4E : The Quantum Universe

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Ch 3 : Quantum Theory Of Light

- What is the nature of light ?
  - When it propagates ?
  - When it interacts with Matter?
- What is Nature of Matter ?
  - When it interacts with light ?
  - As it propagates ?

- Revolution in Scientific Thought
  - A firestorm of new ideas (NOT steady dragged out progress)
    - Old concepts violently demolished , new ideas born
      - Rich interplay of experimental findings & scientific reason
- One such revolution happened at the turn of 20^{th} Century
  - Led to the birth of Quantum Theory & Modern Physics

Classical Picture of Light : Maxwell's Equations

Maxwell's Equations:

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$\oint_S \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I + \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

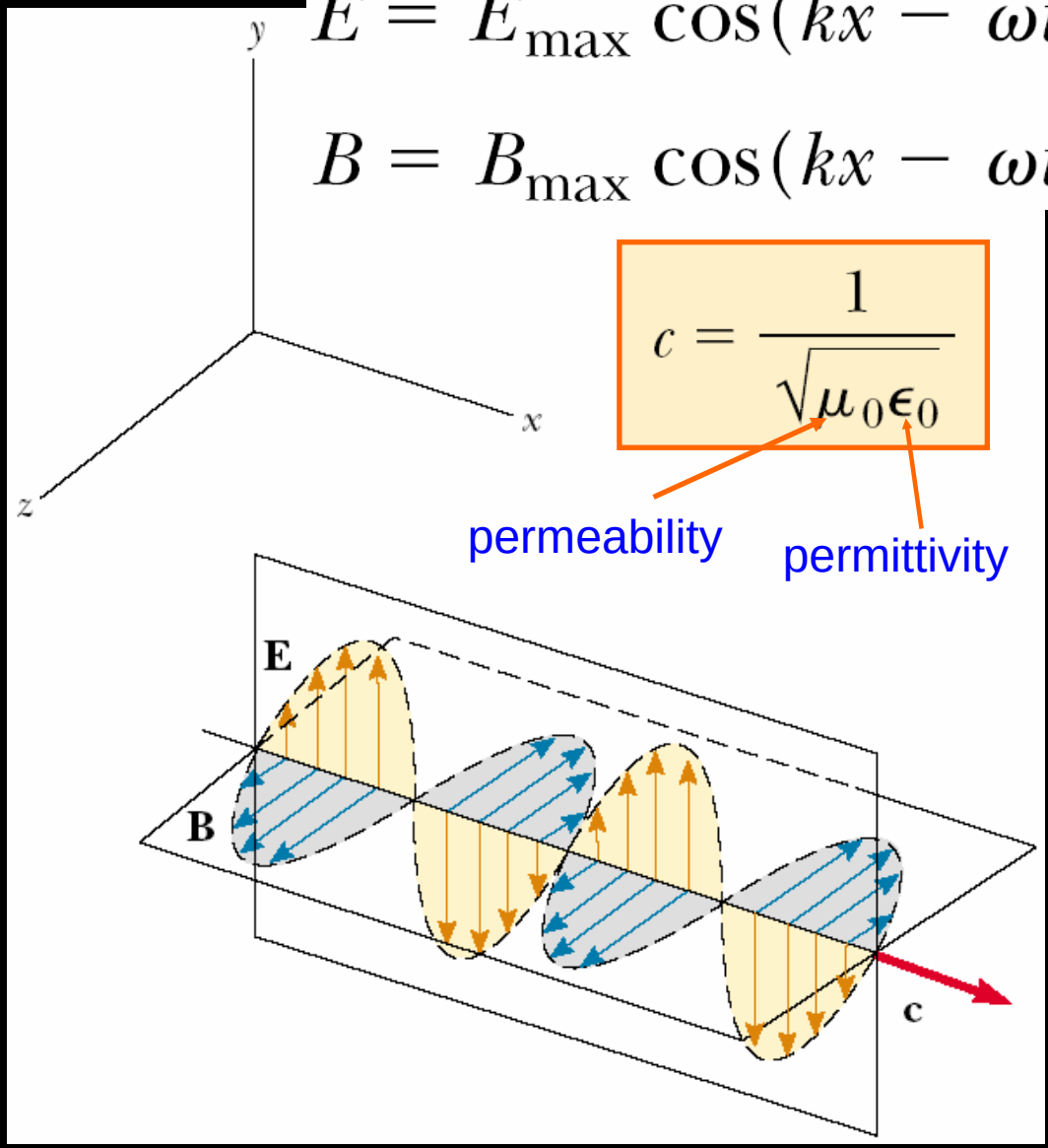
$$E = E_{\max}\cos(kx - \omega t)$$

$$B = B_{\max}\cos(kx - \omega t)$$

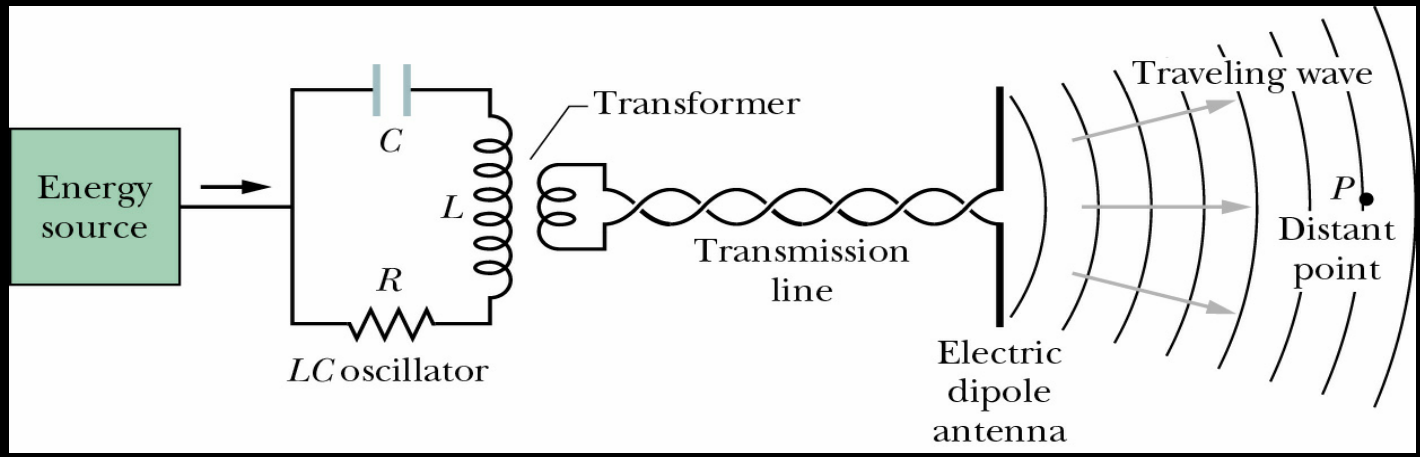
$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

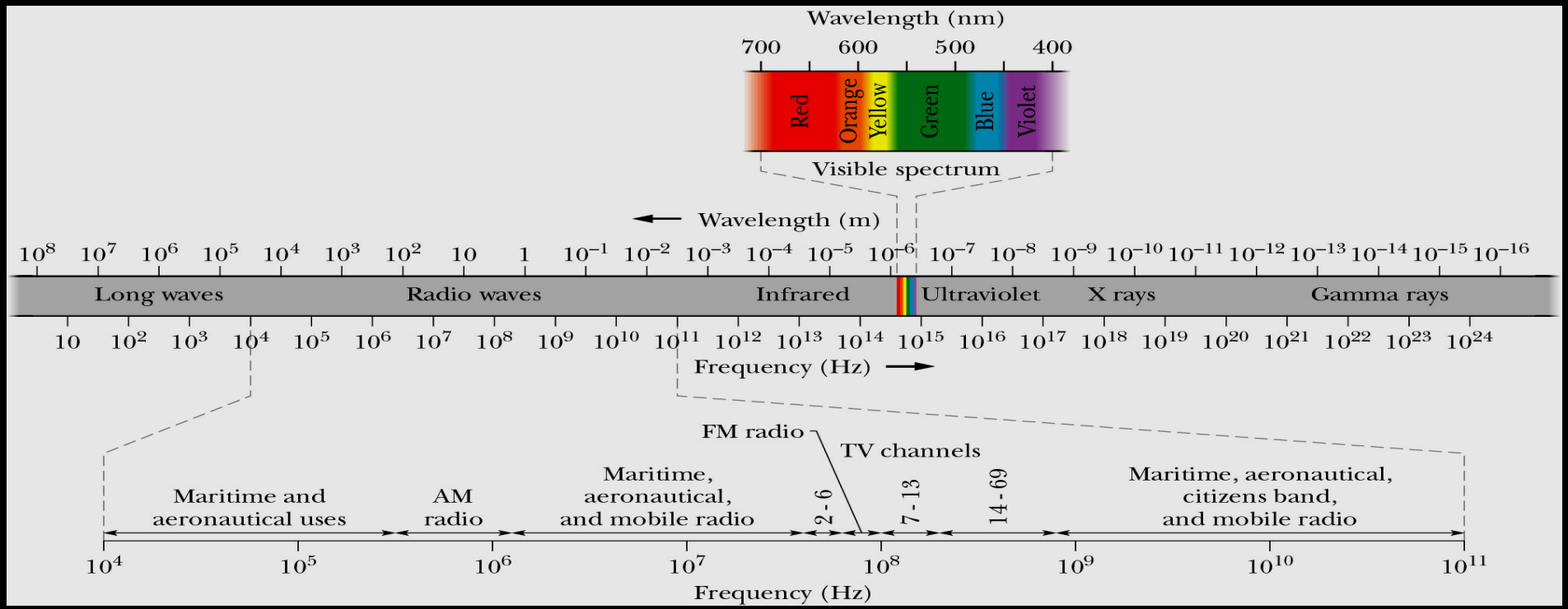
permeability

permittivity

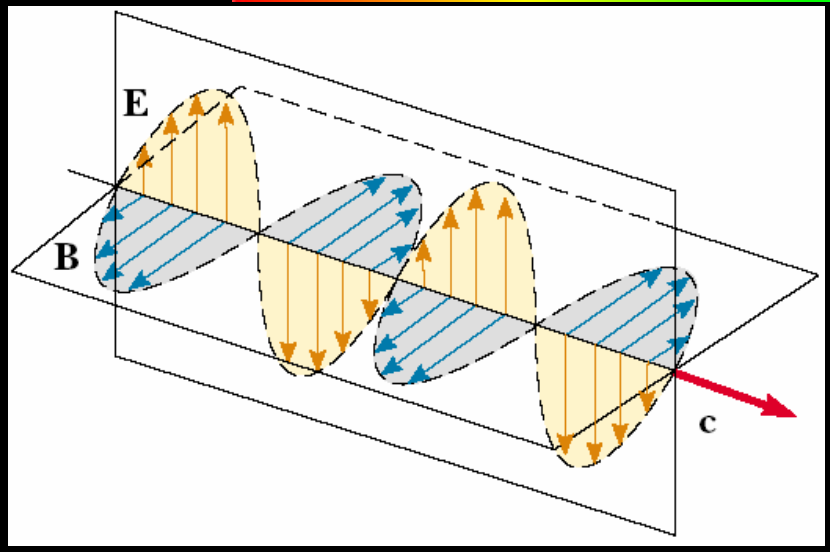


Hertz & Experimental Demonstration of Light as EM Wave





Properties of EM Waves: Maxwell's Equations



Energy Flow in EM Waves

Poynting Vector $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$

Power incident on an area A $= \vec{S}.\vec{A} = \frac{1}{\mu_0}\left(AE_0 B_0 Sin^2(kx - \omega t) \right)$

Intensity of Radiation $I = \frac{1}{2\mu_0 c} E_0^2$

Larger the amplitude of Oscillation
More intense is the radiation

Disasters in Classical Physics (~1899-1922)

Disaster ➔ Experimental observation that could not be explained by Classical theory

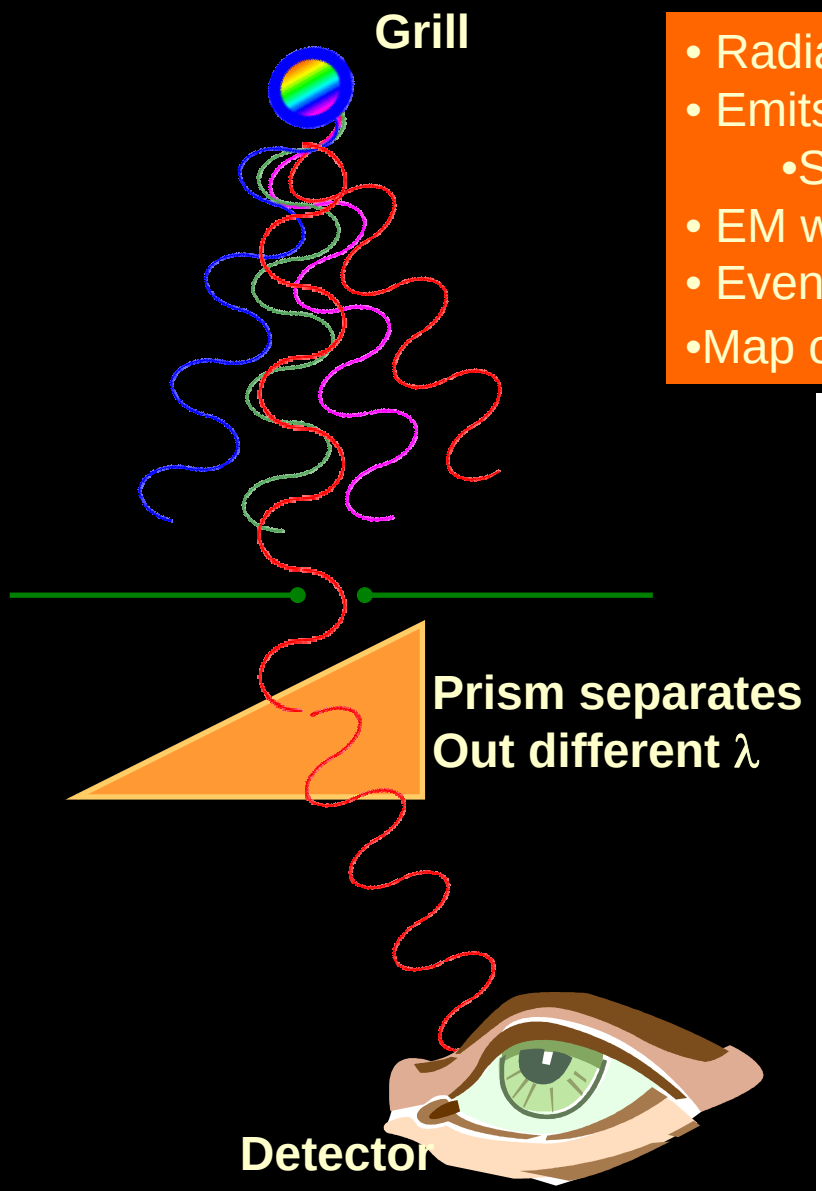
- Disaster # 1 : Nature of Blackbody Radiation from your BBQ grill
- Disaster # 2: Photo Electric Effect
- Disaster # 3: Scattering light off electrons (Compton Effect)

Resolution of Experimental Observation will require radical changes in how we think about nature

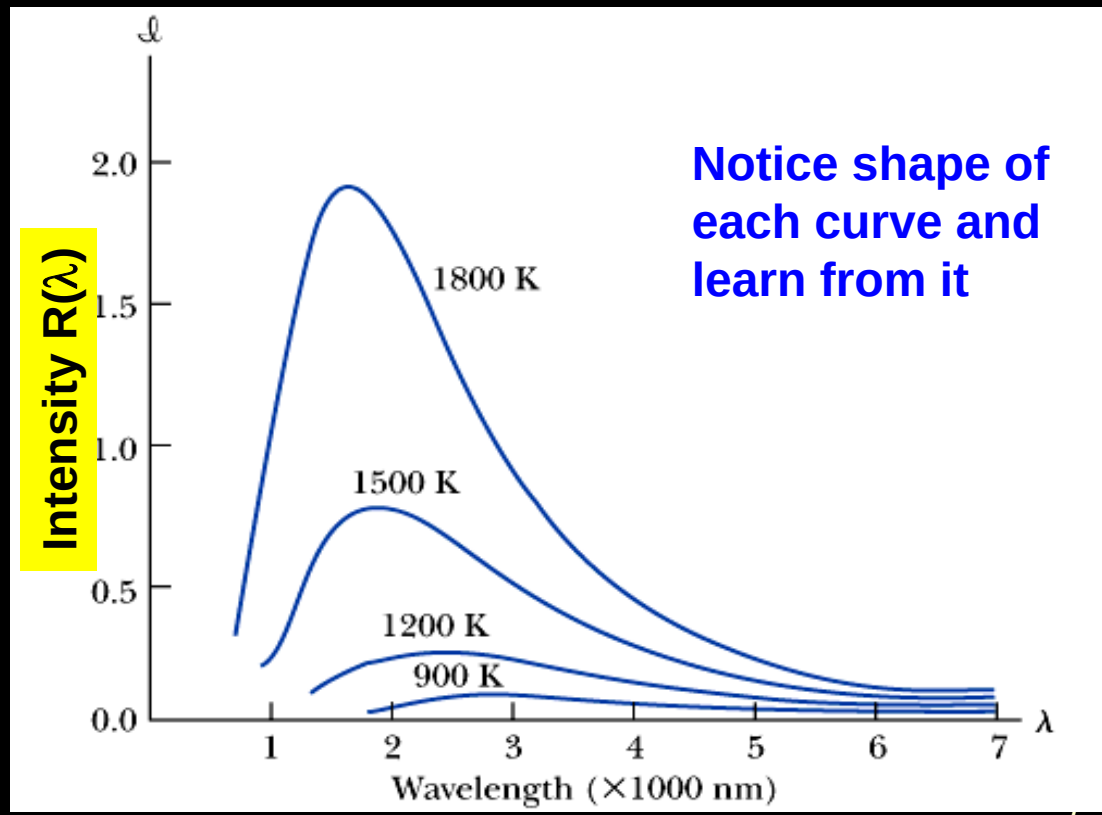
- ➔ QUANTUM PHYSICS: The Art of Conversation with Subatomic Particles

Nature of Radiation: An Expt with BBQ Grill

Question : Distribution of Intensity of EM radiation Vs T & λ



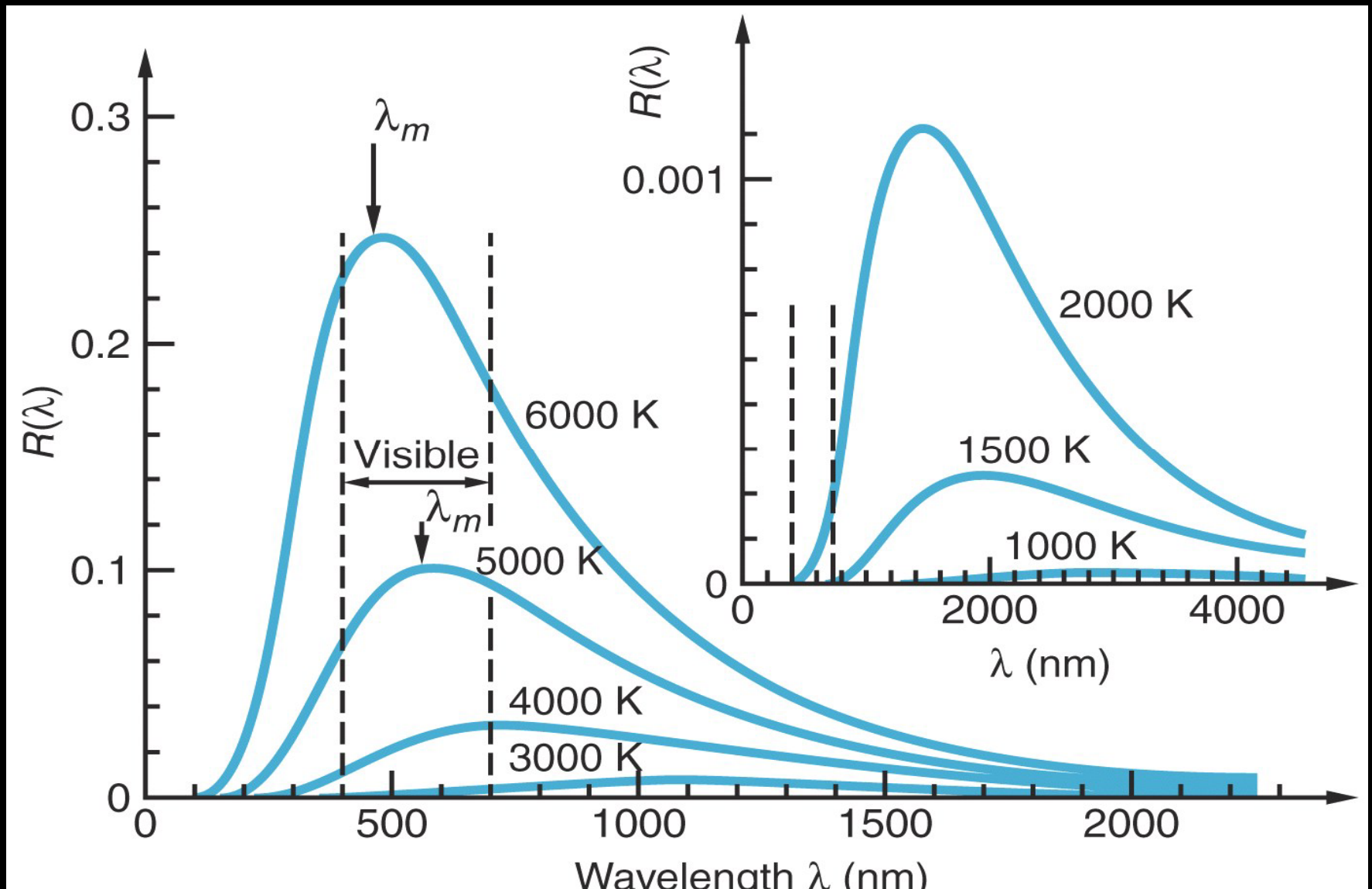
- Radiator (BBQ grill) at some temp T
- Emits variety of wavelengths
  - Some with more intensity than others
- EM waves of diff. λ bend differently within prism
- Eventually recorded by a detector (eye)
- Map out emitted Power / area Vs λ



Notice shape of each curve and learn from it

Radiation From a Blackbody at Different Temperatures

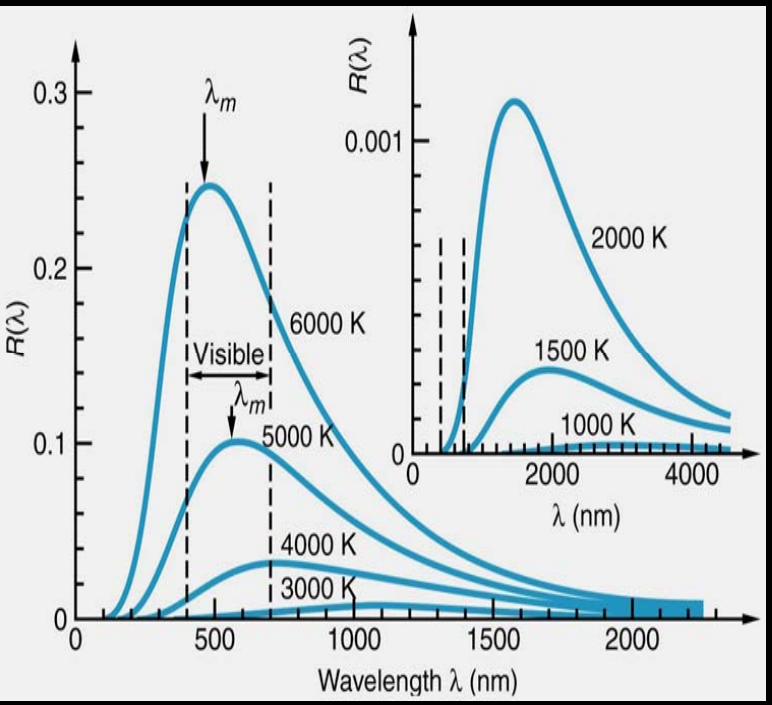
Radiancy is Radiation intensity per unit λ



(a) Intensity of Radiation $I = \int R(\lambda) d\lambda \propto T^4$

$I = \sigma T^4$ (Area under curve)

Stephan-Boltzmann Constant $\sigma = 5.67\ 10^{-8}\ W / m^2 K^4$



(b) Higher the temperature of BBQ
Lower is the λ of PEAK intensity

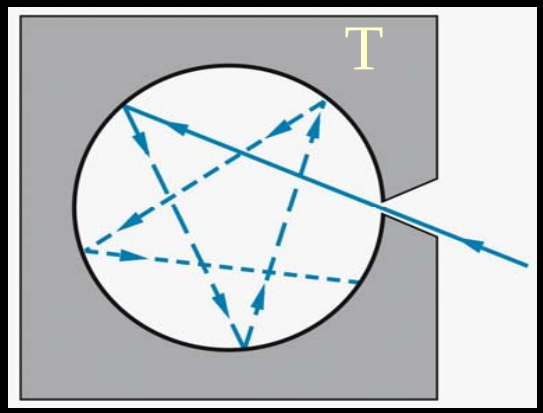
$I_{MAX} \propto 1 / T$

$\lambda_{MAX}\ T = const$
$= 2.898\ 10^{-3}\ mK$

As a body gets hotter it gets more RED then White : Wein's Law

Reason for different shape of R(λ) Vs λ for different temperature?
Can one explain in on basis of Classical Physics ??

Blackbody Radiator: An Idealization



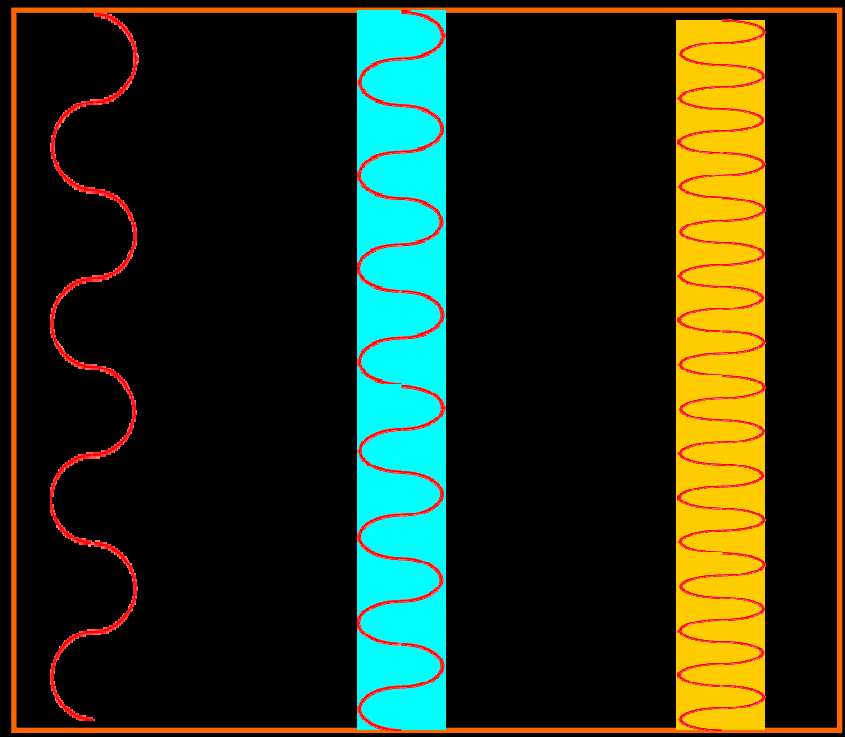
Classical Thought:
- **Box is filled with EM standing waves**
- **Radiation reflected back-and-forth between walls**
- **Radiation in thermal equilibrium with walls of Box**
- **How may waves of wavelength λ can fit inside the box ?**

Blackbody Absorbs everything
Reflects nothing
All light entering opening gets absorbed (ultimately) by the cavity wall

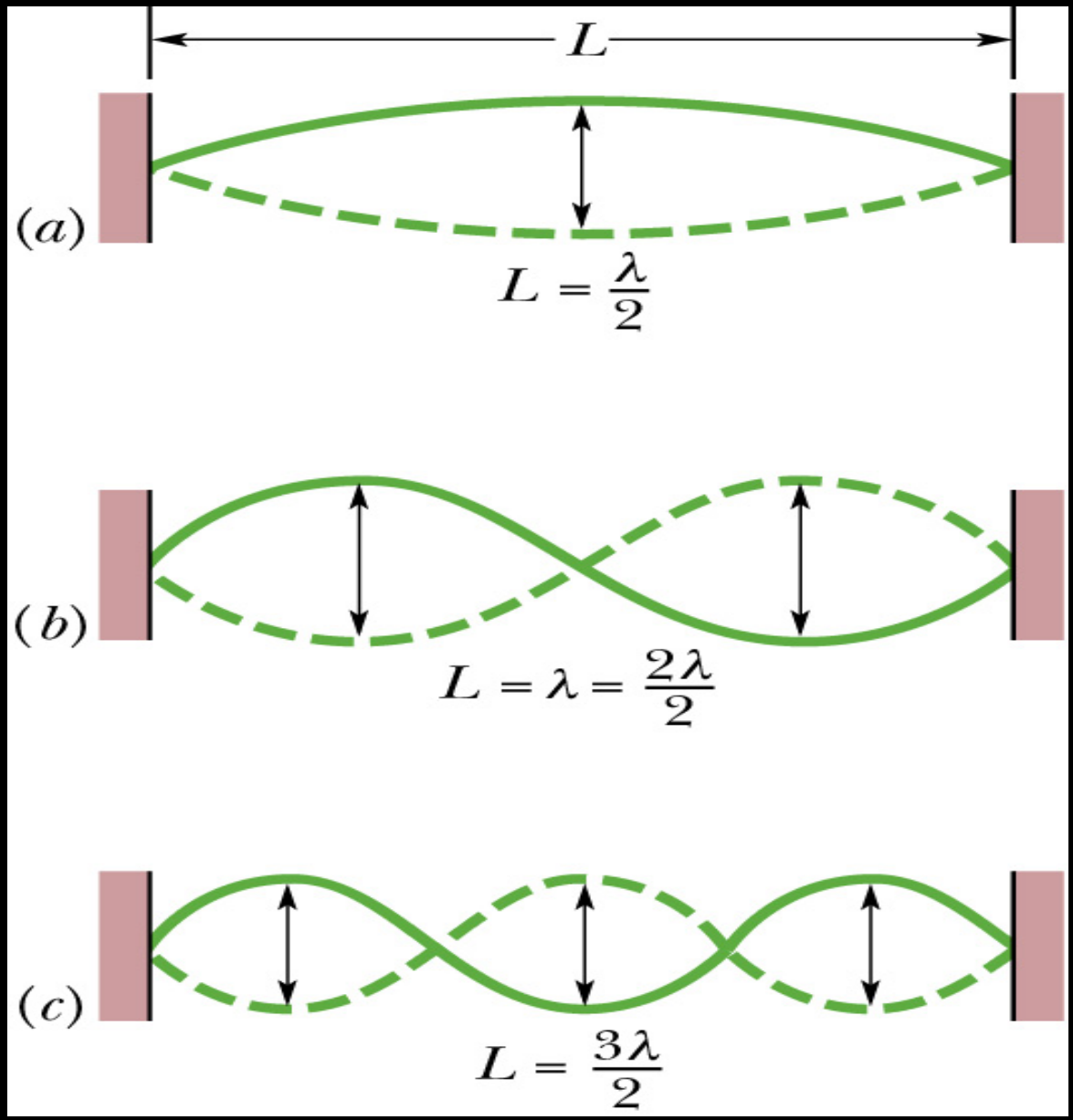
Cavity in equilibrium T w.r.t. surrounding. So it radiates everything It absorbs

Emerging radiation is a sample of radiation inside box at temp T

Predict nature of radiation inside Box ?



Standing Waves



Classical Calculation

of standing waves between Wavelengths λ and $\lambda+d\lambda$ are

$$N(\lambda)d\lambda = \frac{8\pi V}{\lambda^4} \bullet d\lambda \; ; \; V = \text{Volume of box} = L^3$$

Each standing wave contributes energy $E = kT$ to radiation in Box

Energy density $u(\lambda)$ = [# of standing waves/volume] × Energy/Standing Wave

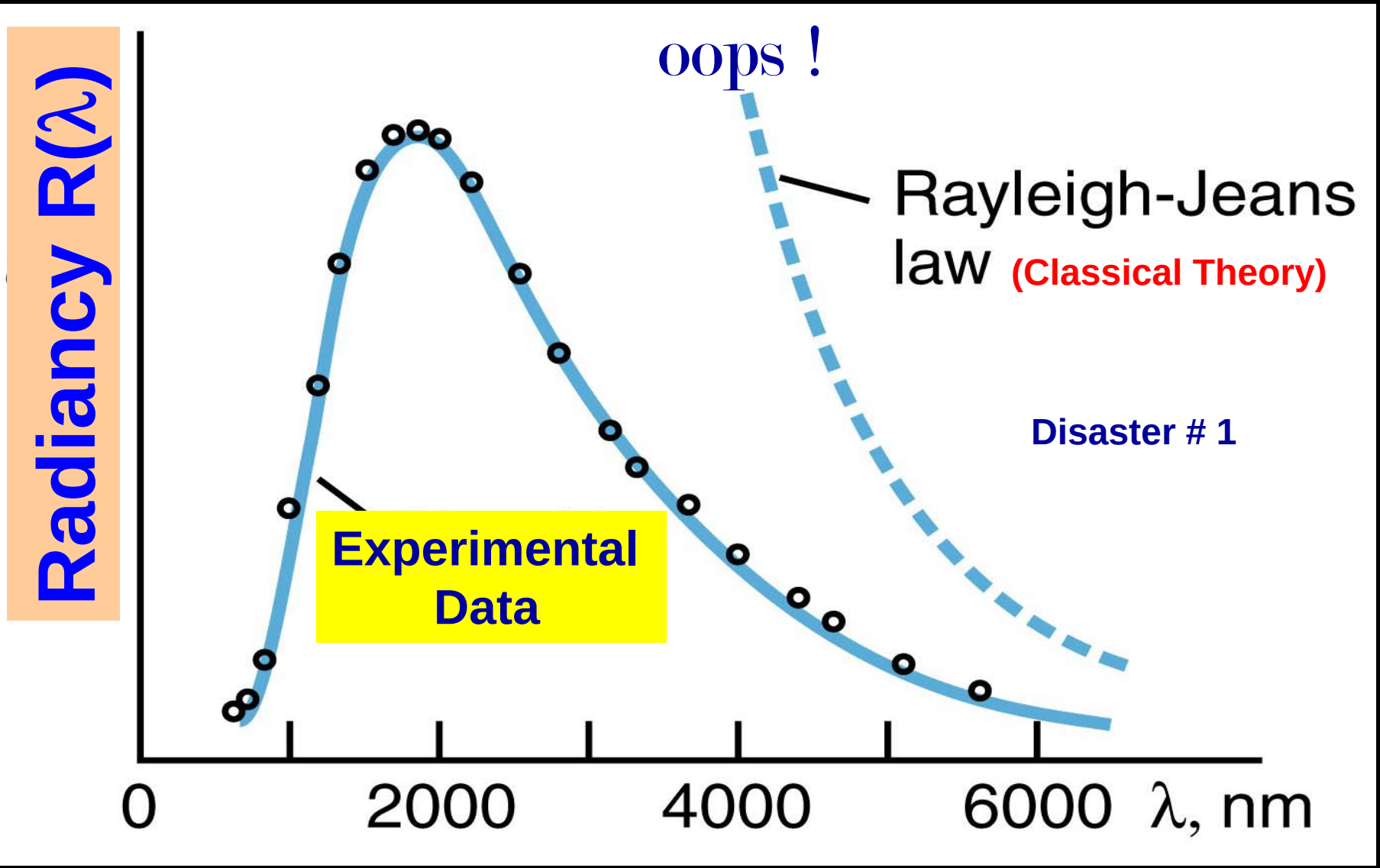
$$= \frac{8\pi V}{\lambda^4} \times \frac{1}{V} \times kT = \frac{8\pi}{\lambda^4} kT$$

$$\text{Radiancy } R(\lambda) = \frac{c}{4} u(\lambda) = \frac{c}{4}\frac{8\pi}{\lambda^4} kT = \frac{2\pi c}{\lambda^4} kT$$

Radiancy is Radiation intensity per unit λ interval: Lets plot it

Prediction : as $\lambda \rightarrow 0$ (high frequency f), $R(\lambda) \rightarrow$ Infinity !
Oops !

Ultra Violet (Frequency) Catastrophe

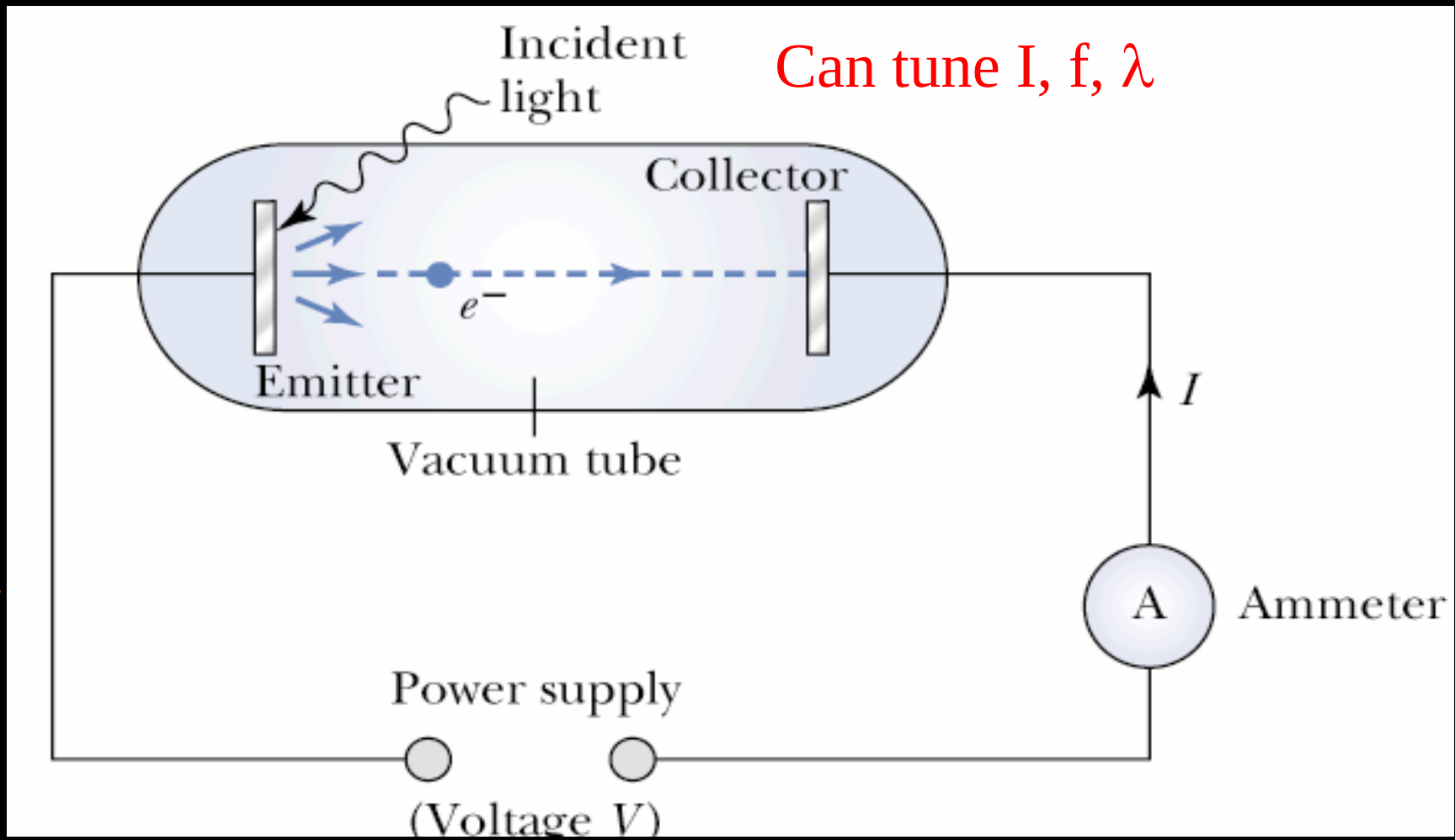


That was a Disaster !

(#1)

Disaster # 2 : Photo-Electric Effect

Light of intensity I, wavelength λ and frequency ν incident on a photo-cathode

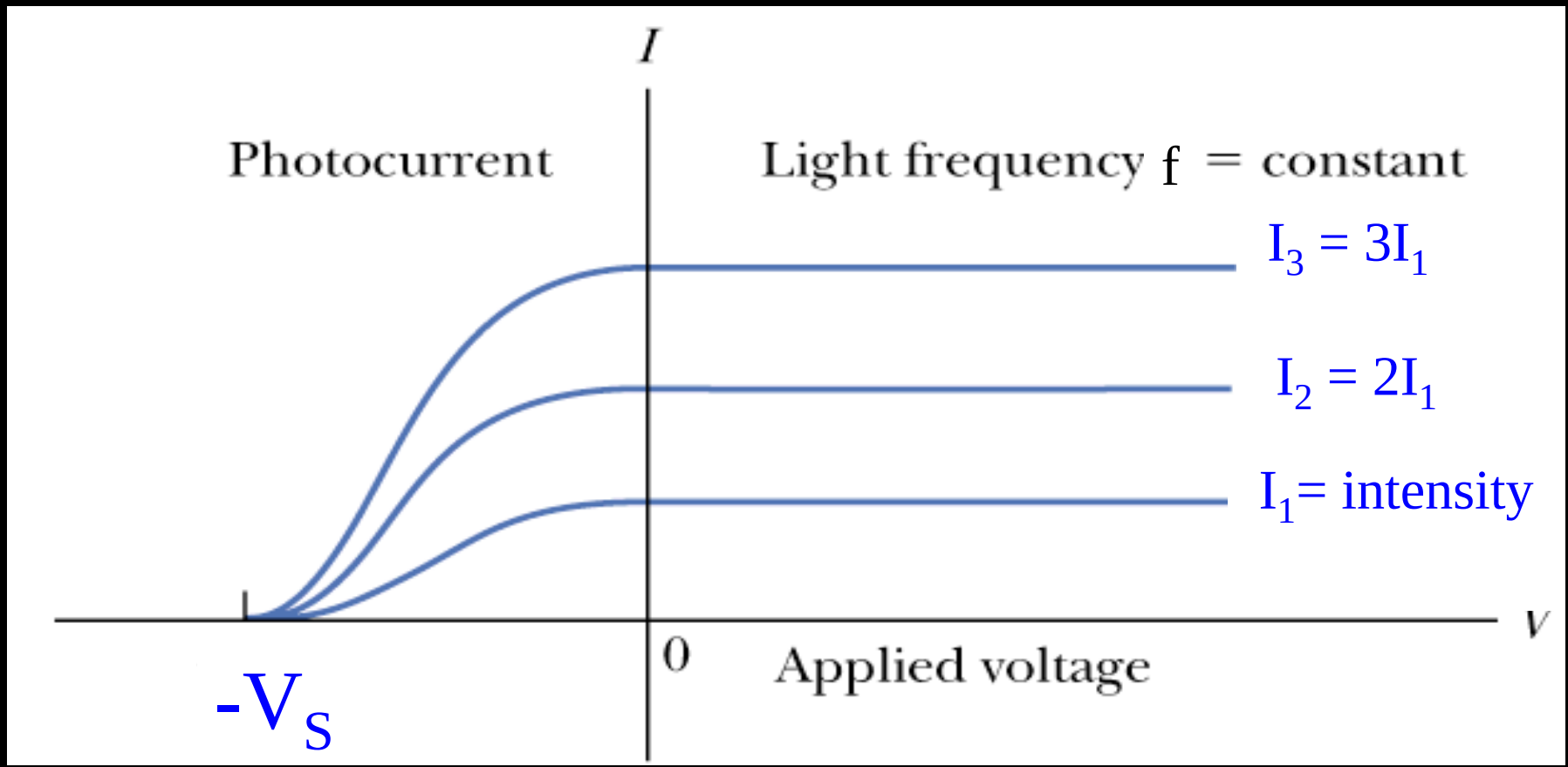


Measure characteristics of current in the circuit as a fn of I, f, λ

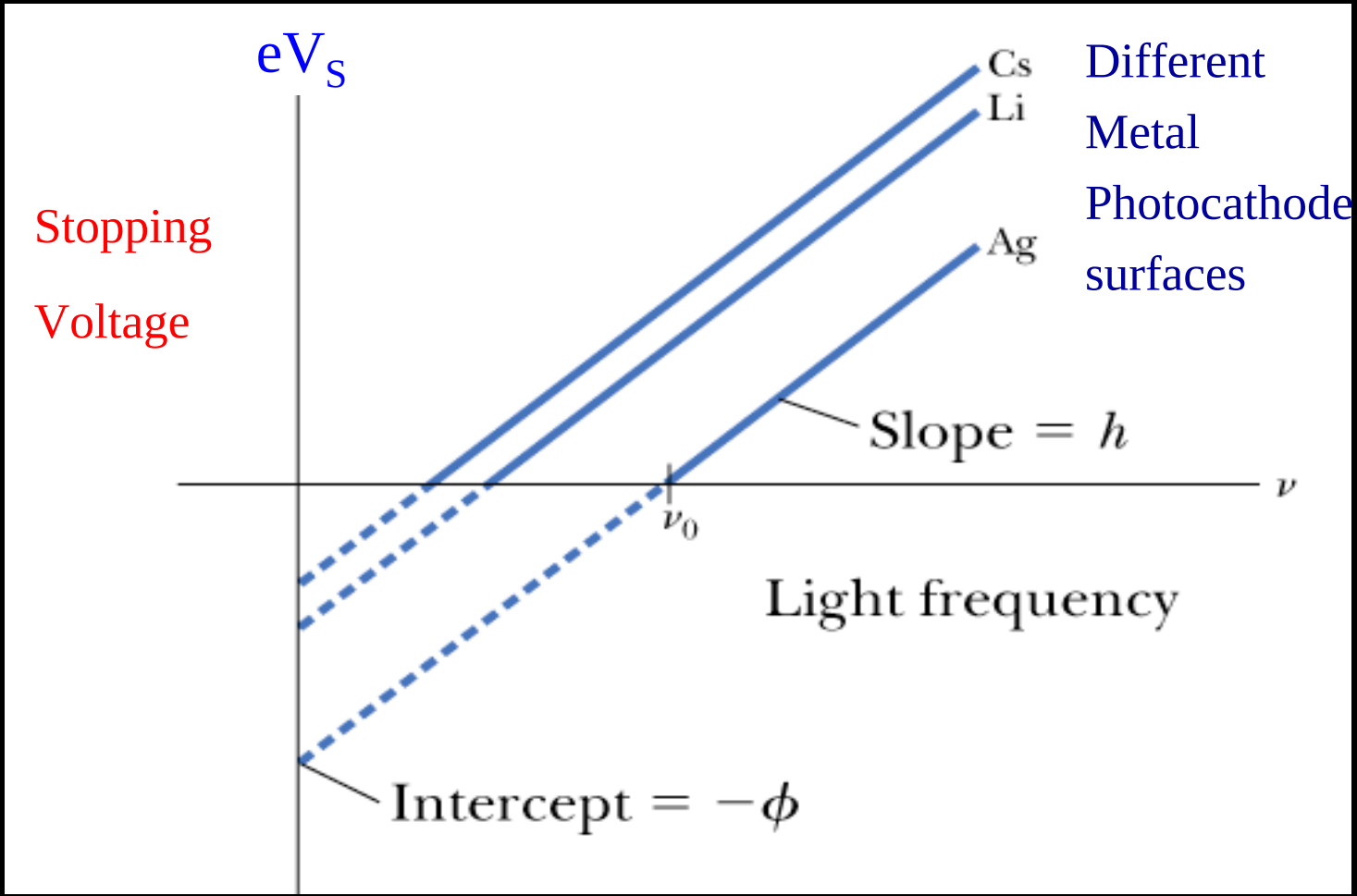
Photo Electric Effect: Measurable Properties

- Rate of electron emission from cathode
  - From current *i* seen in ammeter
- Maximum kinetic energy of emitted electron
  - By applying retarding potential on electron moving towards Collector plate
    - **$K_{MAX} = eV_S$ (V_S = Stopping voltage)**
    - **Stopping voltage → no current flows**
- Effect of different types of photo-cathode metal
- Time between shining light and first sign of photo-current in the circuit

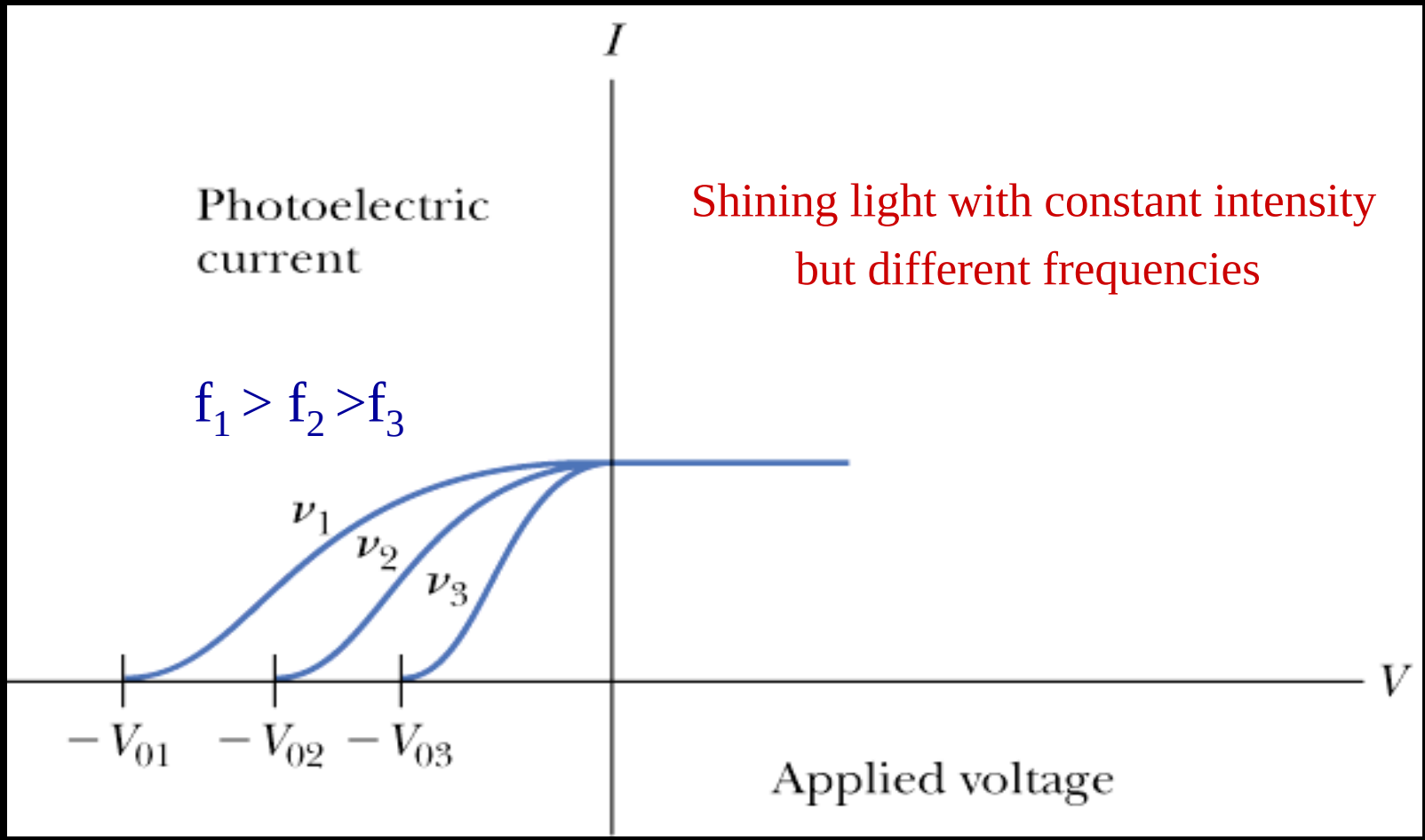
Observations : Current Vs Frequency of Incident Light



Stopping Voltage V_s Vs Incident Light Frequency



Retarding Potential Vs Light Frequency



Conclusions from the Experimental Observation

- Max Kinetic energy K_{MAX} independent of Intensity I for light of same frequency
- No photoelectric effect occurs if light frequency f is below a threshold no matter how high the intensity of light
- For a particular metal, light with $f > f_0$ causes photoelectric effect IRRESPECTIVE of light intensity.
  - f_0 is characteristic of that metal
- Photoelectric effect is instantaneous !...not time delay

Can one Explain all this Classically !

Classical Explanation of Photo Electric Effect

- As light Intensity increased $\Rightarrow$ field amplitude larger
  - E field and electrical force seen by the "charged subatomic oscillators" Larger
    - $\vec{F} = e\vec{E}$
    - More force acting on the subatomic charged oscillator
    - $\Rightarrow$ More energy transferred to it
    - $\Rightarrow$ Charged particle "hooked to the atom" should leave the surface with more Kinetic Energy KE !! The intensity of light shining rules !
- As long as light is <u>intense enough</u> , light of ANY frequency f should cause photoelectric effect
- Because the Energy in a Wave is uniformly distributed over the Spherical wavefront incident on cathode, should be a noticeable time lag ΔT between time is incident & the time a photo-electron is ejected : Energy absorption time
  - How much time ? Lets calculate it classically.

Classical Physics: Time Lag in Photo-Electric Effect

- Electron absorbs energy incident on a surface area where the electron is confined ≅ size of atom in cathode metal
- Electron is "bound" by attractive Coulomb force in the atom, so it must absorb a minimum amount of radiation before its stripped off
- Example : Laser light Intensity I = 120W/m^2 on Na metal
  - Binding energy = 2.3 eV= "Work Function"
  - Electron confined in Na atom, size ≅ 0.1nm ..how long before ejection ?
  - Average Power Delivered P_{AV} = **I . A**, A= $\pi r^2 \cong 3.1 \times 10^{-20}$ m^2
  - If all energy absorbed then $\Delta E = P_{AV} \cdot \Delta T \Rightarrow \Delta T = \Delta E / P_{AV}$

$$\Delta T = \frac{(2.3eV)(1.6\times10^{-19}\,J/eV)}{(120W/m^2)(3.1\times10^{-20}\,m^2)} = 0.10\ S$$

  - Classical Physics predicts measurable delay even by the primitive clocks of 1900
  - But in experiment, the effect was observed to be instantaneous !!
  - Classical Physics fails in explaining all results

That was a Disaster !

(# 2)

Beginning of a search for a hero or an explanation or both !