4E : The Quantum Universe

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Transition Between States In Quantum Systems

In formulating the Hydrogen Atom, Bohr was obliged to postulate that the frequency of radiation emitted by an atom dropping from energy level E_m to a lower level E_n is:

$$\boxed{f = \frac{E_m - E_n}{h}}$$

This relationship rises naturally in Quantum Mechanics, consider for simplicity a system in which an electron only in the x direction:The time-dependent Wavefunction $\Psi_n(x,t)=\psi_n(x)e^{-i\frac{E_n}{\hbar}t}$;

$$<x>=\int_{-\infty}^{\infty} x \ \psi_n^* \psi_n dx = \text{ constant in time, does not oscillate, no radiation occurs}$$

But, due to an external perturbation lasting some time, electron shifts from one state (m) to another(n)

In this period wavefunction of electron is a linear superposition of two possible states

$\Psi = a\Psi_n + b\Psi_m$; a^*a= prob. of electron in state n and b^*b= prob. of electron in state m; $\boxed{a^*a+b^*b=1}$

Initially a=1,b=0 and finally a=0,b=1. While the electron is in either state there is no radiation but when it is in the midst of transition from $m \rightarrow n$, both a and b have non-vanishing values and radiation is produced.

Expectation value for compostive wavefunction $<x> = \int x\Psi^*\Psi dx$;

$$<x>=\int_{-\infty}^{\infty} x(a^2\Psi_n^*\Psi_n + b^*a\Psi_m^*\Psi_n + a^*b\Psi_n^*\Psi_m + b^2\Psi_m^*\Psi_m)dx$$

Transition Between States In Quantum Systems

$$\langle x\rangle = \int_{-\infty}^{\infty} x(a^2\Psi_n^*\Psi_n + b^* a\Psi_m^*\Psi_n + a^* b\Psi_n^*\Psi_m + b^2\Psi_m^*\Psi_m)\,dx$$

$$\langle x\rangle = a^2\int x\psi_n^*\psi_n dx + b^2\int x\psi_m^*\psi_m dx$$

$$+ ab^*\int x\psi_m^* e^{+i(E_m/\hbar)t}\psi_n e^{-i(E_n/\hbar)t} dx + a^* b\int x\psi_n^* e^{+i(E_n/\hbar)t}\psi_m e^{-i(E_m/\hbar)t} dx$$

Use $e^{i\theta} = \cos\theta + i\sin\theta$ and $e^{-i\theta} = \cos\theta - i\sin\theta$ in the above and consider just the REAL part of expression for the last two terms, it varies with time as

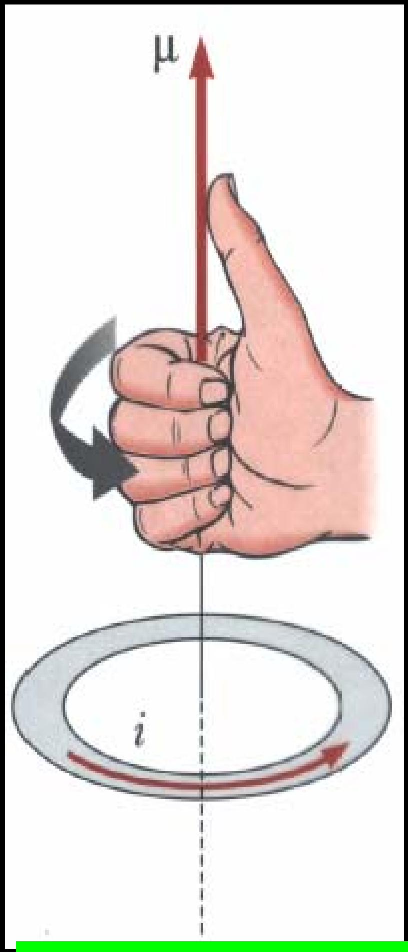
$$\cos\left(\frac{E_m - E_n}{\hbar}\right)t = \cos 2\pi\left(\frac{E_m - E_n}{h}\right)t = \cos 2\pi f t$$

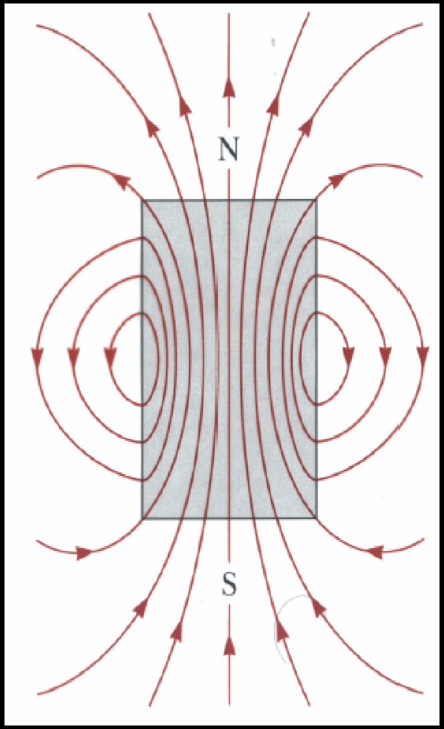
So the <x> of the electron oscillates with frequency f and one has a nice electric dipole analogy $\Rightarrow$ Hence radiative transtions !

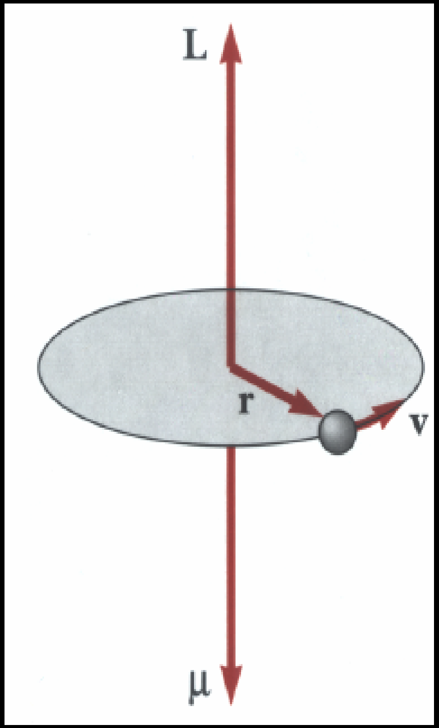
Similarly for particle in an infinite well or harmonic oscillator ...

What's So "Magnetic" about m_l ?

Precessing electron → Current in loop → Magnetic Dipole moment μ



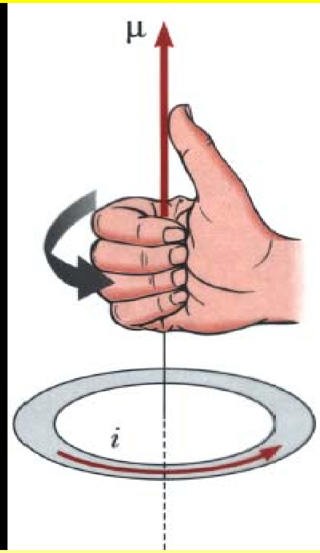


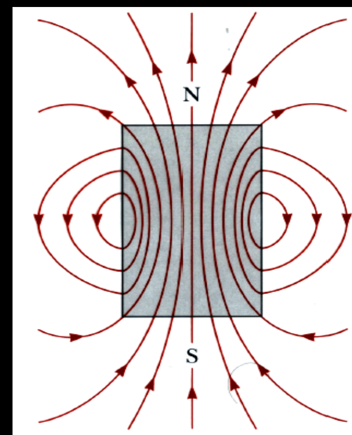


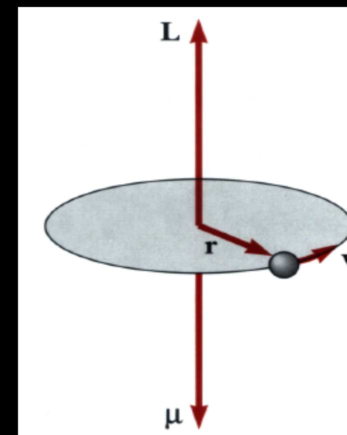
The electron's motion → hydrogen atom is a dipole magnet

The "Magnetism" of an Orbiting Electron

Precessing electron → Current in loop → Magnetic Dipole moment μ







Electron in motion around nucleus $\Rightarrow$ circulating charge $\Rightarrow$ curent i

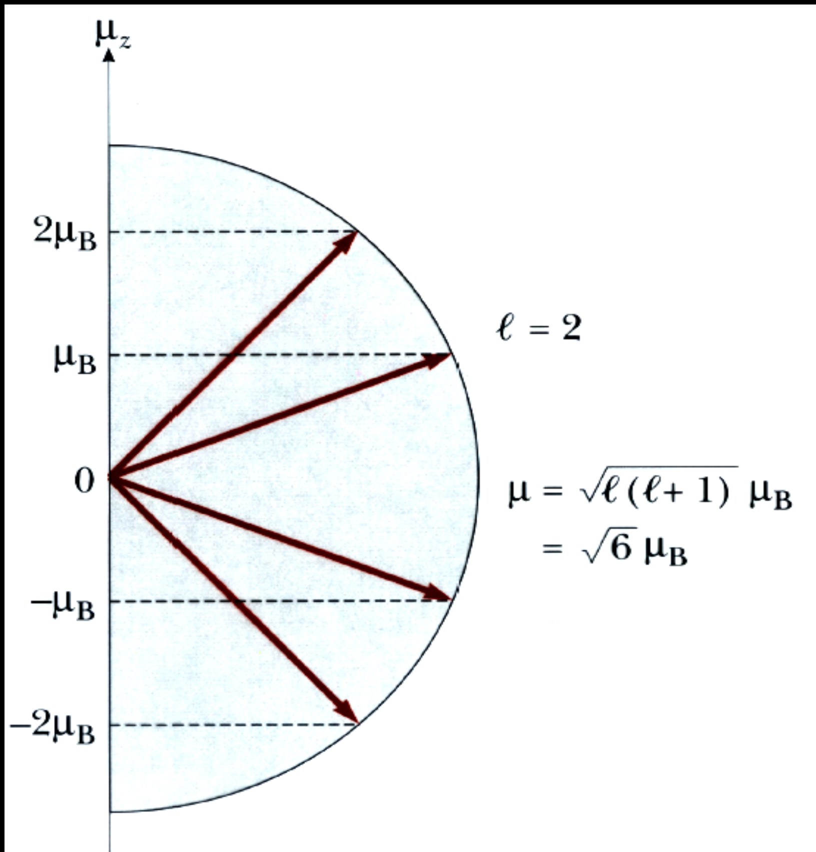
$$i = \frac{-e}{T} = \frac{-e}{\frac{2\pi r}{v}} = \frac{-ep}{2\pi m r};$$ Area of current loop $A = \pi r^2$

Magnetic Moment $|\mu| = iA = \left(\frac{-e}{2m}\right) rp;$ $$\vec{\mu} = \left(\frac{-e}{2m}\right)\vec{r}\times\vec{p} = \left(\frac{-e}{2m}\right)\vec{L}$$

Like the $\vec{L}$, magnetic moment $\vec{\mu}$ also precesses about "z" axis

$$\text{z component, } \mu_z = \left(\frac{-e}{2m}\right)L_z = \left(\frac{-e\hbar}{2m}\right)m_l = -\mu_B m_l = \textit{quantized}\,!$$

Quantized Magnetic Moment



$$\mu_z = \left(\frac{-e}{2m}\right) L_z = \left(\frac{-e\hbar}{2m}\right) m_l$$

$$= -\mu_B m_l$$

$$\mu_B = \text{Bohr Magnetron}$$

$$= \left(\frac{e\hbar}{2m_e}\right)$$

Why all this ? Need to find a way to break the Energy Degeneracy & get electron in each (n, l, m_l) state to identify itself, so we can "talk" to it and make it do our bidding:

" Walk this way, talk this way!"