4E : The Quantum Universe

Lecture 11, April 16

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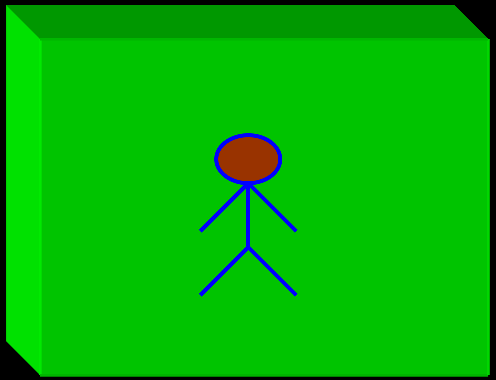
Implications of Uncertainty Principles

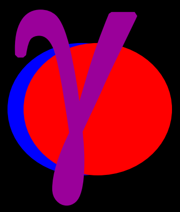
A bound "particle" is one that is confined in some finite region of space.



One of the cornerstones of Quantum mechanics is that bound particles can not be stationary – <u>even at Zero absolute temperature !</u>

There is a non-zero limit on the kinetic energy of a bound particle

Matter-Antimatter Collisions and Uncertainty Principle



Look at Rules of Energy and Momentum Conservation : Are they ?

$E_{before} = mc^2 + mc^2$ and $E_{after} = 2mc^2$

$P_{before} = 0$ but since photon produced in the annihilation → $P_{after} = 2mc$!

Such violation are allowed but must be consumed instantaneously !
Hence the name “virtual” particles

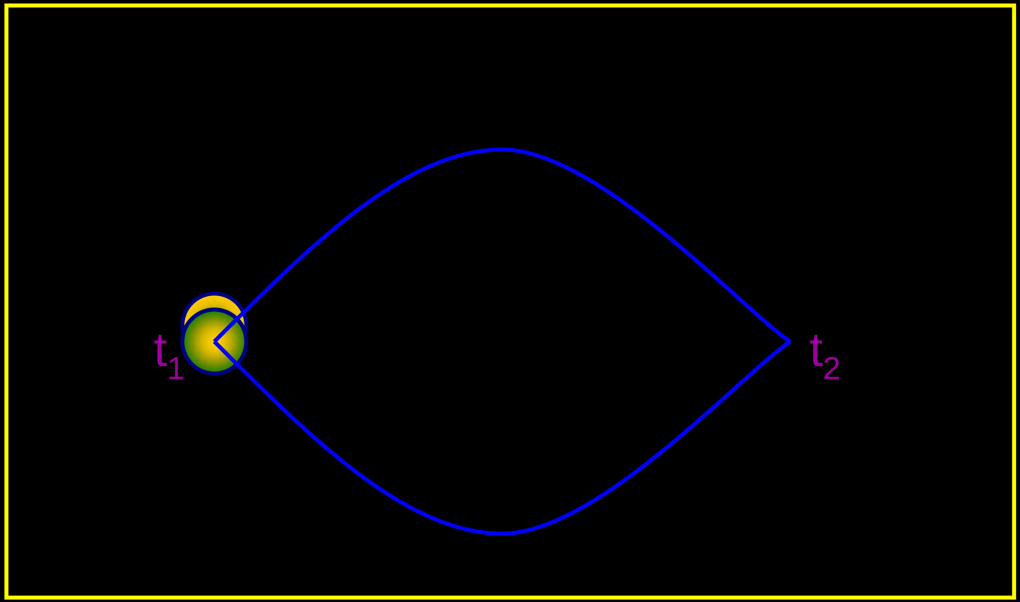
Fluctuations In The Vacuum : Breaking Energy Conservation Rules

Vacuum, at any energy, is bubbling with particle creation and annihilation

$\Delta E \cdot \Delta t \approx h/2\pi$ implies that you can (in principle) pull out an **elephant + anti-elephant** from NOTHING (Vaccum) but for a very very short time Δt !!

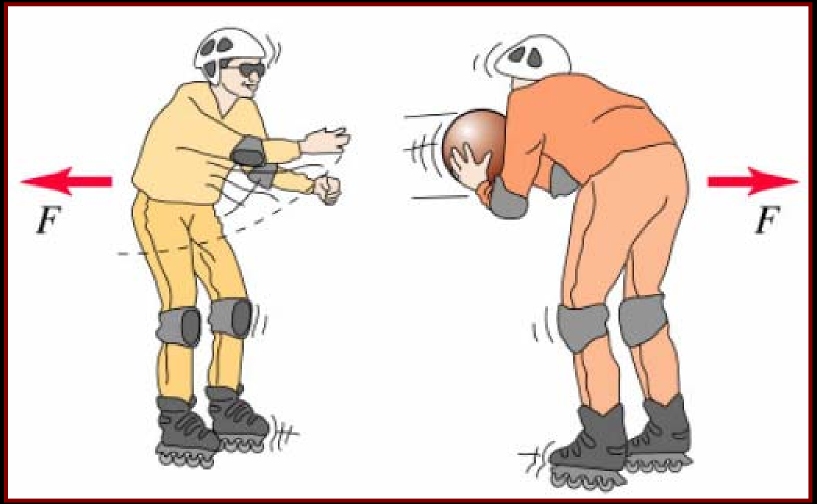
How Much Time : $\Delta t = \dfrac{\hbar}{2Mc^2}$

How cool is that !



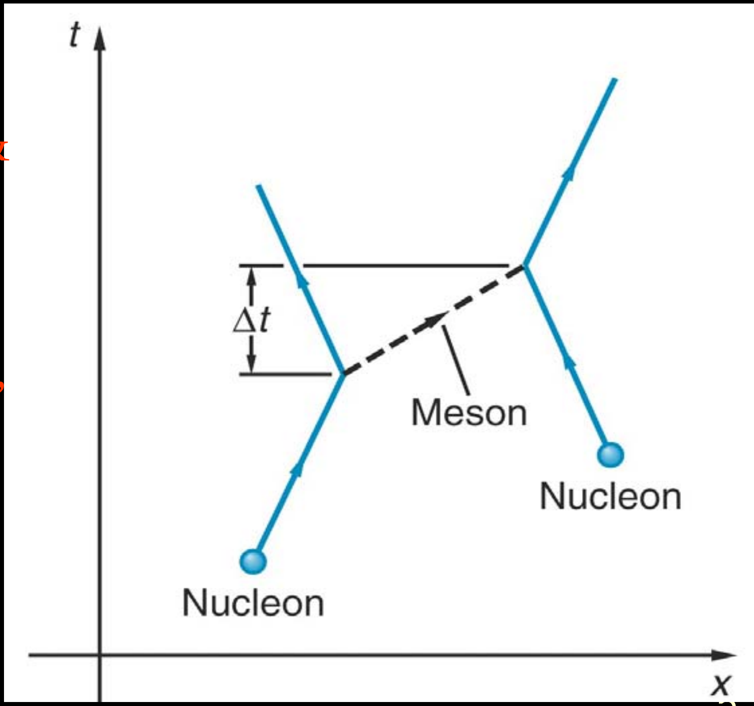
How far can the virtual particles propagate ? Depends on their mass

Strong Force Within Nucleus → Exchange Force and Virtual Particles



Repulsive force

- Strong Nuclear force can be modeled as exchange of virtual particles called $\pi^{\pm}$ mesons by nucleons (protons & neutrons)
- $\pi^{\pm}$ mesons are emitted by proton and reabsorbed by a neutron
- The short range of the Nuclear force is due to the "large" mass of the exchanged meson
- $M_{\pi} = 140\ \mathrm{MeV}/c^2$

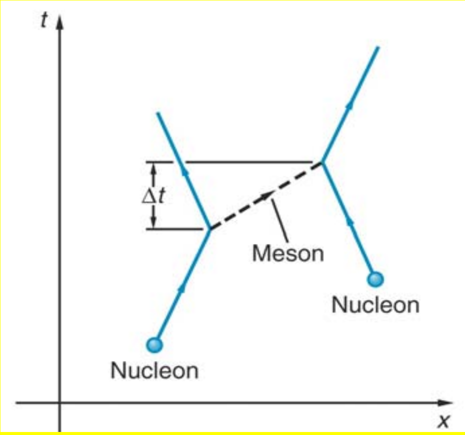


Range of Nuclear Exchange Force

How long can the emitted virtual particle last?

$\Delta E \times \Delta t \geq \hbar$

The virtual particle has rest mass + kinetic energy



$\Rightarrow$ Its energy $\boxed{\Delta E \geq Mc^2}$

$\Rightarrow$ Particle can not live for more than $\boxed{\Delta t \leq \hbar / Mc^2}$

Range R of the meson (and thus the exchange force)

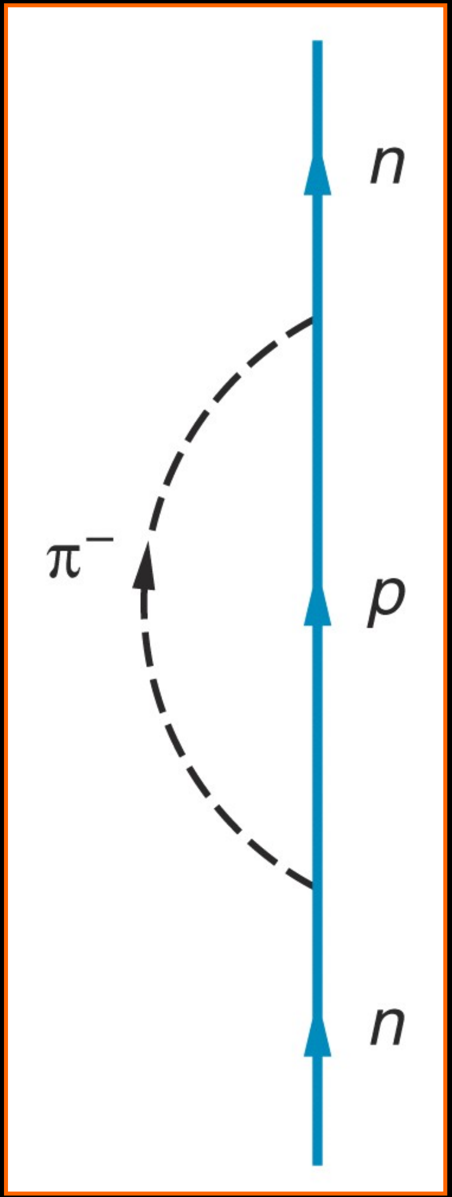
$R = c\Delta t = c\hbar / Mc^2 = \hbar / Mc$

For M=140 MeV/c^2 $\Rightarrow R \simeq \dfrac{1.06 \times 10^{-34} J.s}{(140 MeV/c^2) \times c^2 \times (1.60 \times 10^{-13} J/MeV)}$

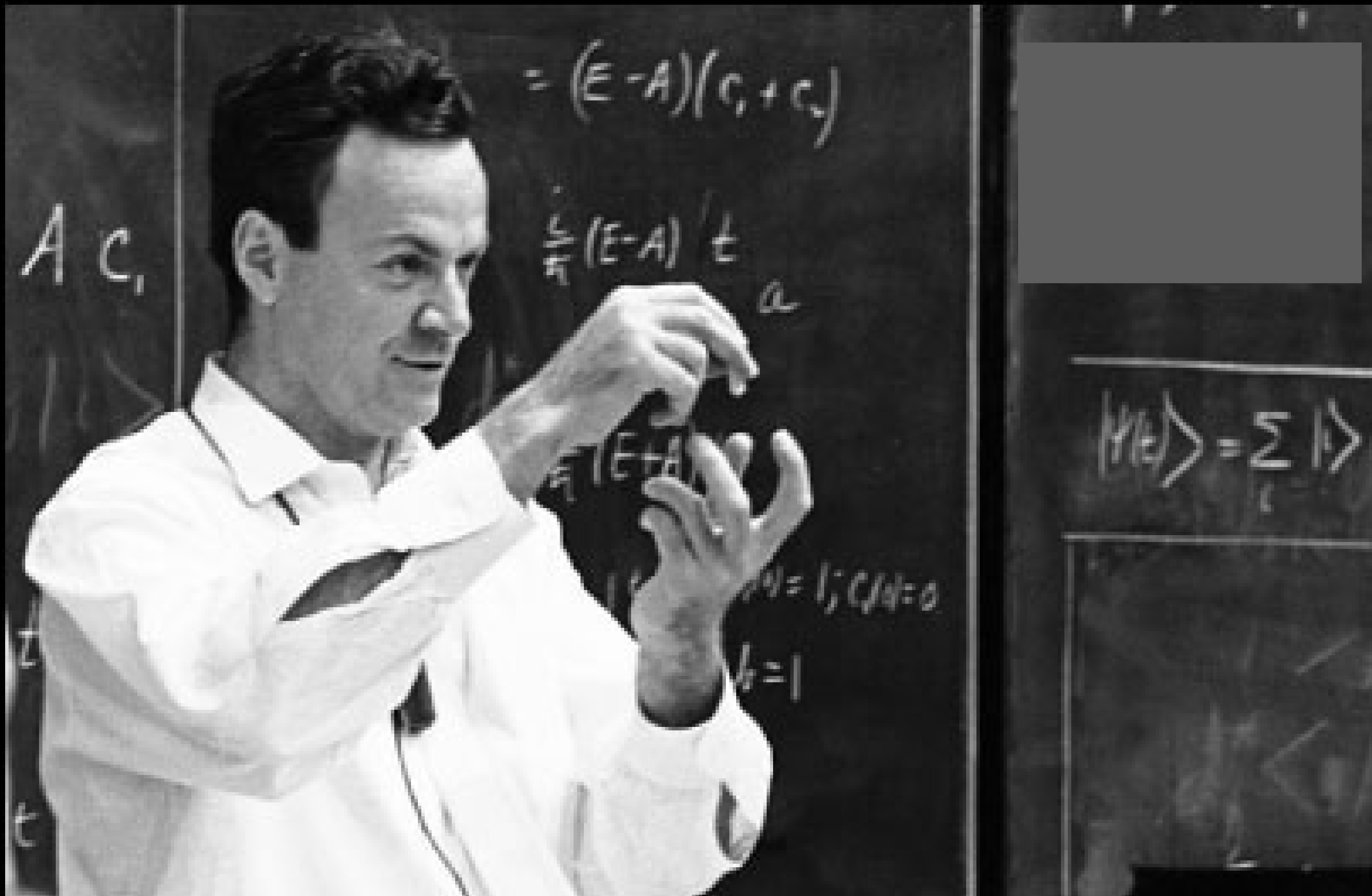
$\boxed{R \simeq 1.4 \times 10^{-15} m = 1.4\, fm}$

Subatomic Cinderella Act

- Neutron emits a charged pion for a time Δt and becomes a (charged) proton
- After time Δt , the proton reabsorbs charged pion particle (π^-) to become neutron again
- But in the time Δt that the positive proton and π^- particle exist, they can interact with other charged particles
- After time Δt strikes , the Cinderella act is over !



Quantum Behavior : Richard Feynman



See Chapters 1 & 2 of Feynman Lectures in Physics   Vol III

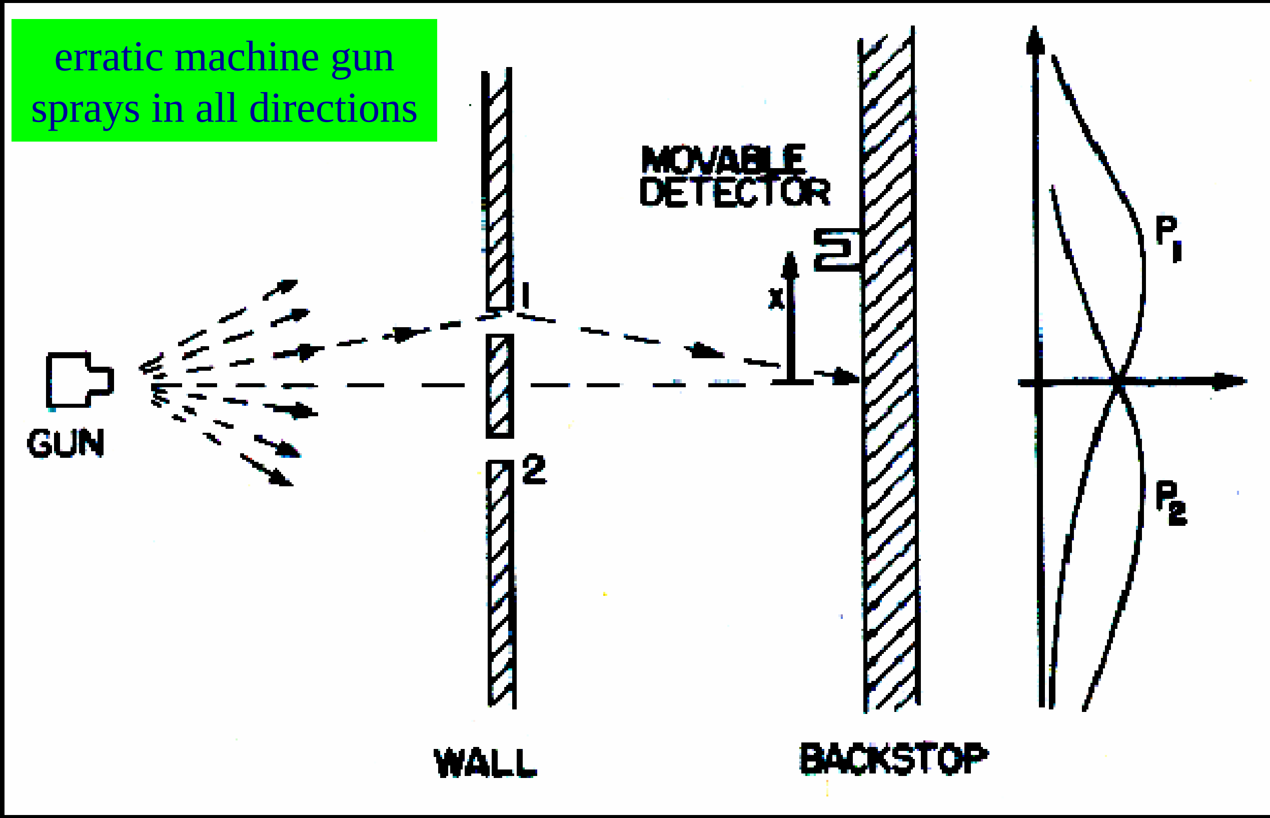
Or   Six Easy Pieces by Richard Feynman : Addison Wesley Publishers

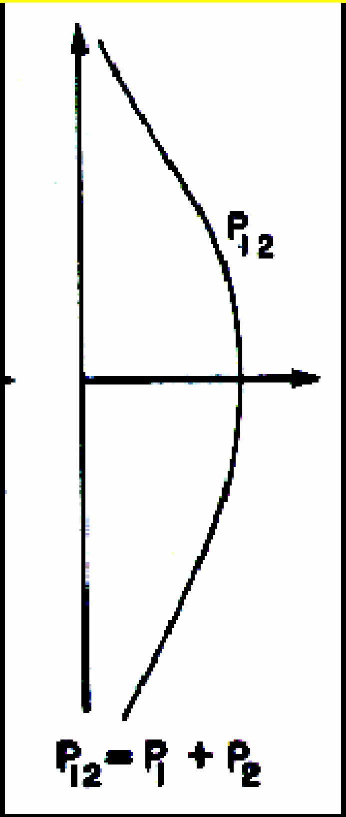
An Experiment with Indestructible Bullets

Prob.when one or other hole open

Probability P_{12} when both holes open ?

erratic machine gun sprays in all directions





made of armor plate

sandbox

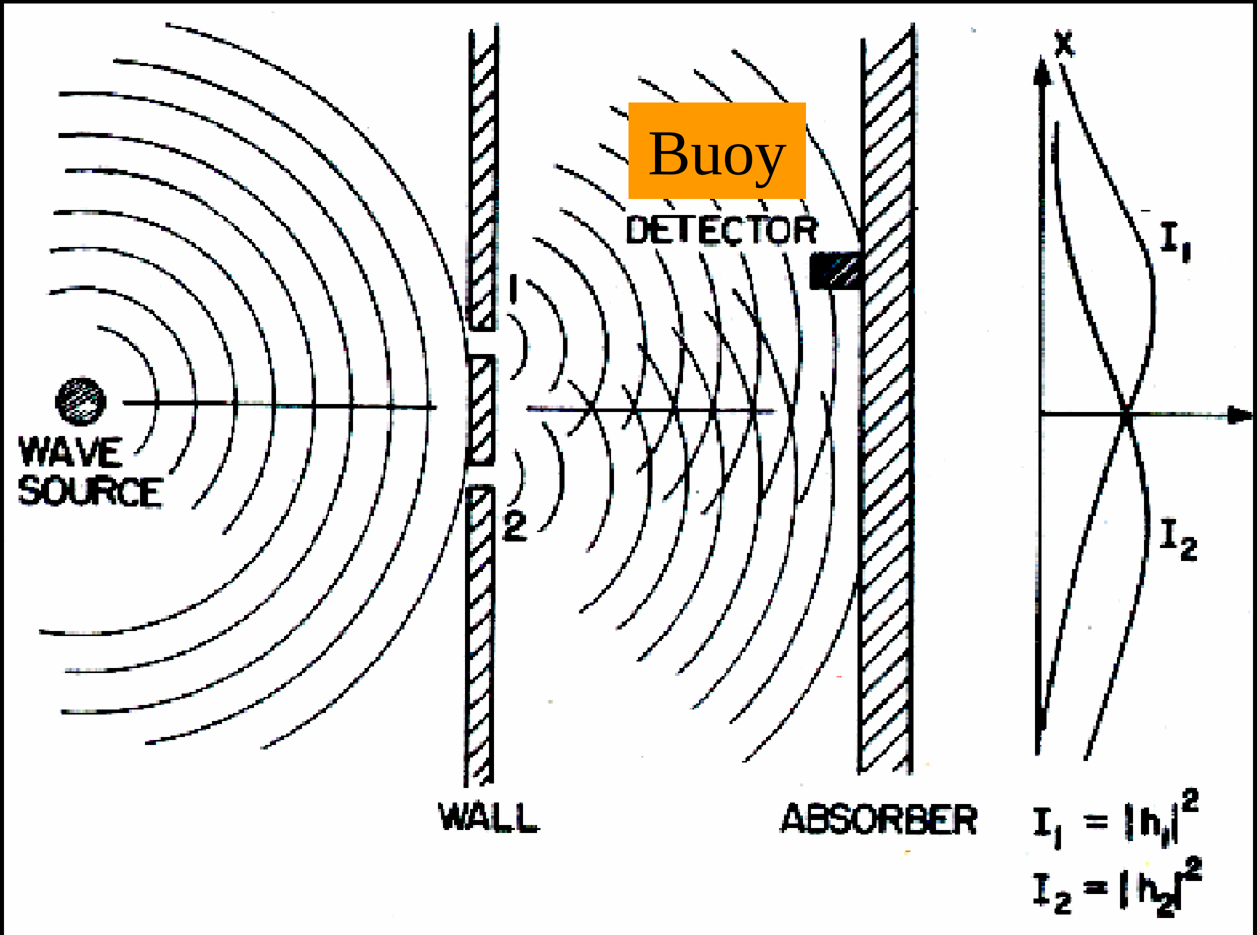
$P_{12} = P_1 + P_2$

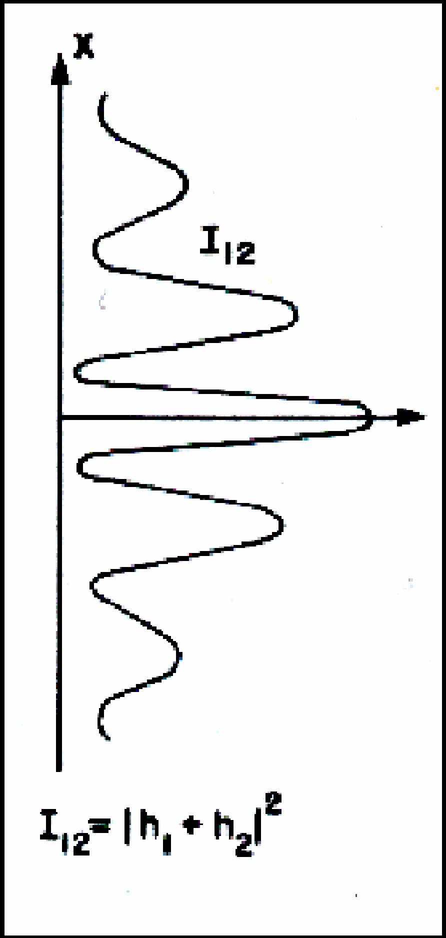
An Experiment With Water Waves

Measure Intensity of Water Waves
(by measuring detector displacement)

when one or other hole open

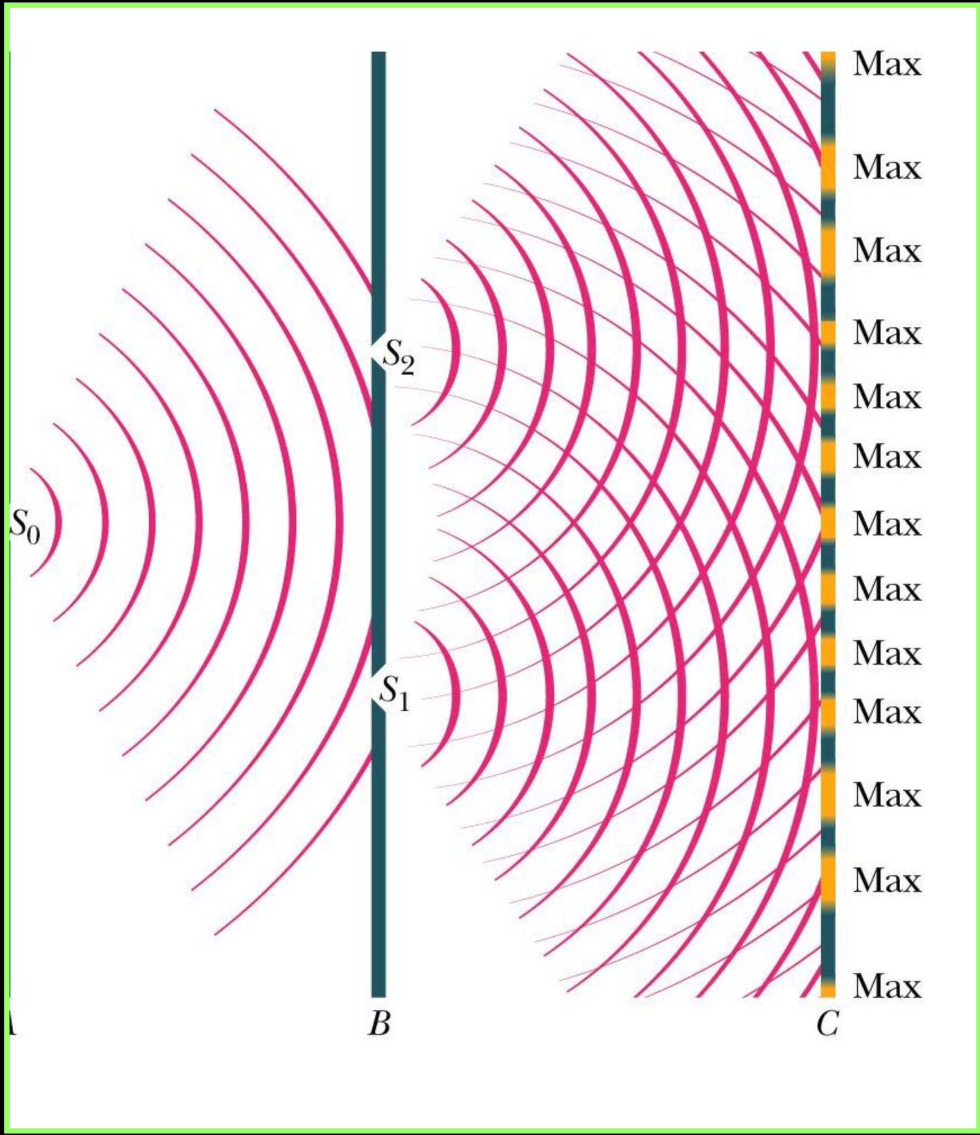
Intensity I_{12} when Both holes open



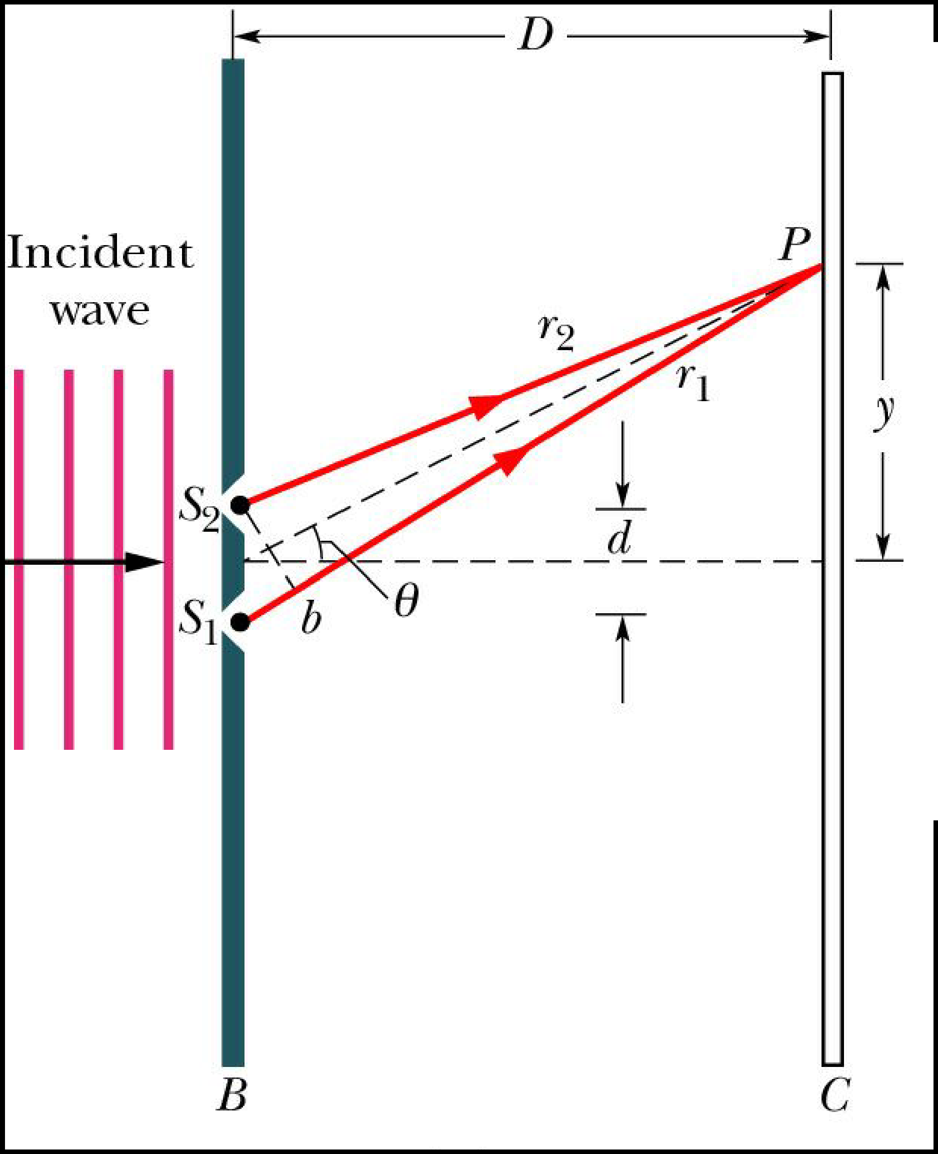


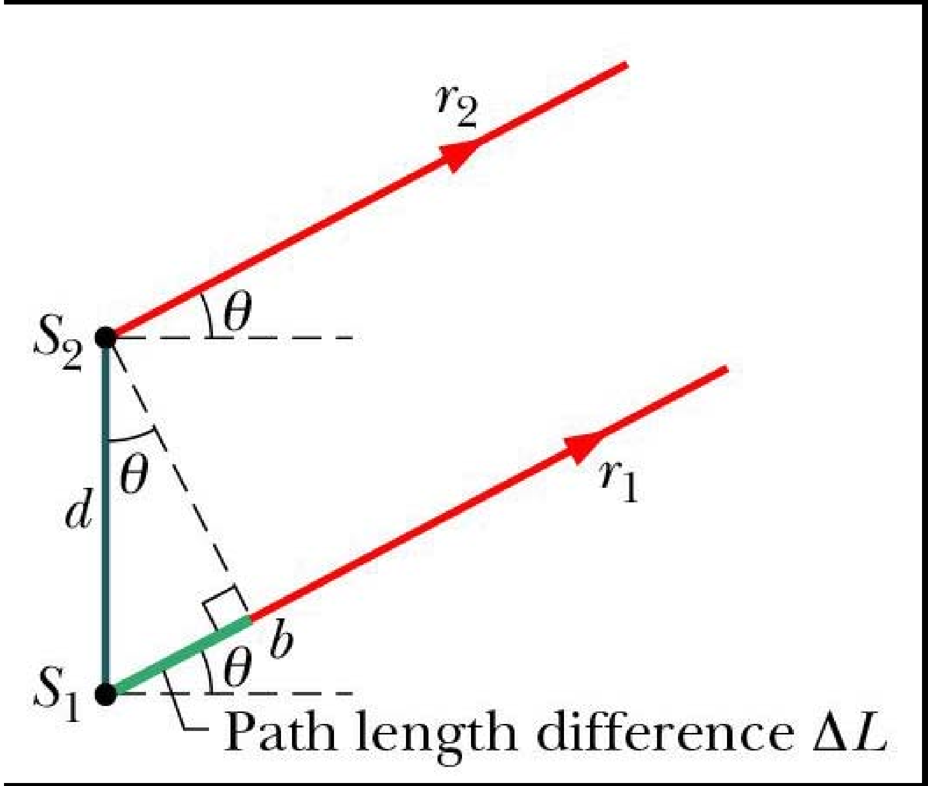
$$I_{12} = |h_1 + h_2|^2 = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

Wave Phenomena → Interference and Diffraction



Interference Phenomenon in Waves



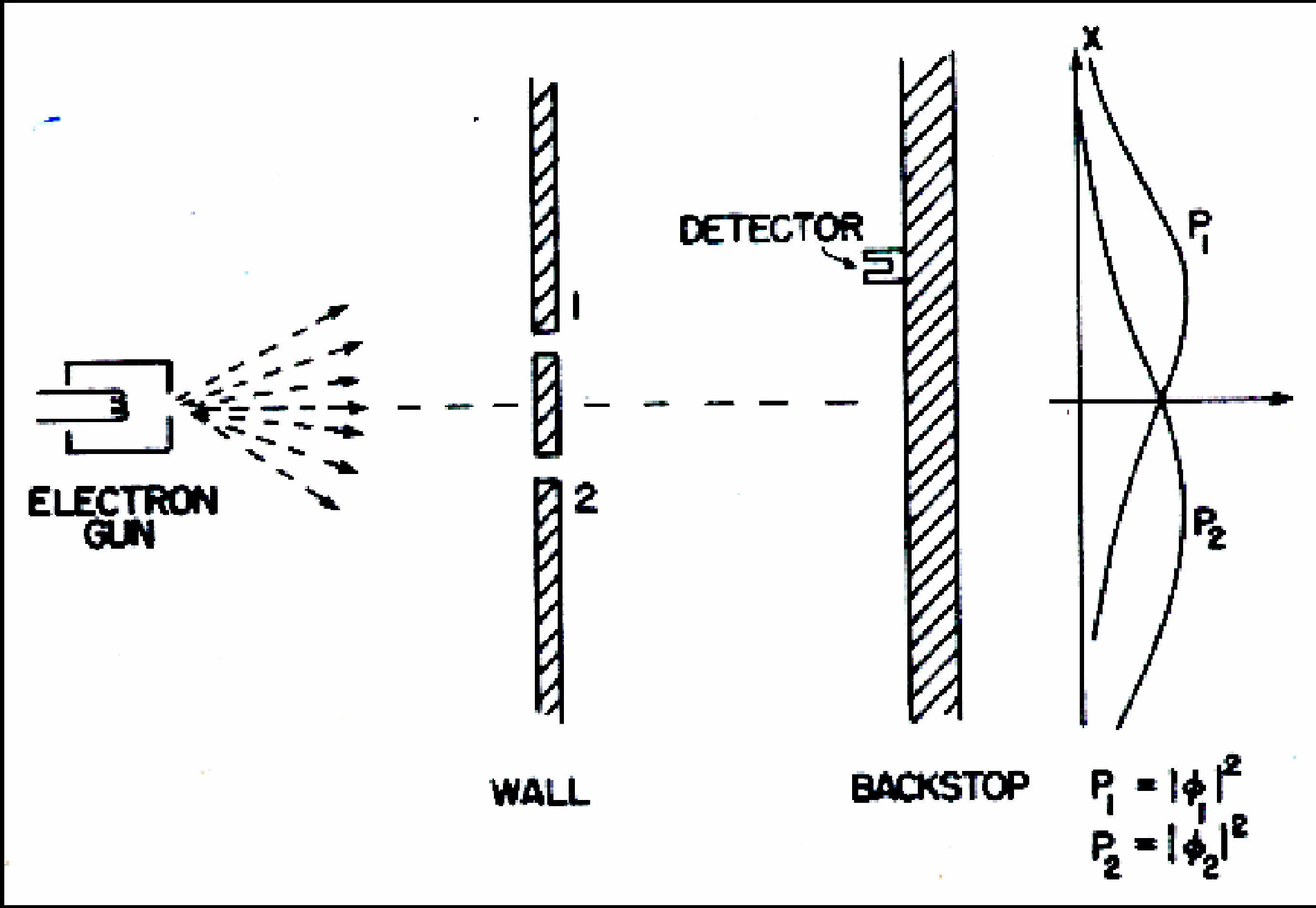


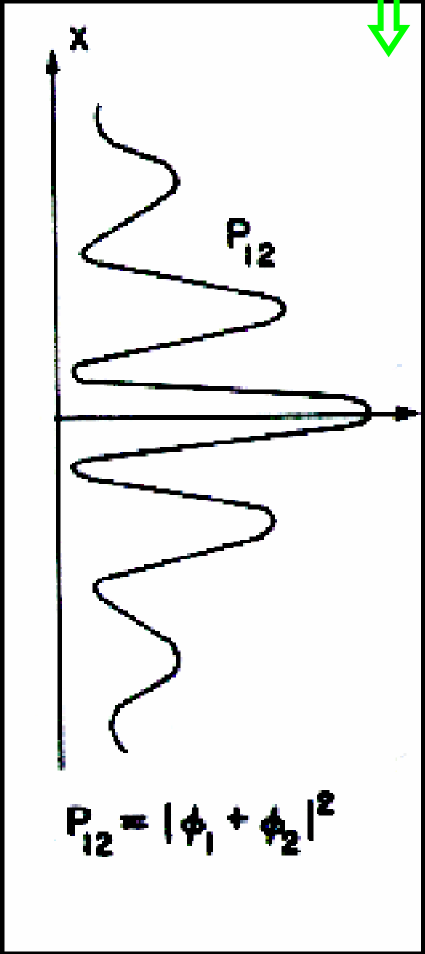
$$n\lambda = d\sin\theta$$

An Experiment With (indestructible) Electrons

when one or other hole open

Probability P_{12} when both holes open



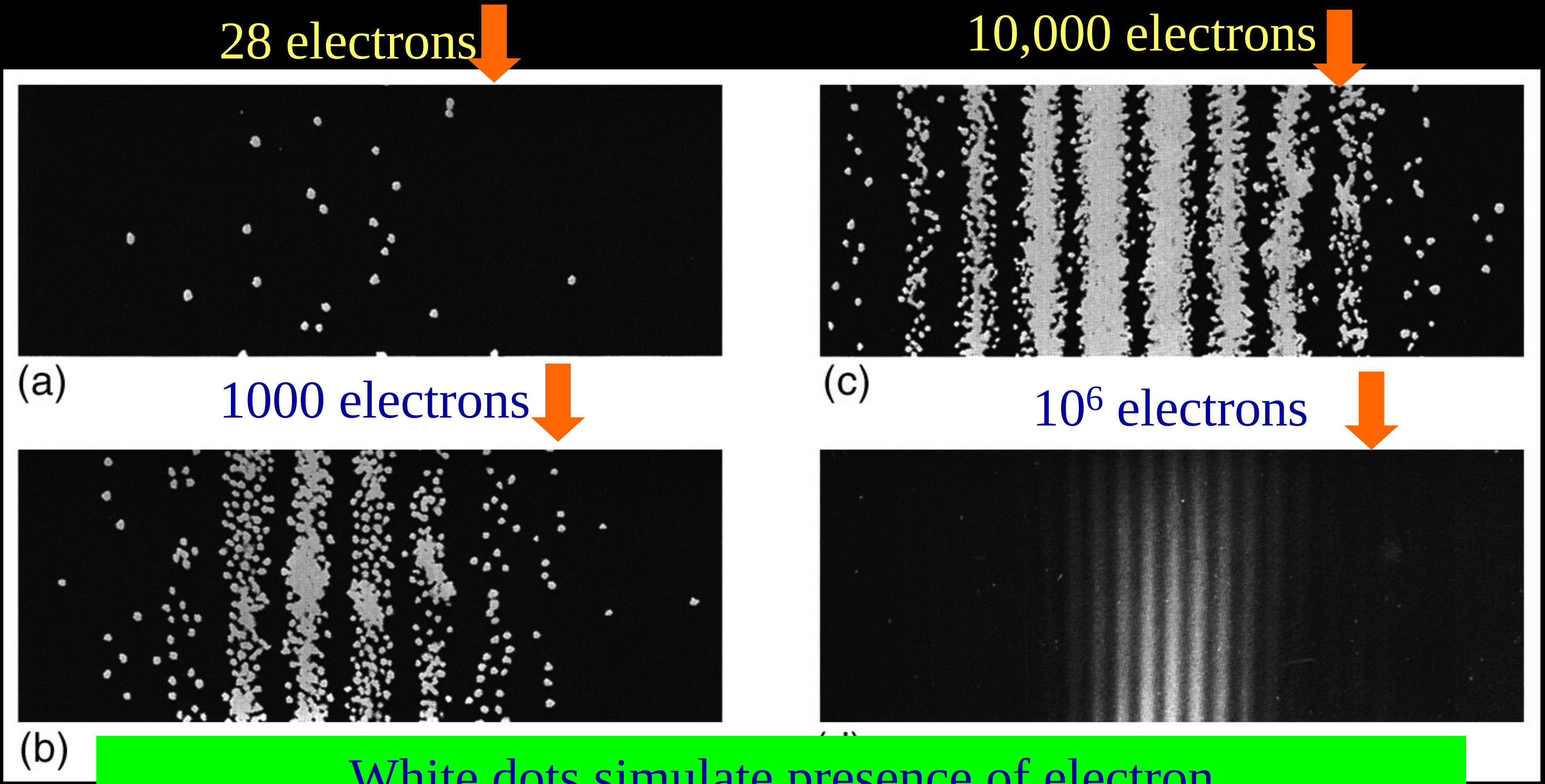


$$P_{12} \neq P_1 + P_2$$

Interference Pattern of Electrons When Both slits open

Growth of 2-slit Interference pattern thru different exposure periods

photographic plate (screen) struck by :



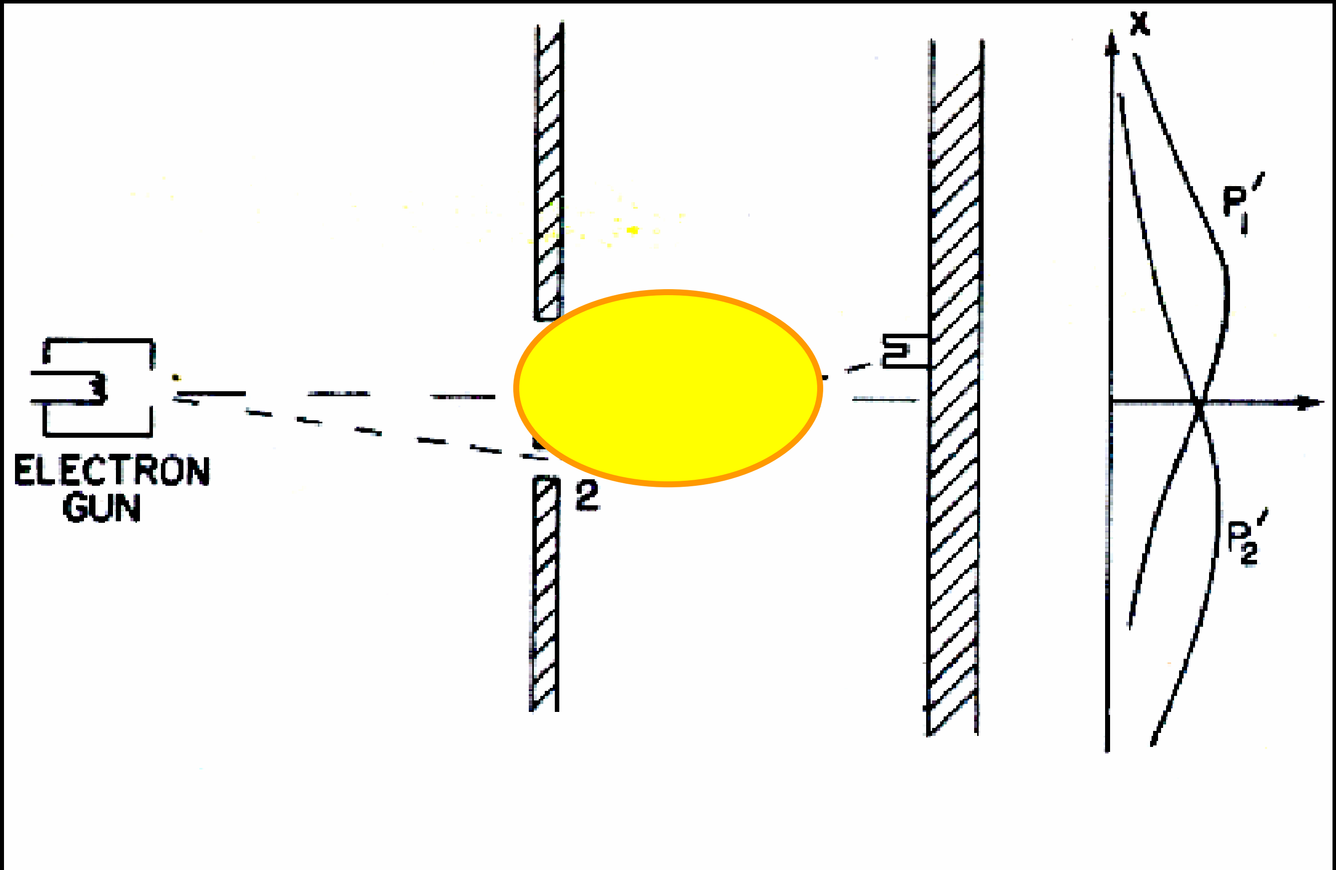
White dots simulate presence of electron

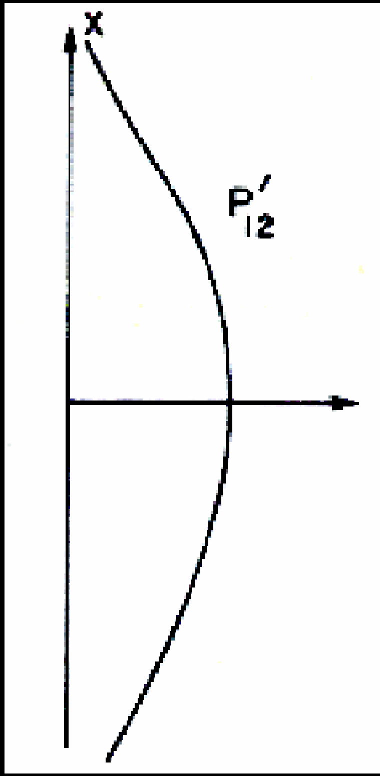
No white dots at the place of destructive Interference (minima)

*Watching Which Hole Electron Went Thru By Shining **Intense** Light*

when one or other hole open

Probability P_{12} when both holes open and I *see* which hole the electron came thru



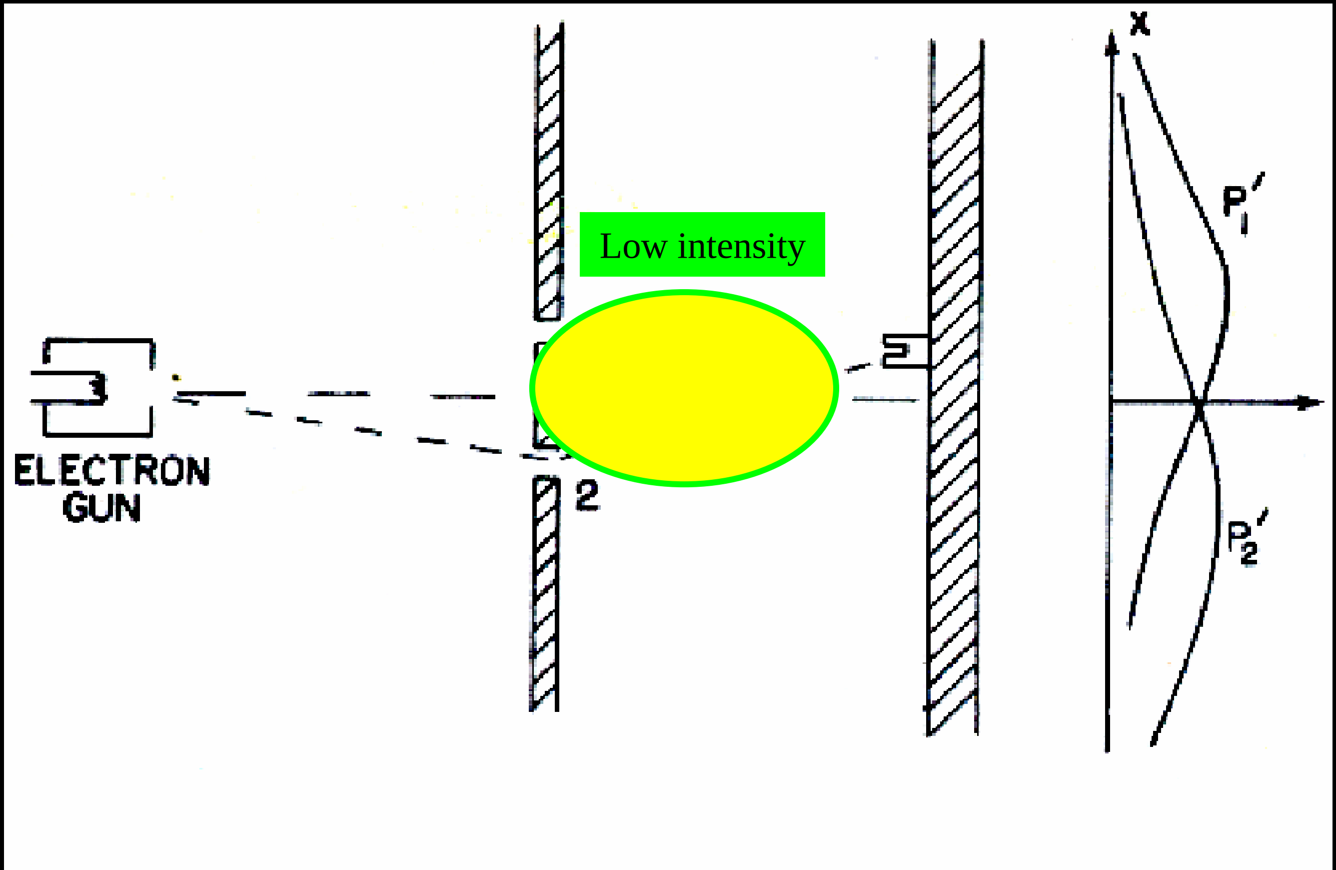


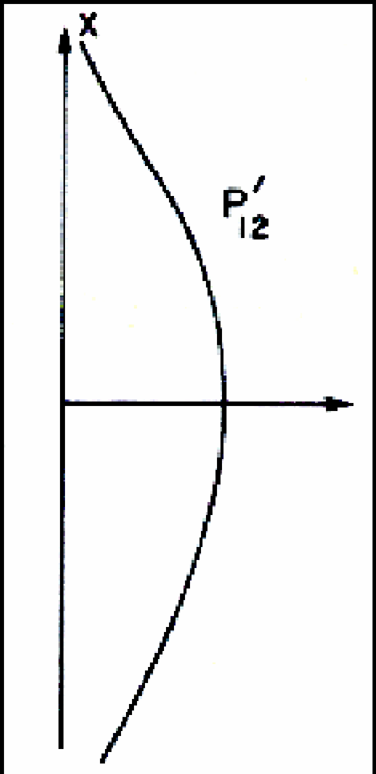
$P'_{12} = P'_1 + P'_2$

Watching electrons with dim light: See light flash & hear detector clicks

Low intensity light → Not many photons incident
Maybe a photon hits the electron (See flash, hear click)
Or Maybe the photon misses the electron (no flash, only click)

Probability P_{12} when both holes open and I see the flash and hear the detector click

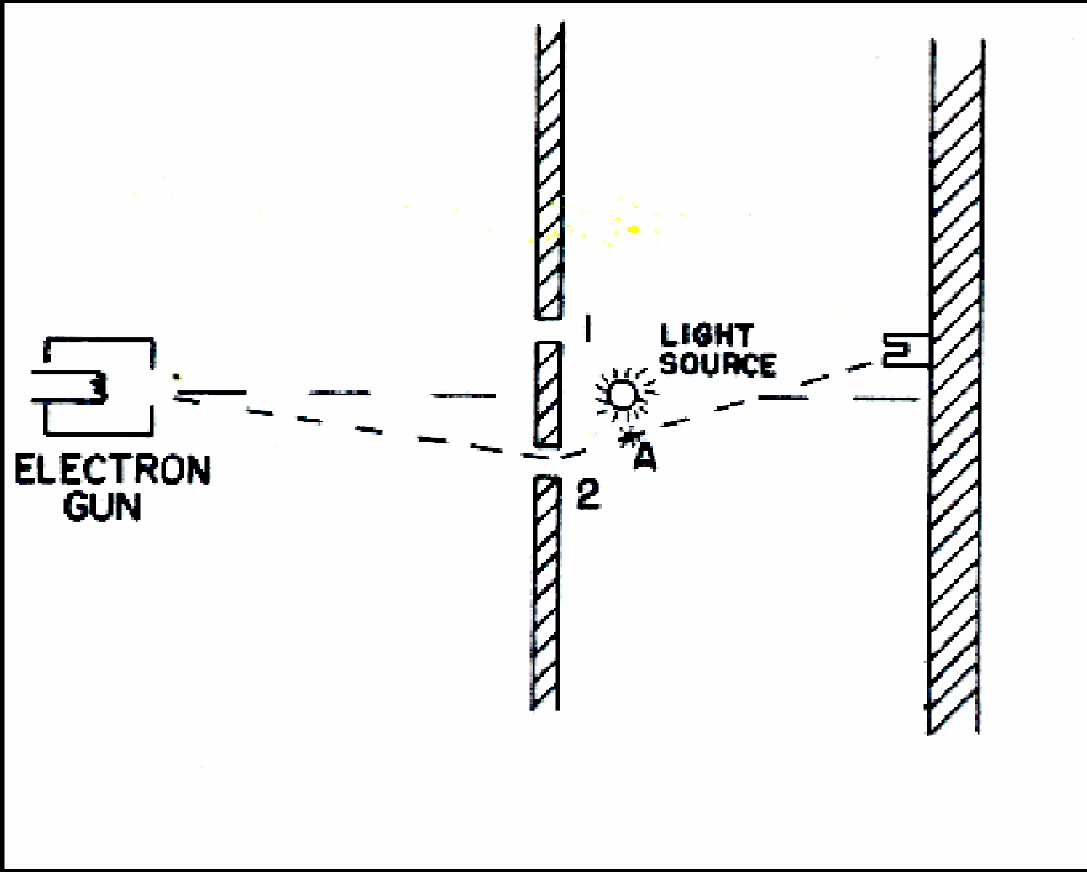


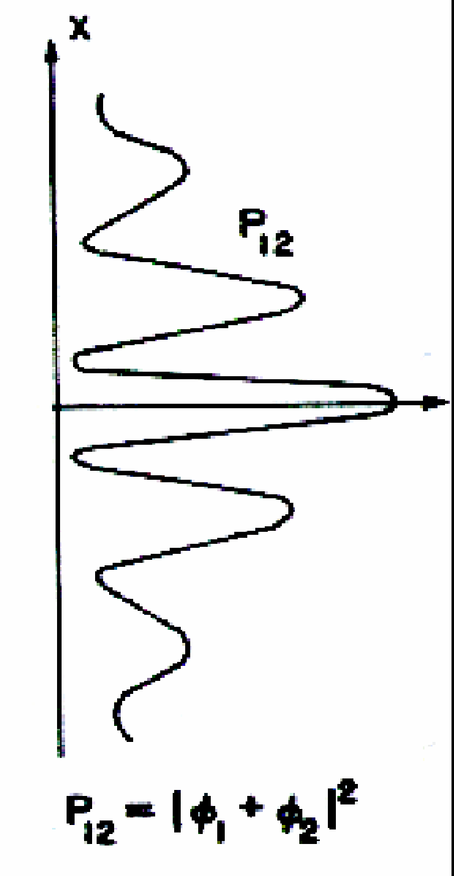


$$P'_{12} = P'_1 + P'_2$$

Watching electrons in dim light: <u>don't see</u> flash but hear detector clicks

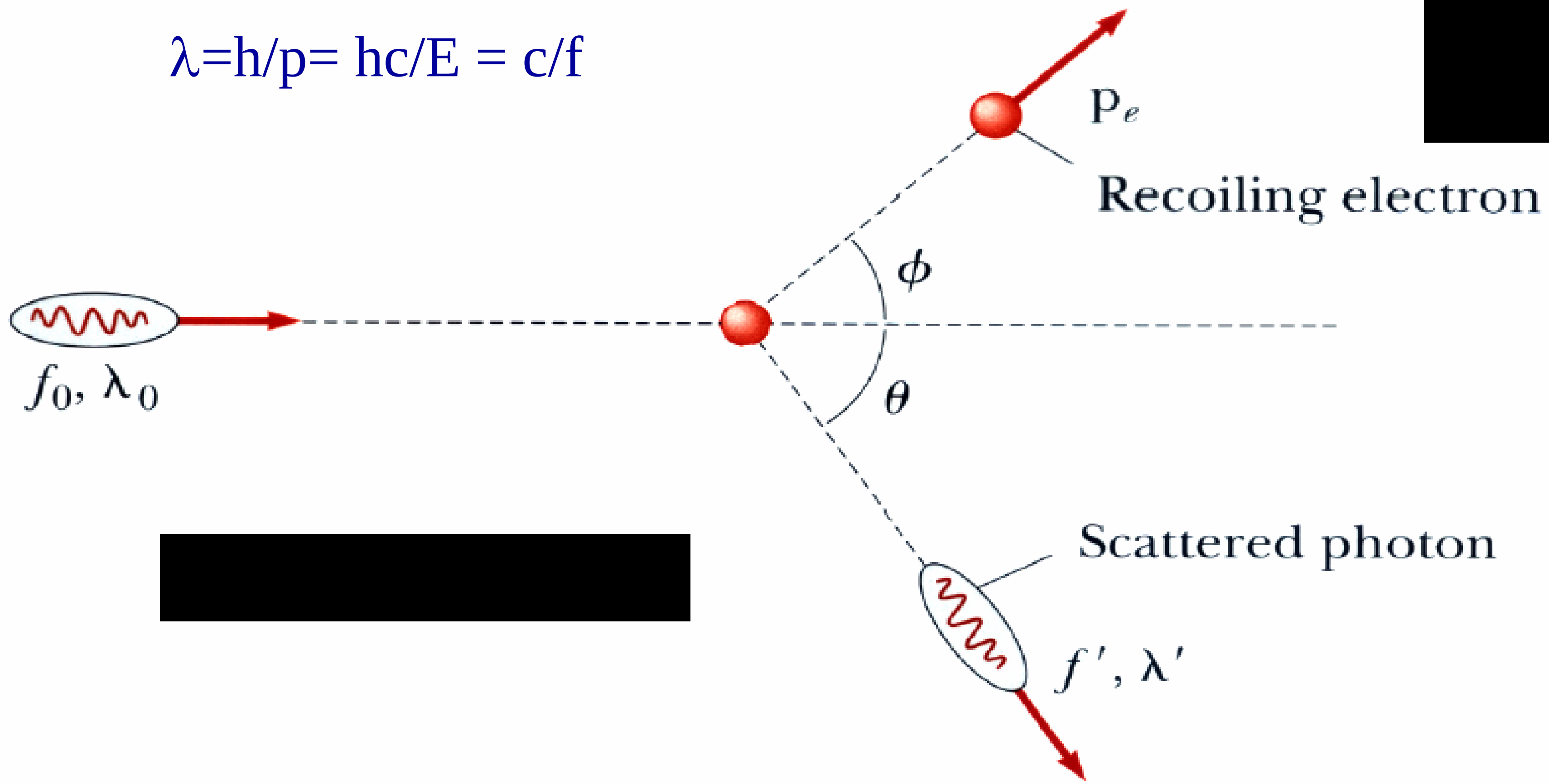
Probability P_{12} when both holes open and I ***dont*** see the flash but hear the detector click





Compton Scattering: Shining light to observe electron

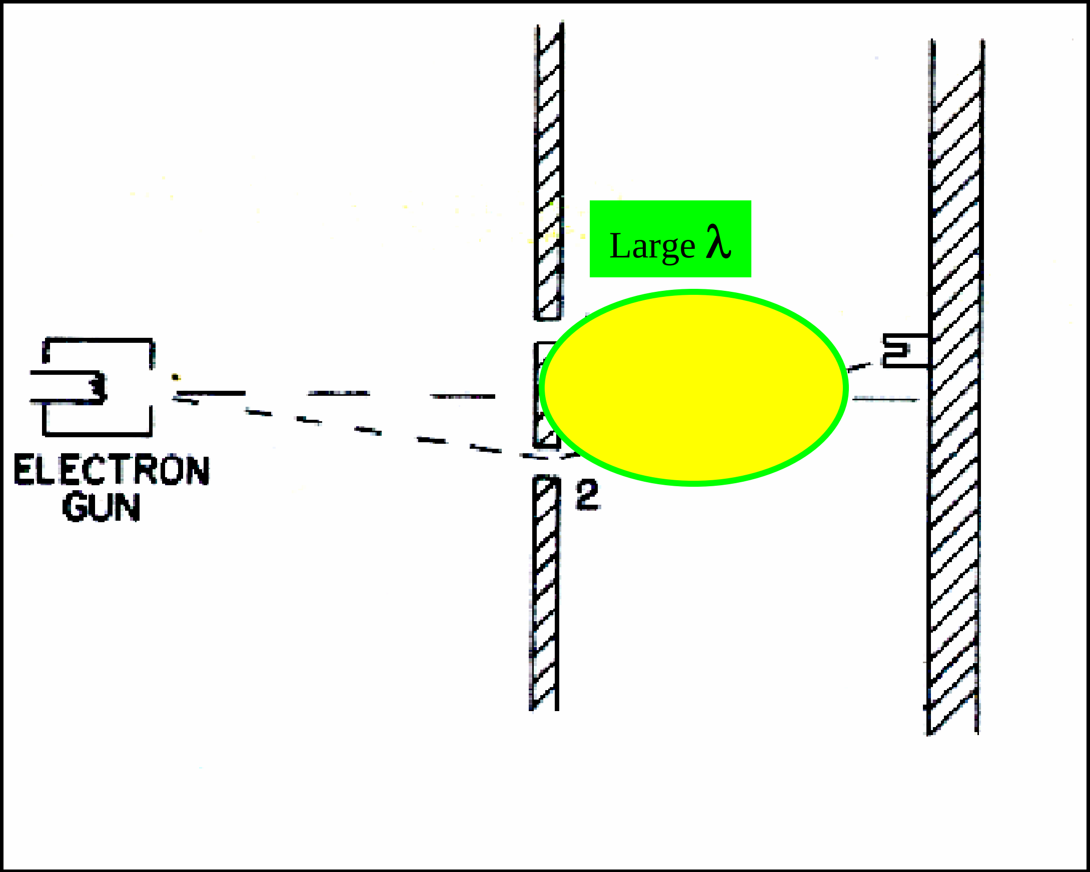
$\lambda = h/p = hc/E = c/f$

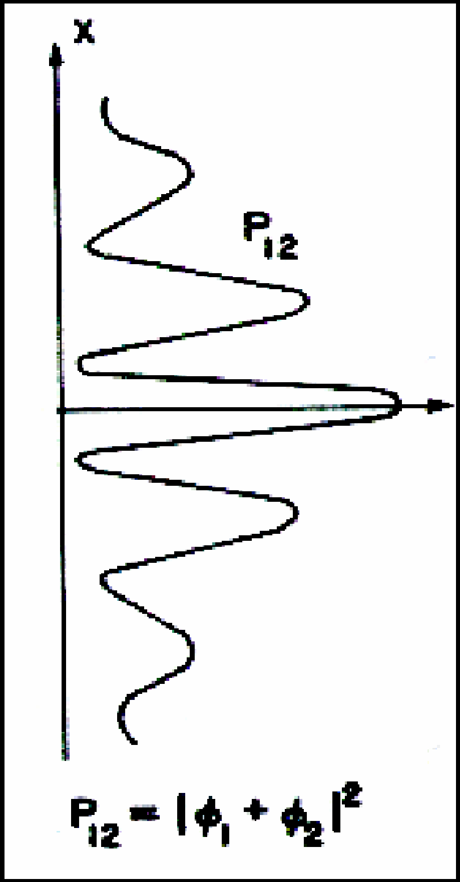


The act of Observation DISTURBS
the object being watched,
here the electron moves away from
where it was originally

Watching Electrons With Light of λ >> slit size but High Intensity

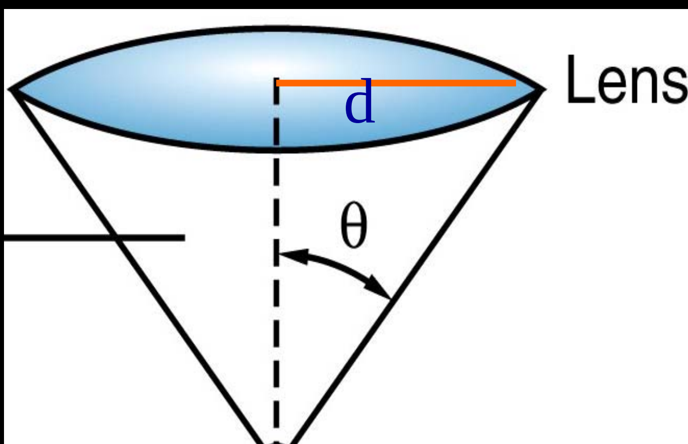
Probability P_{12} when both holes open but can't tell, from the location of flash, which hole the electron came thru





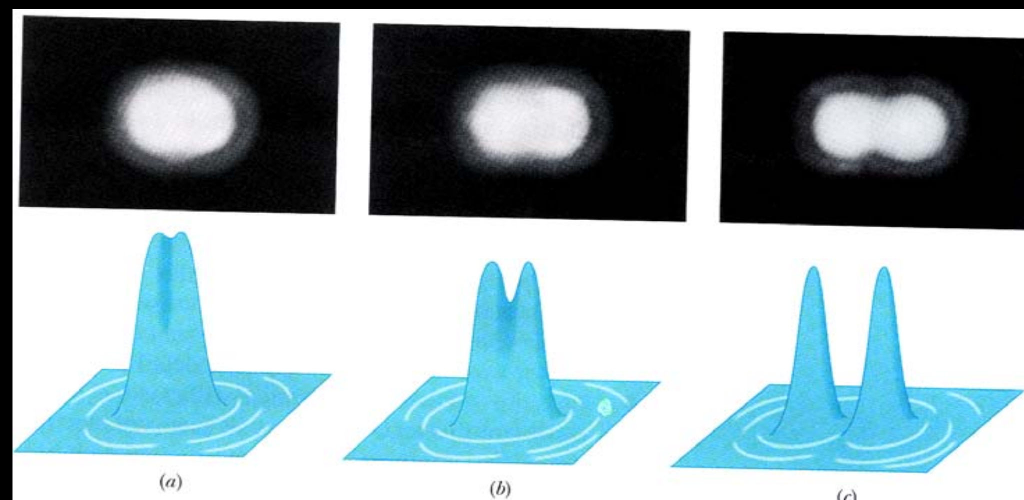
Why Fuzzy Flash? → Resolving Power of Light

Image of 2 separate point sources formed by a converging lens of diameter d, ability to resolve them depends on λ & d because of the Inherent diffraction in image formation



0_0

ΔX



Not resolved | barely resolved | resolved

$$\text{Resolving power } \Delta x \simeq \frac{\lambda}{2\sin\theta}$$

Summary of Experiments So Far

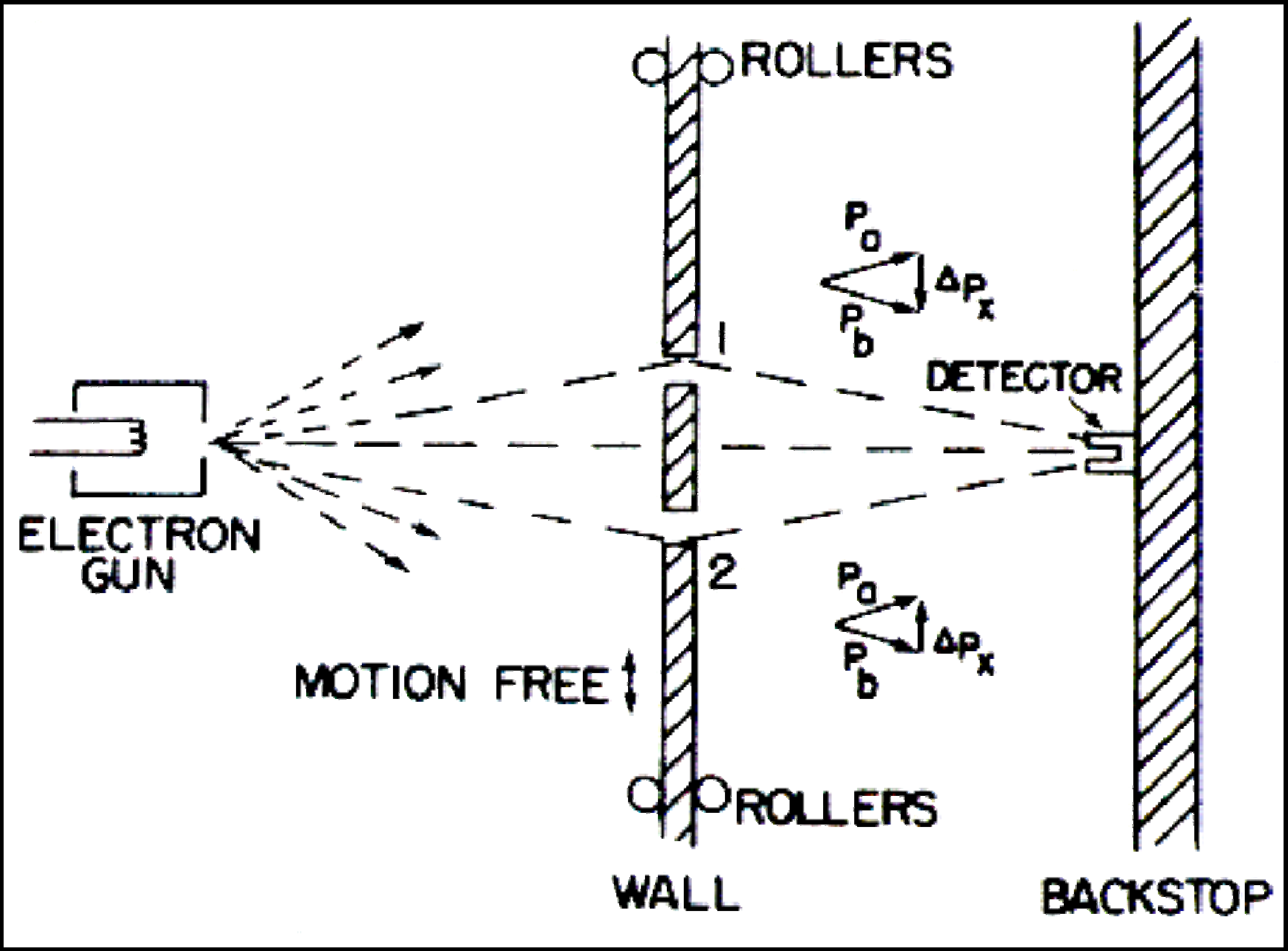
1. Probability of an event is given by the square of amplitude of a complex # Ψ: Probability Amplitude
2. When an event occurs in several alternate ways, probability amplitude for the event is sum of probability amplitudes for each way considered seperately. There is interference:
   - $\Psi = \Psi_1 + \Psi_2$

     $P_{12} = |\Psi_1 + \Psi_2|^2$
3. If an experiment is done which is capable of determining whether one or other alternative is actually taken, probability for event is just sum of each alternative
   - Interference pattern is LOST !

Is There No Way to Beat Uncertainty Principle?

- How about NOT watching the electrons!
- Let's be a bit crafty !!
- Since this is a Thought experiment→ ideal conditions
  - Mount the wall on rollers, put a lot of grease → frictionless
  - Wall will move when electron hits it
  - Watch recoil of the wall containing the slits when the electron hits it
  - By watching whether wall moved up or down I can tell
    -
    -
- Will my ingenious plot succeed?

Measuring The Recoil of The Wall → Not Watching Electron !



?

Losing Out To Uncertainty Principle

- To measure the RECOIL of the wall $\Rightarrow$
  - must know the initial momentum of the wall before electron hit it
  - Final momentum after electron hits the wall
  - Calculate vector sum → recoil
- Uncertainty principle :
  - To do this $\Rightarrow$ $\Delta P = 0$ → $\Delta X = \infty$ [can not know the position of wall exactly]
  - If don't know the wall location, then down know where the holes are
  - Holes will be in different place for every electron that goes thru
  - → The center of interference pattern will have different (random) location for each electron
  - Such random shift is just enough to Smear out the pattern so that no interference is observed !

Summary

- Probability of an event in an ideal experiment is given by the square of the absolute value of a complex number Ψ which is call probability amplitude
  - P = probability
  - Ψ= probability amplitude, $P=|\Psi|^2$

- When an even can occur in several alternative ways, the probability amplitude for the event is the sum of the probability amplitudes for each way considered separately. There is interference:
  - $\Psi = \Psi_1 + \Psi_2$
  - $P=|\Psi_1 + \Psi_2|^2$

- If an experiment is performed which is capable of determining whether one or other alternative is actually taken, the probability of the event is the sum of probabilities for each alternative. The interference is lost: $P = P_1 + P_2$

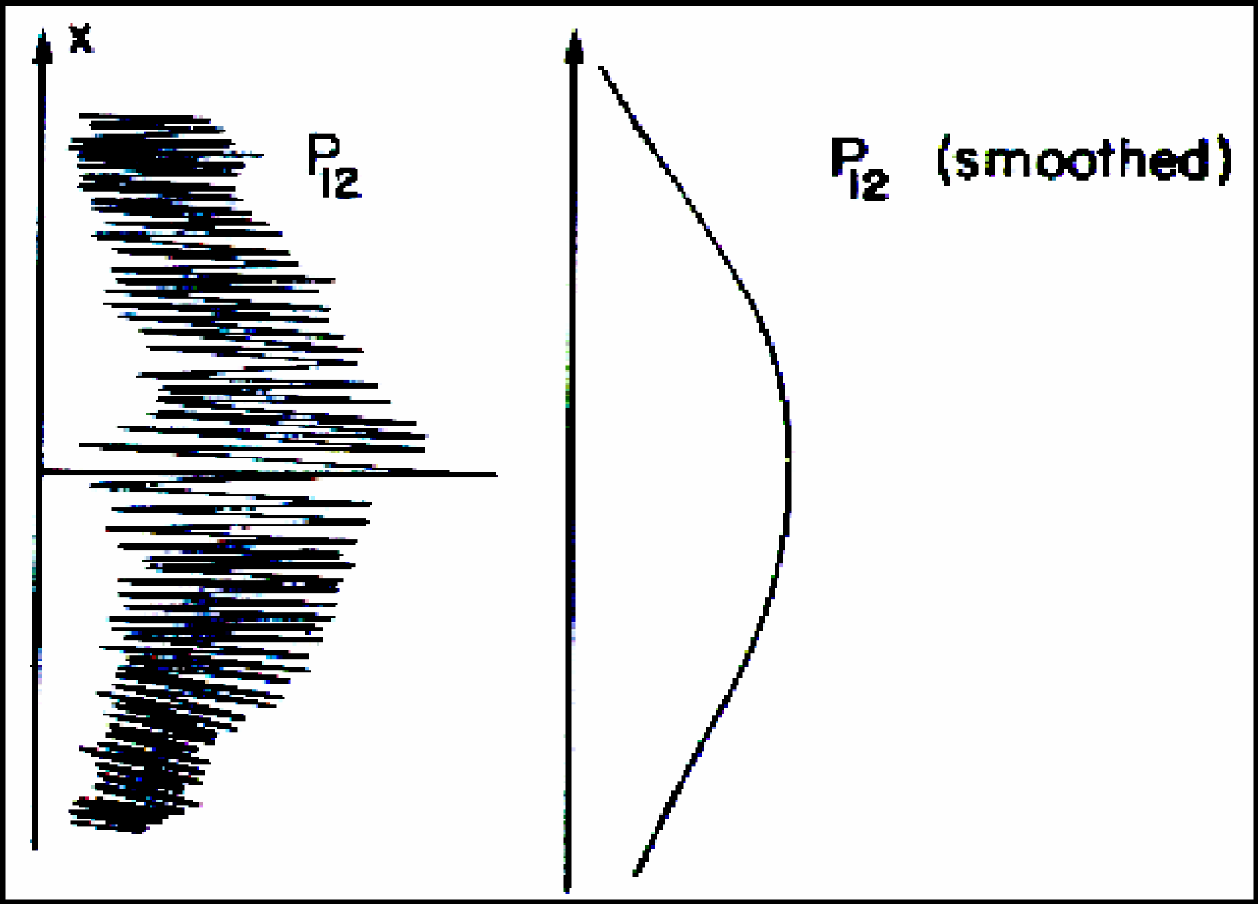
The Lesson Learnt

- In trying to determine which slit the particle went through, we are examining particle-like behavior
- In examining the interference pattern of electron, we are using wave like behavior of electron

Bohr’s Principle of Complementarity:

It is not possible to simultaneously determine physical observables in terms of both particles and waves

The Bullet Vs The Electron: Each Behaves the Same Way



Quantum Mechanics of Subatomic Particles

- Act of Observation destroys the system (No watching!)
- If can't watch then all conversations can only be in terms of Probability P
- Every particle under the influence of a force is described by a Complex wave function $\Psi(x,y,z,t)$
- Ψ is the ultimate DNA of particle: contains all info about the particle under the force (in a potential e.g Hydrogen)
- Probability of per unit volume of finding the particle at some point (x,y,z) and time t is given by
  - $P(x,y,z,t) = \Psi(x,y,z,t) . \Psi^*(x,y,z,t) =| \Psi(x,y,z,t) |^2$
- When there are more than one path to reach a final location then the probability of the event is
  - $\Psi = \Psi_1 + \Psi_2$
  - $P = | \Psi^* \Psi| = |\Psi_1|^2 + |\Psi_2|^2 +2 |\Psi_1| |\Psi_2| \cos\phi$