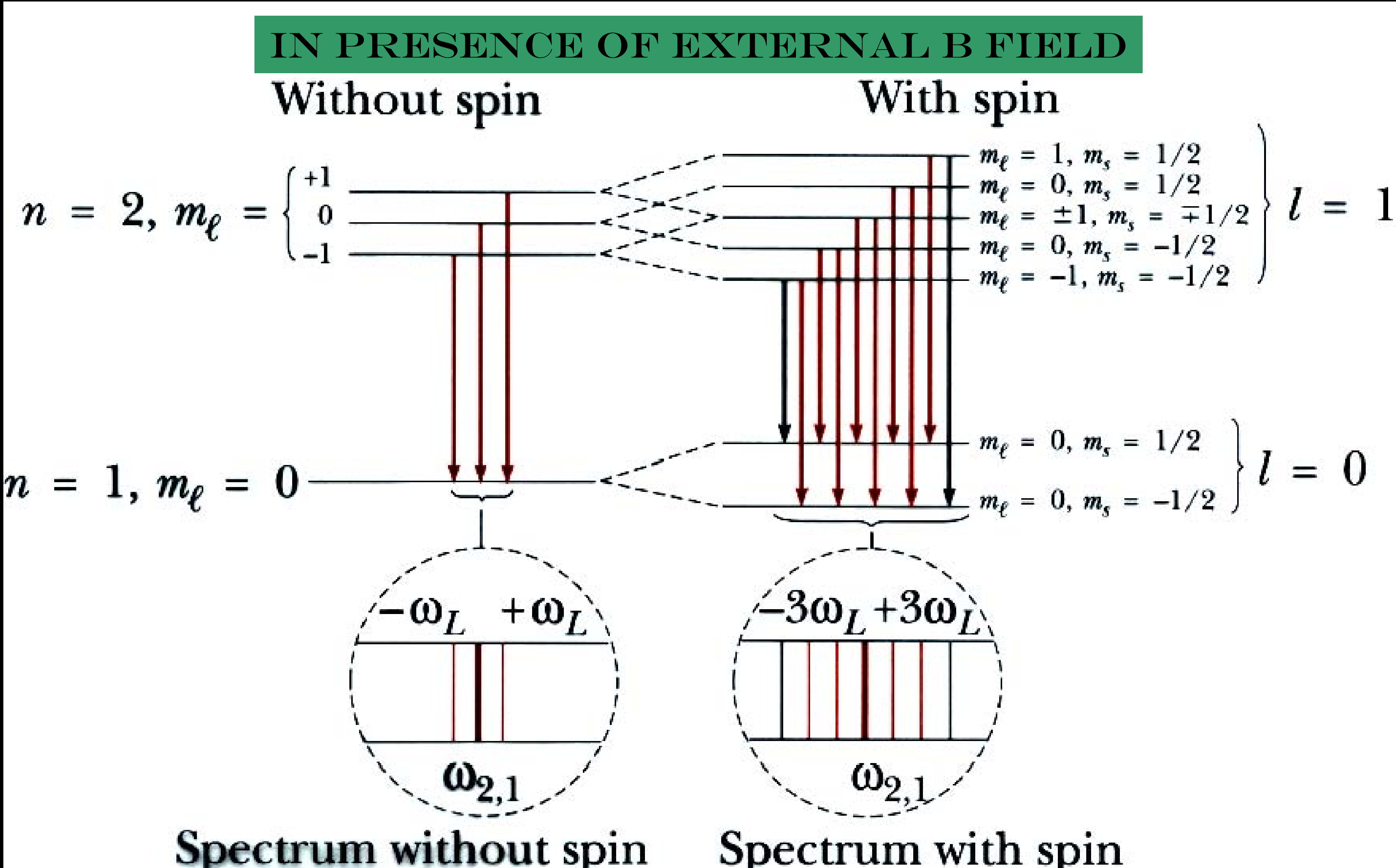


Notice that the net dipole moment vector $\vec{\mu}$ is not || to $\vec{J}$

(There are many such "ubiquitous" quantum numbers for elementary particles!)

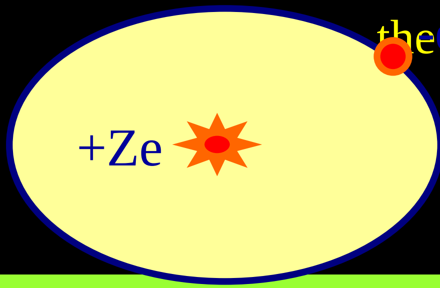
Doubling of Energy Levels Due to Spin Quantum Number

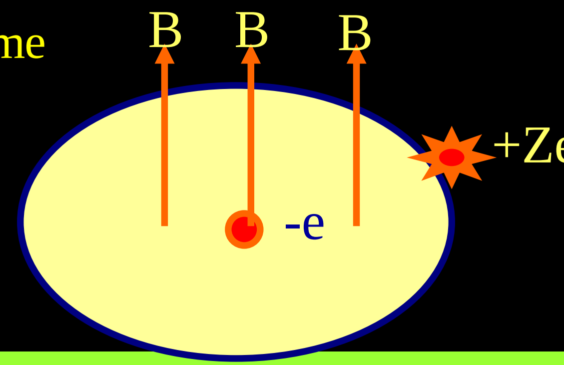
Under Intense B field, each {n , m_l } energy level splits into two depending on spin up or down



Spin-Orbit Interaction: Angular Momenta are Linked Magnetically

Electron revolving around Nucleus finds itself in a "internal" B field because in its ref. frame the nucleus is orbiting around it

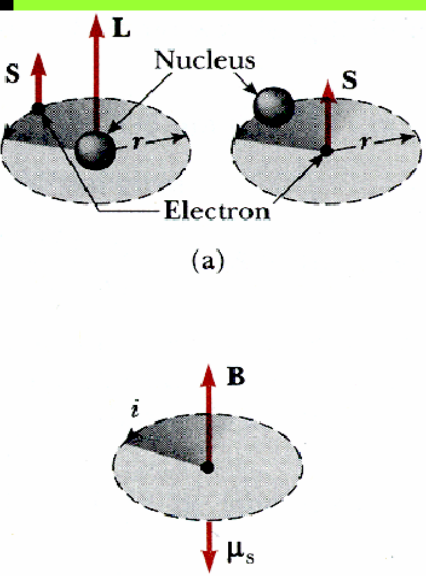




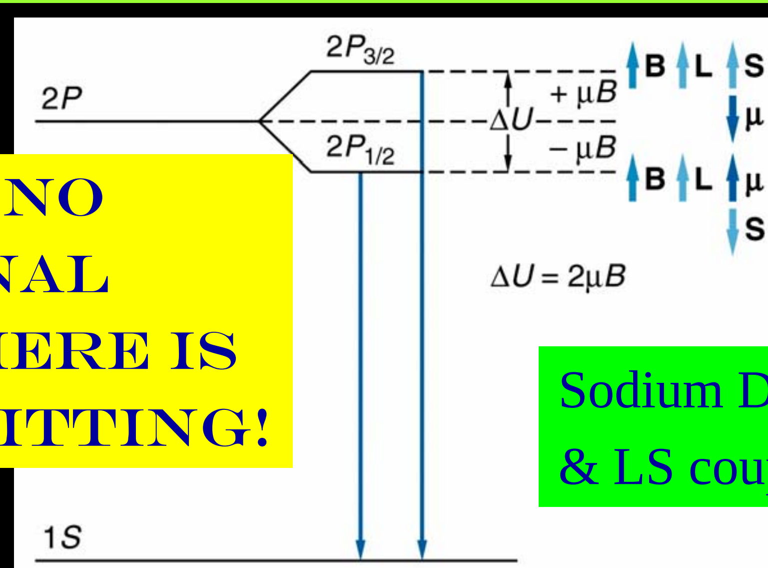
This B field, due to orbital motion, interacts with electron's spin dipole moment $\vec{\mu}_s$

$U_m = -\vec{\mu}.\vec{B} \Rightarrow$ Energy larger when $\vec{S} \parallel \vec{B}$, smaller when anti-parallel

$\Rightarrow$ States with same (n, l, m_l) but diff. spins $\Rightarrow$ energy level splitting/doubling due to $\vec{S}$



UNDER NO EXTERNAL B FIELD THERE IS STILL A SPLITTING!



Sodium Doublet & LS coupling

Vector Model For Total Angular Momentum J

Coupling of Orbital & Spin magnetic moments $\Rightarrow$

Neither Orbital nor Spin angular Momentum are conserved seperately!

$\vec{J} = \vec{L} + \vec{S}$ is conserved so long as there are no external torques present

Rules for Total Angular Momentum Quantization :

$|J| = \sqrt{j(j+1)}\,\hbar$ with $j = |l+s|,\ l+s-1,\ l+s-2 \ldots\ldots, \ldots.,|l-s|$

$J_z = m_j \hbar$ with $m_j = j, j-1, j-2 \ldots\ldots, -j$

Example: state with $(l = 1, s = \frac{1}{2})$

$j = 3/2 \Rightarrow m_j = -3/2, -1/2, 1/2, 3/2$

$j = 1/2 \Rightarrow m_j = \pm 1/2$

In general m_j takes $(2j+1)$ values

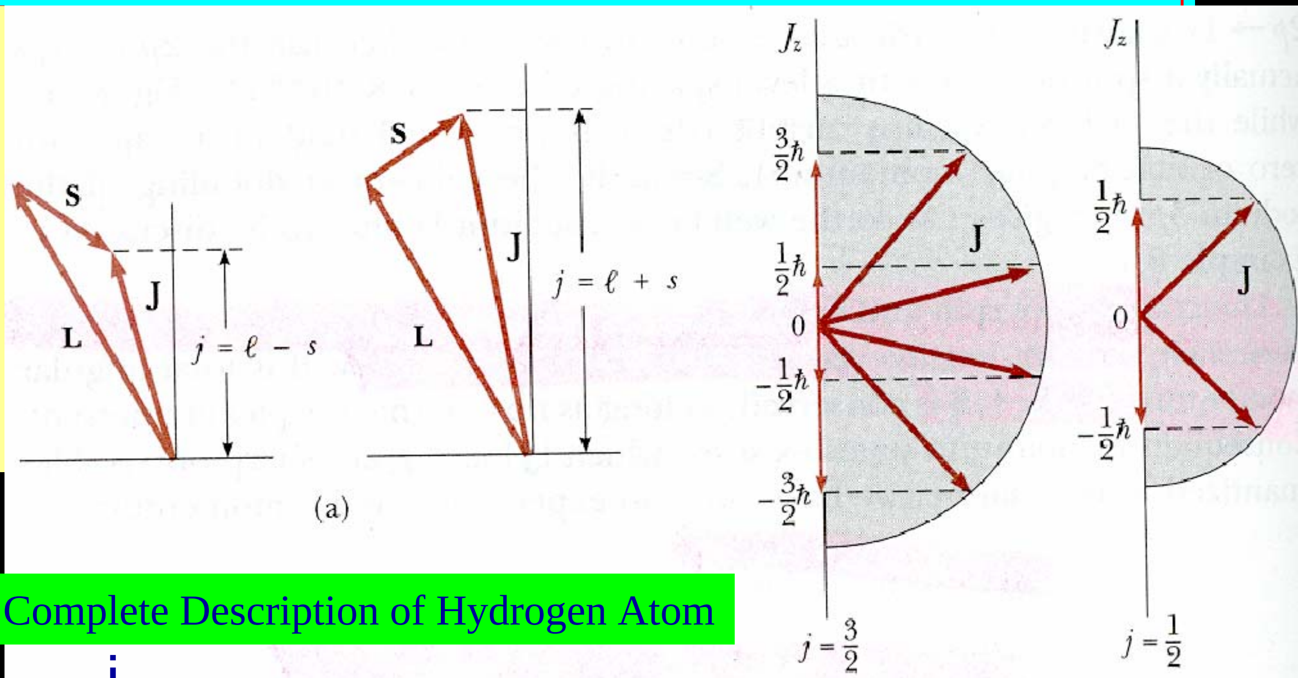
$\Rightarrow$ Even # of orientations

Spectrographic Notation: Final Label

n → $1S_{1/2}$

$2P_{3/2}$ ← j

Complete Description of Hydrogen Atom



Complete Description of Hydrogen Atom

Full description

of the Hydrogen atom:

$\{n, l, m_l, m_s\}$

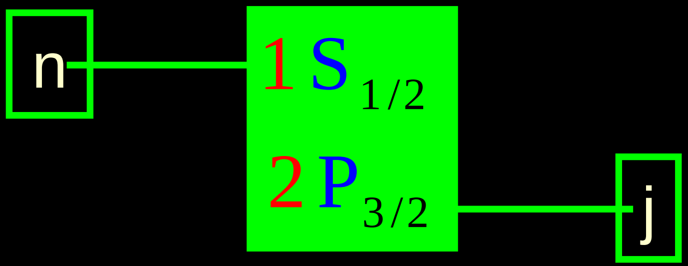
$\Downarrow$

LS Coupling

$\Downarrow$

$\{n, l, j, m_s\}$

corresponding to 4 D.O.F.



How to describe multi-electrons atoms like He, Li etc?

How to order the Periodic table?

- Four guiding principles:
  - Indistinguishable particle & Pauli Exclusion Principle
  - Independent particle model (ignore inter-electron repulsion)
  - Minimum Energy Principle for atom
  - Hund’s “rule” for order of filling vacant orbitals in an atom