4E : The Quantum Universe

Lecture 10, April 14

Vivek Sharma
modphys@hepmail.ucsd.edu

in space x: $\boxed{\Delta k.\Delta x = \pi}$ $\Rightarrow$ since $k = \dfrac{2\pi}{\lambda}$, $p = \dfrac{h}{\lambda}$

$$\Rightarrow \quad \boxed{\Delta p.\Delta x = h/2}$$

usually one writes $\boxed{\Delta p.\Delta x \geq \hbar/2}$ approximate relation

In time t : $\boxed{\Delta w.\Delta t = \pi}$ $\Rightarrow$ since $\omega = 2\pi f, E = hf$

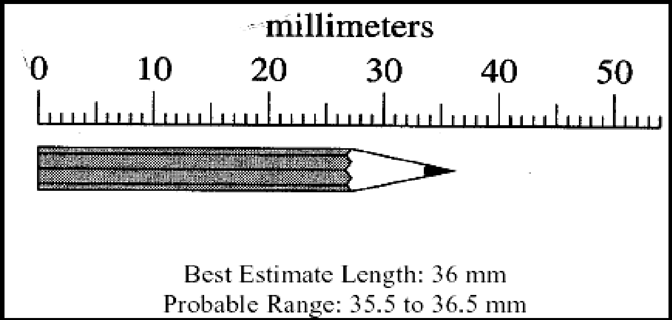
$$\Rightarrow \quad \boxed{\Delta E.\Delta t = h/2}$$

usually one writes $\boxed{\Delta E.\Delta t \geq \hbar/2}$ approximate relation

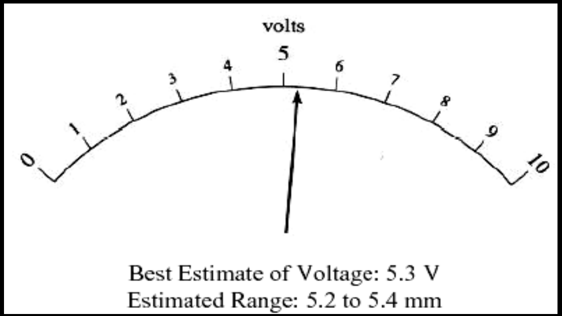
What do these inequalities mean physically?

Know the Error of Thy Ways: Measurement Error → Δ

- Measurements are made by observing something : length, time, momentum, energy
- All measurements have some (limited) precision….no matter the instrument used
- Examples:
  - How long is a desk ? L = (5 ± 0.1) m = L ± ΔL (depends on ruler used)
  - How long was this lecture ? T = (50 ± 1)minutes = T ± ΔT (depends on the accuracy of your watch)
  - How much does Prof. Sharma weigh ? M = (1000 ± 700) kg = m ± Δm
    - Is this a correct measure of my weight ?
      - Correct (because of large error reported) but imprecise
      - My correct weight is covered by the (large) error in observation



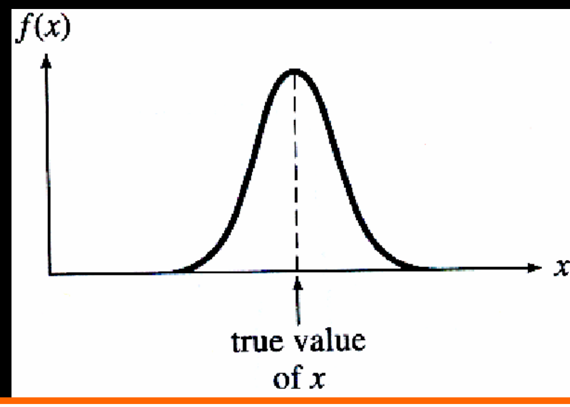
Length Measure



Voltage (or time) Measure

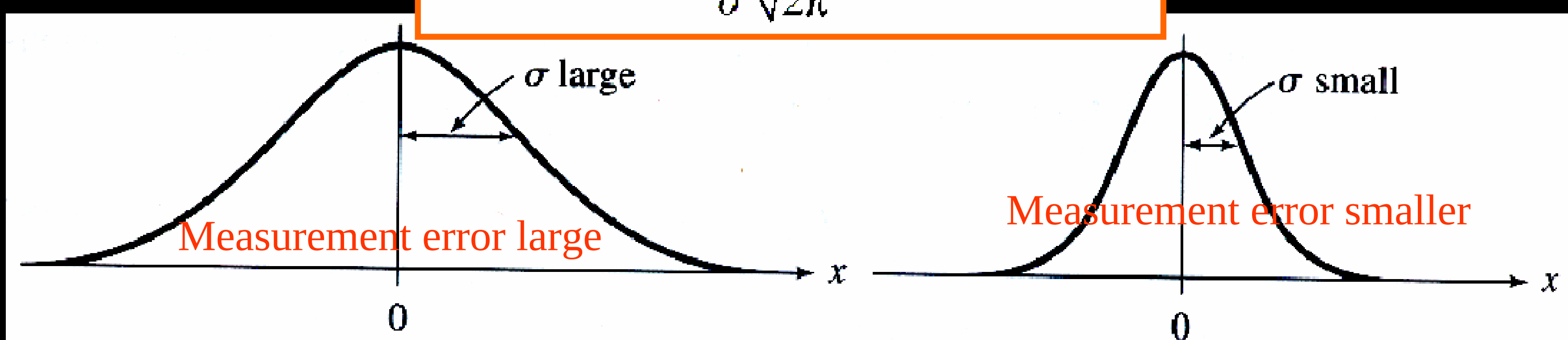
Measurement Error : $x \pm \Delta x$

- Measurement errors are unavoidable since the measurement procedure is an experimental one
- True value of an measurable quantity is an abstract concept
- In a set of repeated measurements with random errors, the distribution of measurements resembles a Gaussian distribution characterized by the parameter σ or Δ characterizing the width of the distribution

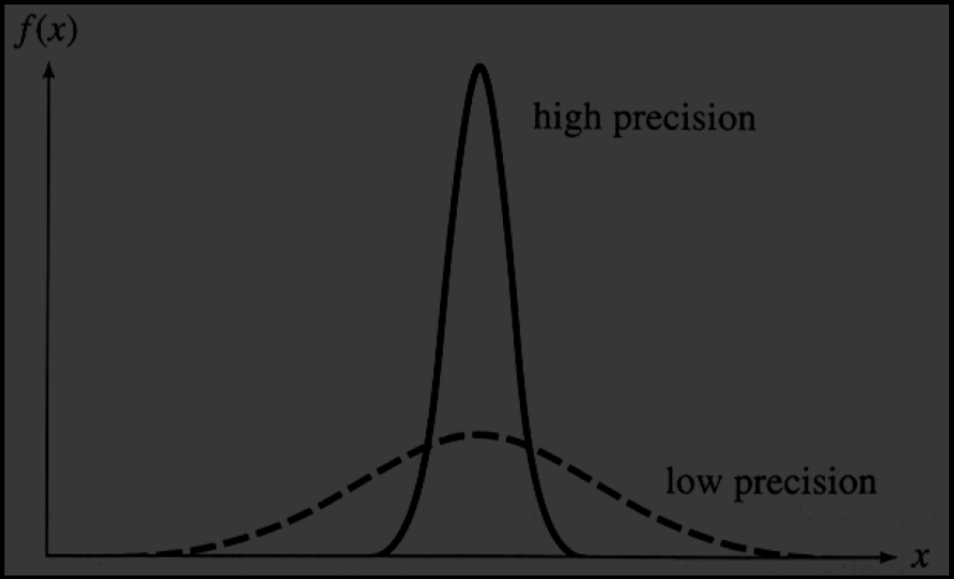


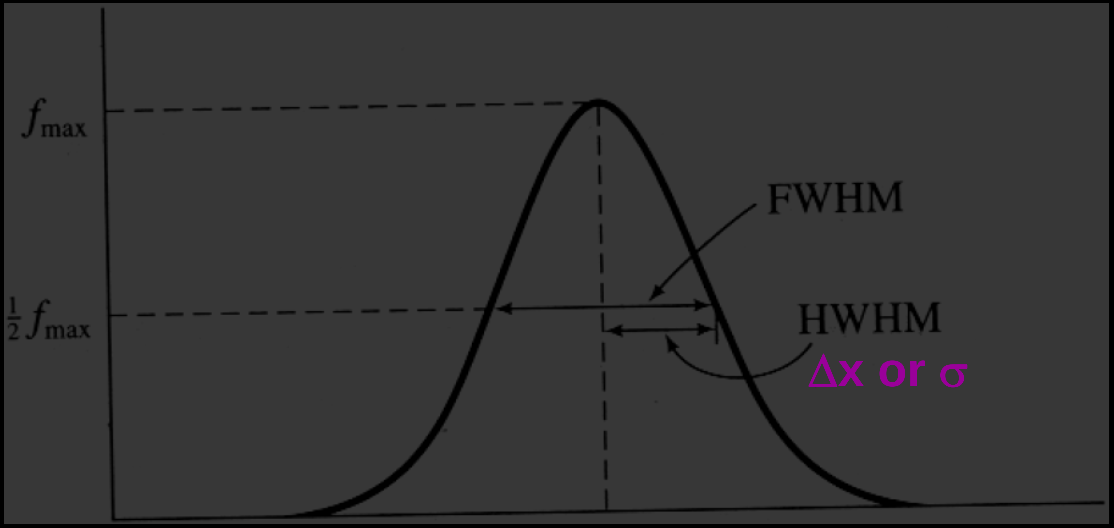
The Gauss, or Normal, Distribution

$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{(x-X)^2/2\sigma^2}.$$



Measurement Error : $x \pm \Delta x$





Interpreting Measurements with random Error : Δ

Will use Δ = σ interchangeably

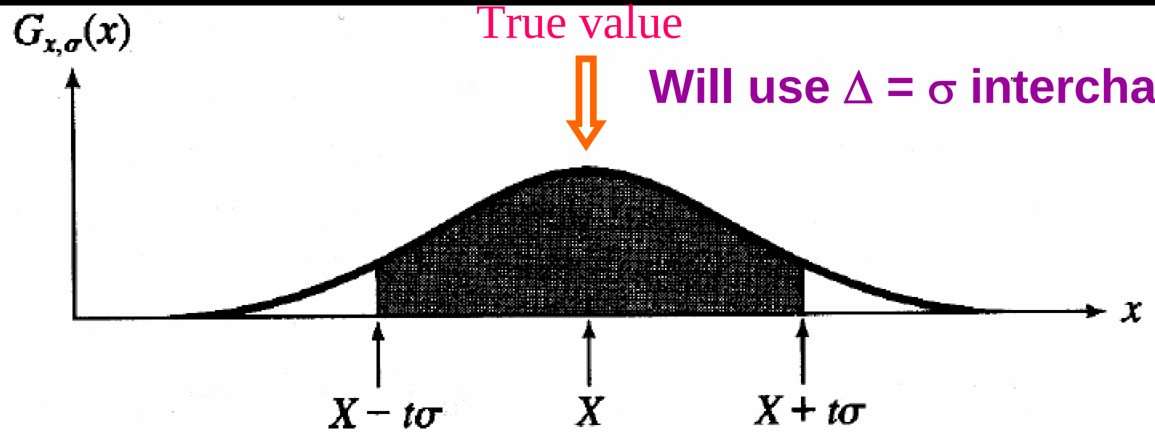
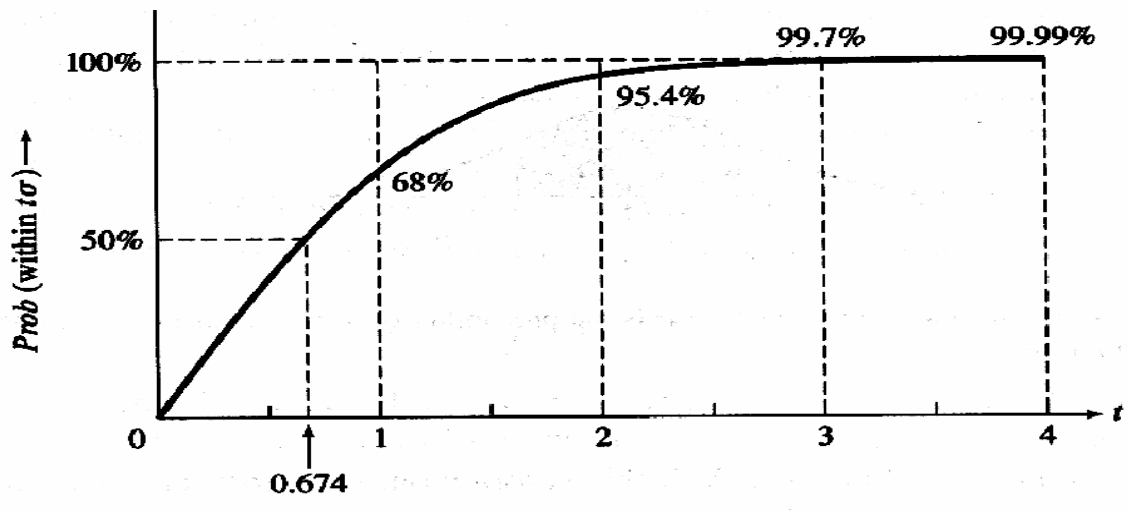


Figure 5.12. The shaded area between $X \pm t\sigma$ is the probability of a measurement within t standard deviations of X.



t	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.5	3.0	3.5	4.0
Prob (%)	0	20	38	55	68	79	87	92	95.4	98.8	99.7	99.95	99.99

Where in the World is Carmen San Diego?

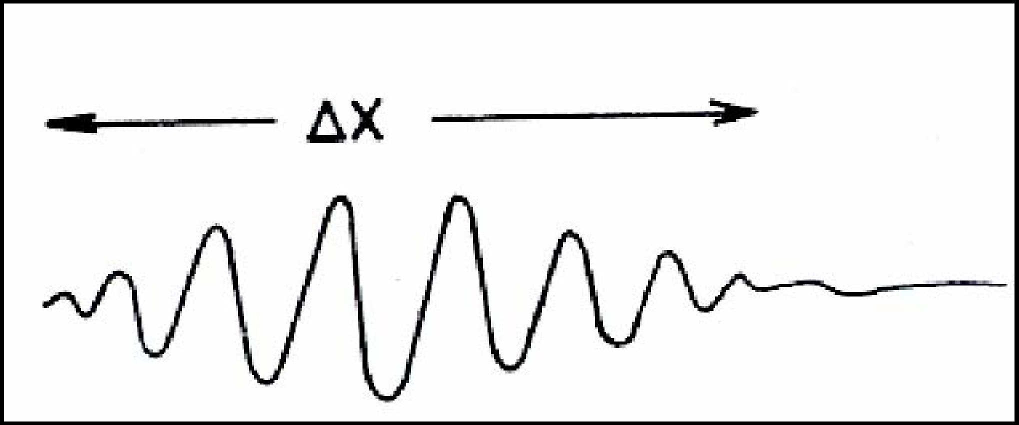
Carmen San Diego hidden inside a big box of length L

Suppose you can't see thru the (blue) box, what is you best estimate of her location inside box (she could be anywhere inside the box)

Your best unbiased measure would be $x = L/2 \pm L/2$

There is no perfect measurement, there are always measurement error

Wave Packets & Matter Waves



- What is the Wave Length of this wave packet?
  - made of waves with $\lambda-\Delta\lambda < \lambda < \lambda+\Delta\lambda$
  - De Broglie wavelength $\lambda = h/p$
    - → Momentum Uncertainty: $p-\Delta p < p < p+\Delta p$
- Similarly for frequency ω or f
  - made of waves with $\omega-\Delta\omega < \omega < \omega+\Delta\omega$

Planck's condition $E= hf = h\omega/2$

→ Energy Uncertainty: $E-\Delta E < E < E + \Delta E$

Back to Heisenberg's Uncertainty Principle

- $\Delta x . \Delta p \geq h/4\pi \Rightarrow$ If the measurement of the position of a particle is made with a precision Δx and a SIMULTANEOUS measurement of its momentum p_x in the X direction , then the product of the two uncertainties (measurement errors) can never be smaller than $\simeq h/4\pi$ irrespective of how precise the measurement tools

- $\Delta E . \Delta t \geq h/4\pi \Rightarrow$ If the measurement of the energy E of a particle is made with a precision ΔE and it took time Δt to make that measurement, then the product of the two uncertainties (measurement errors) can never be smaller than $\simeq h/4\pi$ irrespective of how precise the measurement tools

These rules arise from the way we constructed the wave packets describing Matter "pilot" waves

Perhaps these rules are bogus, can we verify this with some physical picture ??

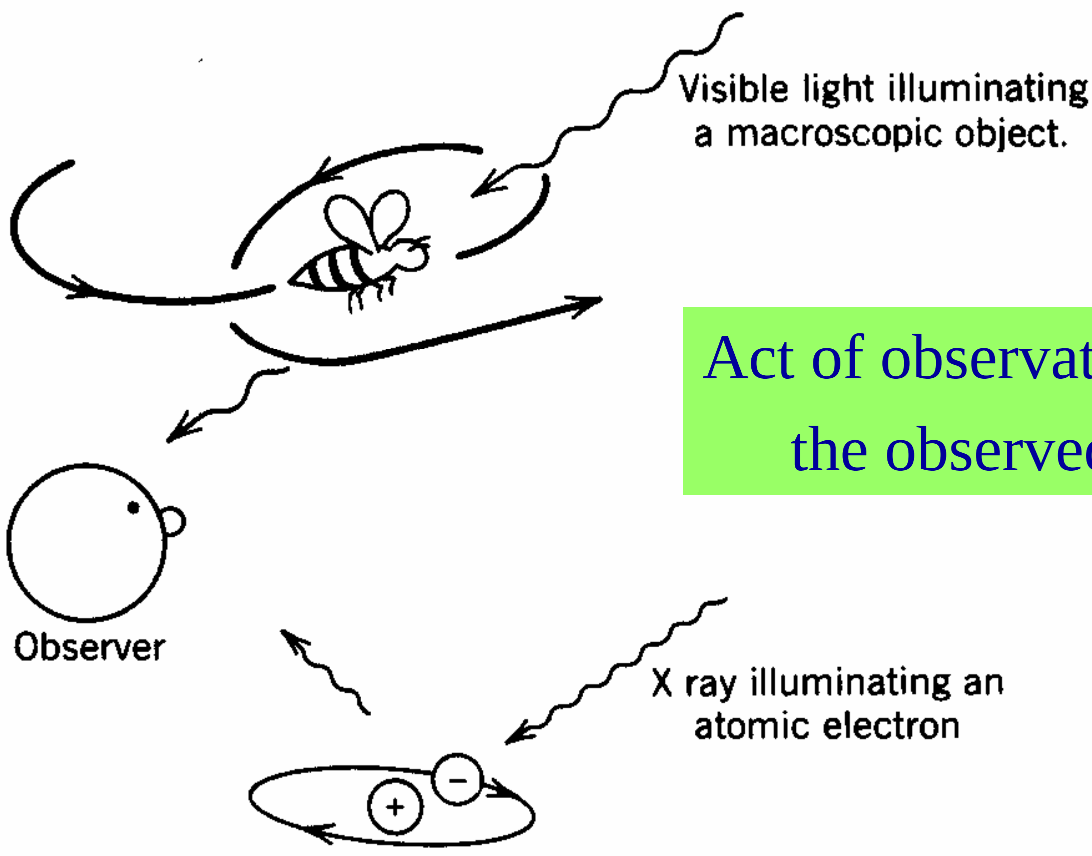
Are You Experienced ?

- What you experience is what you observe
- What you observe is what you measure
- No measurement is perfect, they all have measurement error: question is of the degree
  - Small or large Δ

- Uncertainty Principle and Breaking of Conservation Rules
  - Energy Conservation
  - Momentum Conservation

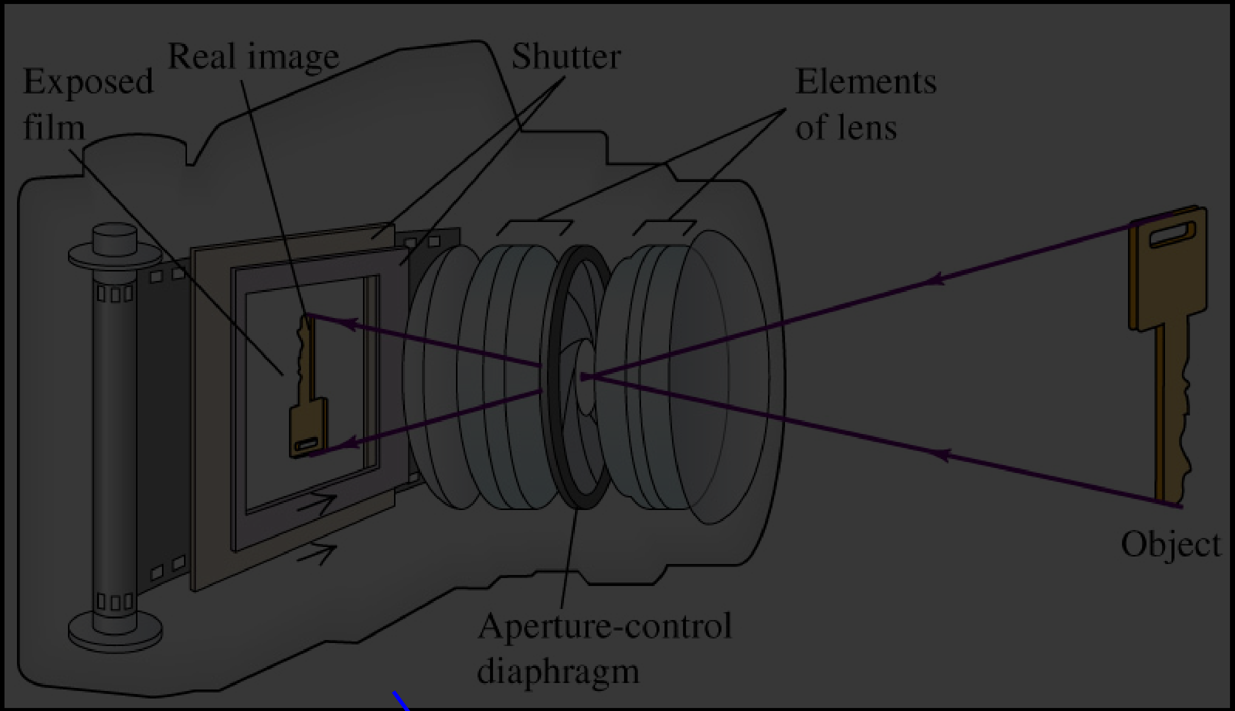
The Act of Observation (Compton Scattering)

Observations of particle motion by means of scattered illumination. When the incident wavelength is reduced to accommodate the size of the particle, the momentum transferred by the photon becomes large enough to disturb the observed motion.

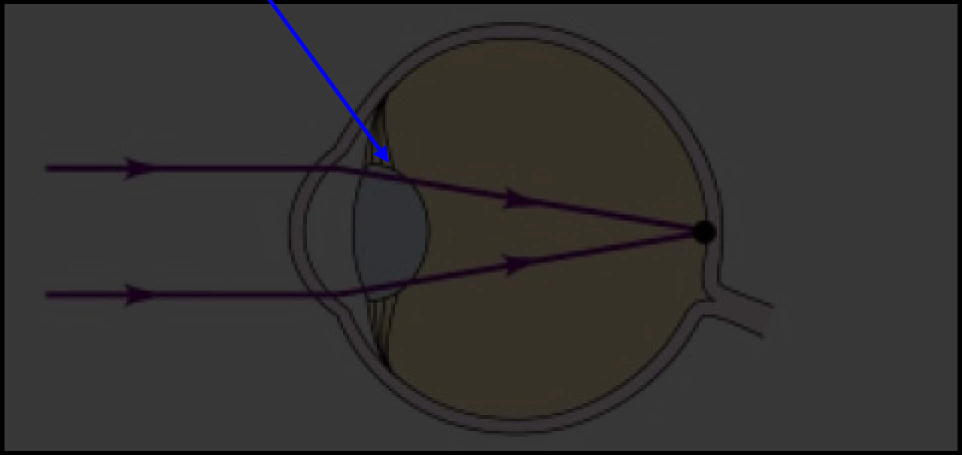


Act of observation disturbs the observed system

Act of Observation Tells All

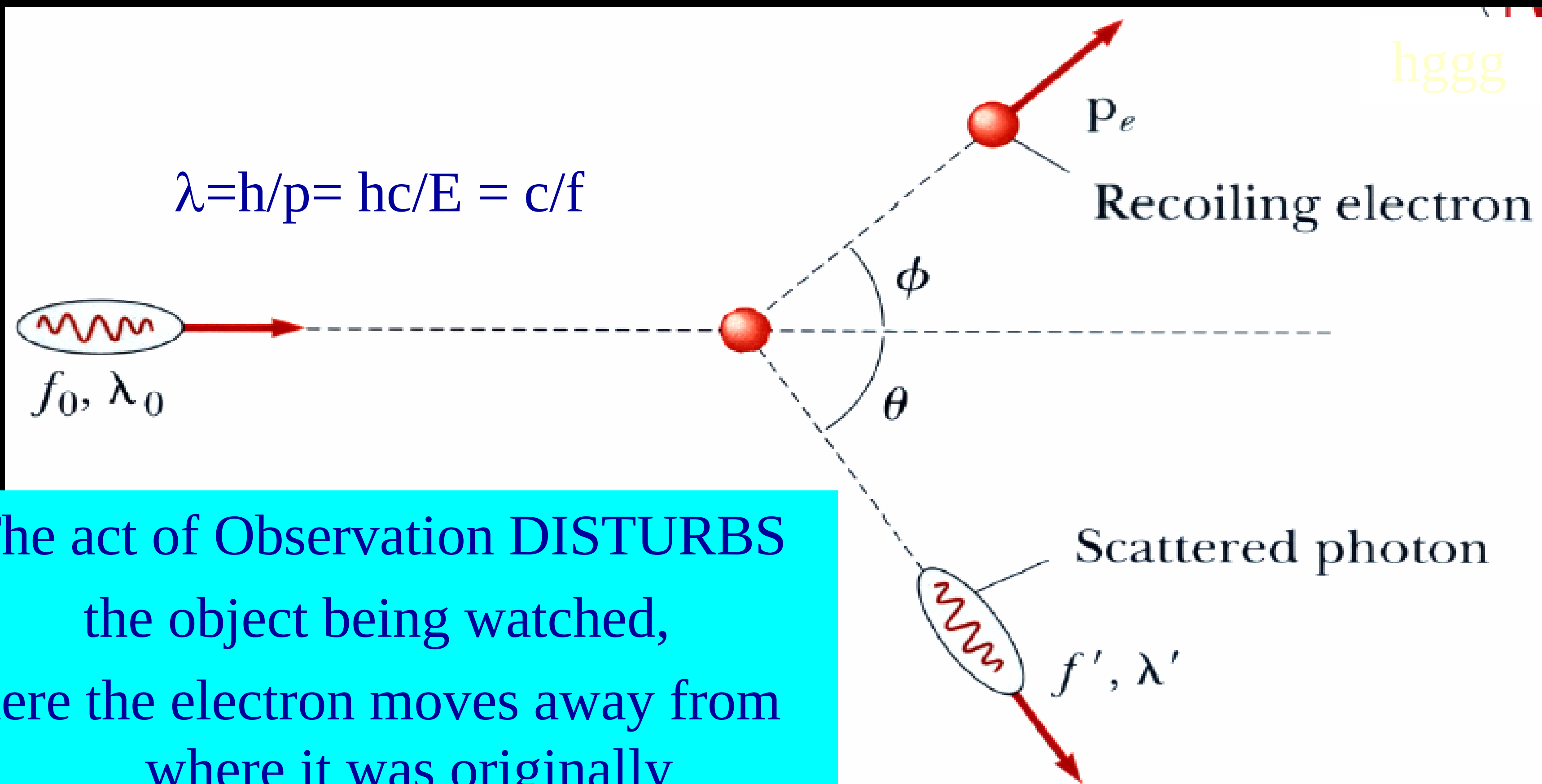


lens



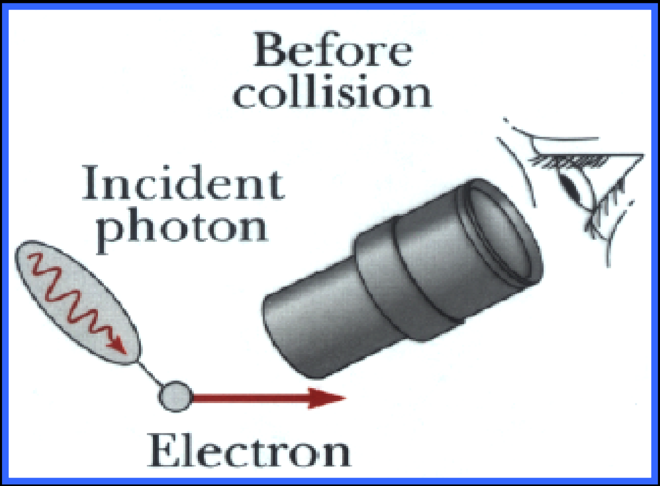
Compton Scattering: Shining light to observe electron

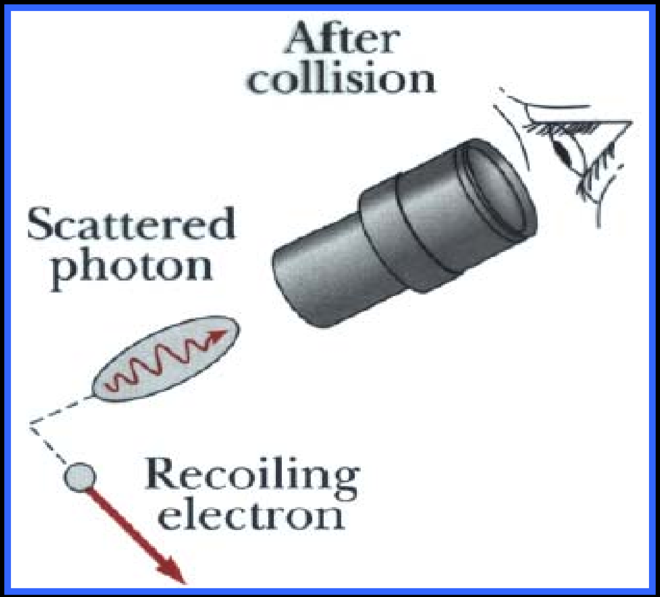
Photon scattering off an electron, Seeing→ the photon enters my eye

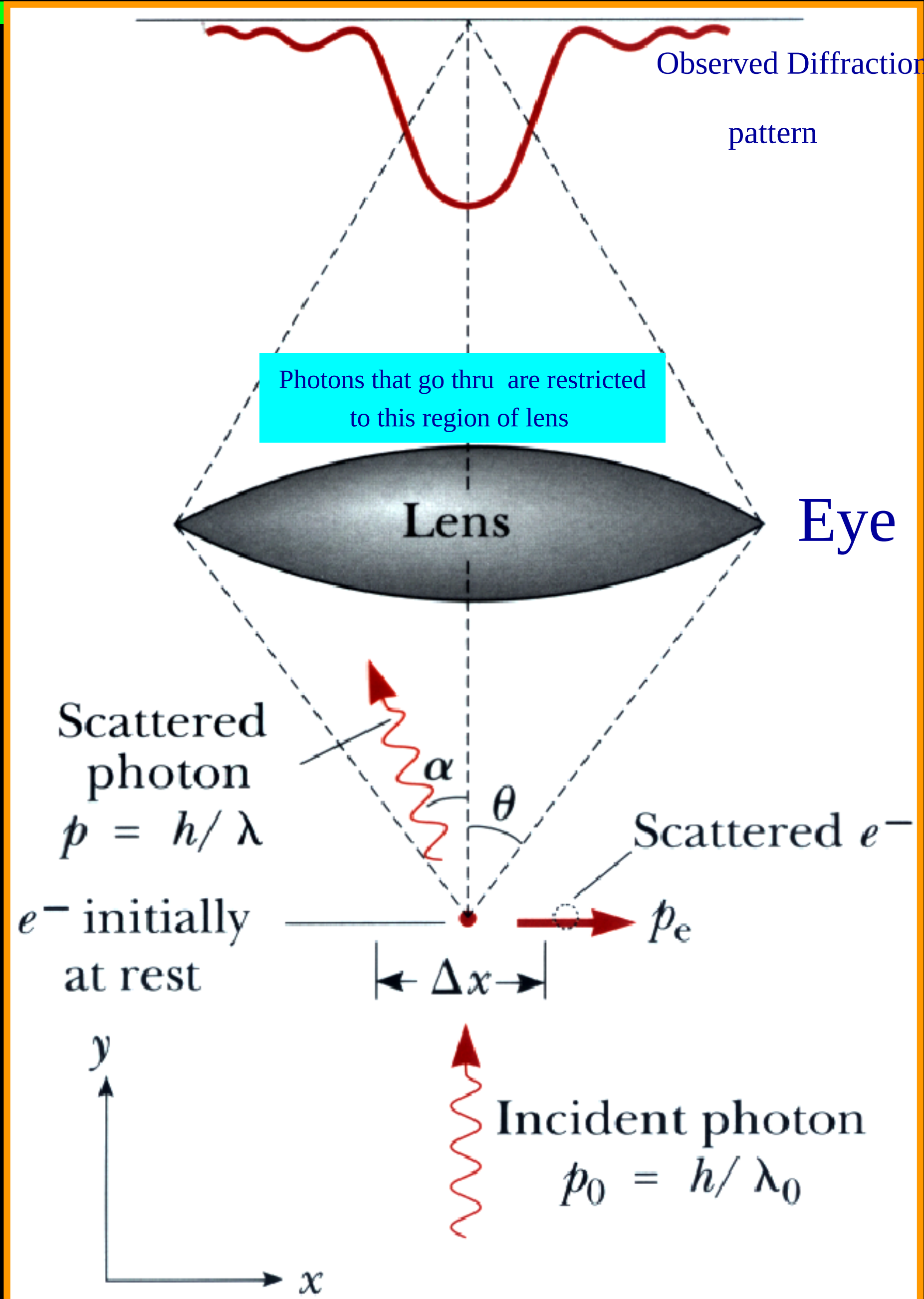


The act of Observation DISTURBS
the object being watched,
here the electron moves away from
where it was originally

Act of Watching: A Thought Experiment







Diffraction By a Circular Aperture (Lens)

See Resnick, Halliday Walker 6th Ed (on S.Reserve), Ch 37, pages 898-900

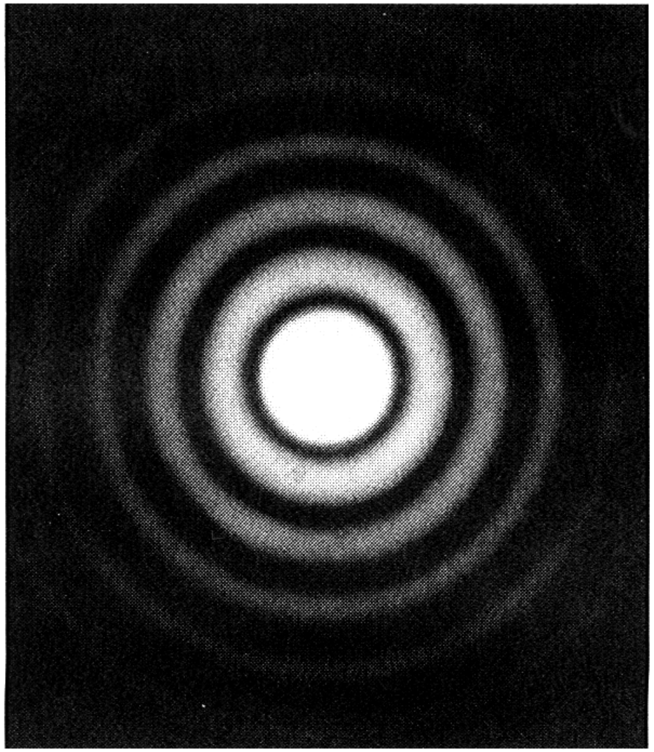


Fig. 37-9 The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

Diffracted image of a point source of light thru a lens (circular aperture of size d)

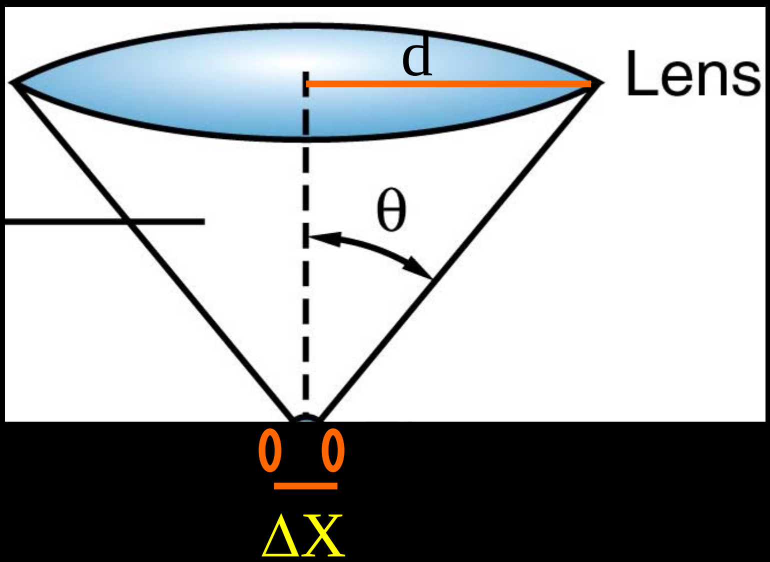
First minimum of diffraction pattern is located by

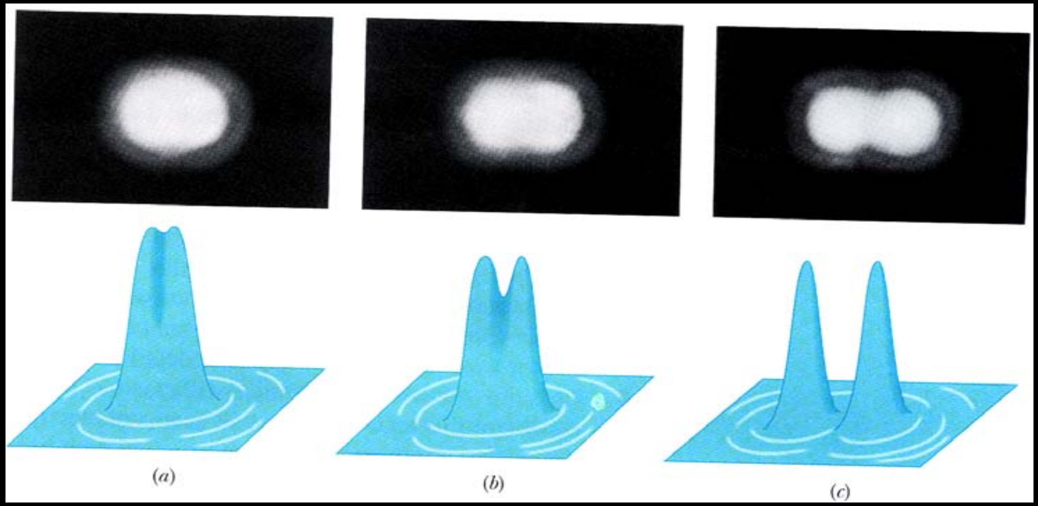
$$\sin\theta = 1.22\frac{\lambda}{d}$$

See previous picture for definitions of ϑ, λ, d

Resolving Power of Light Thru a Lens

Image of 2 separate point sources formed by a converging lens of diameter d, ability to resolve them depends on λ & d because of the Inherent diffraction in image formation



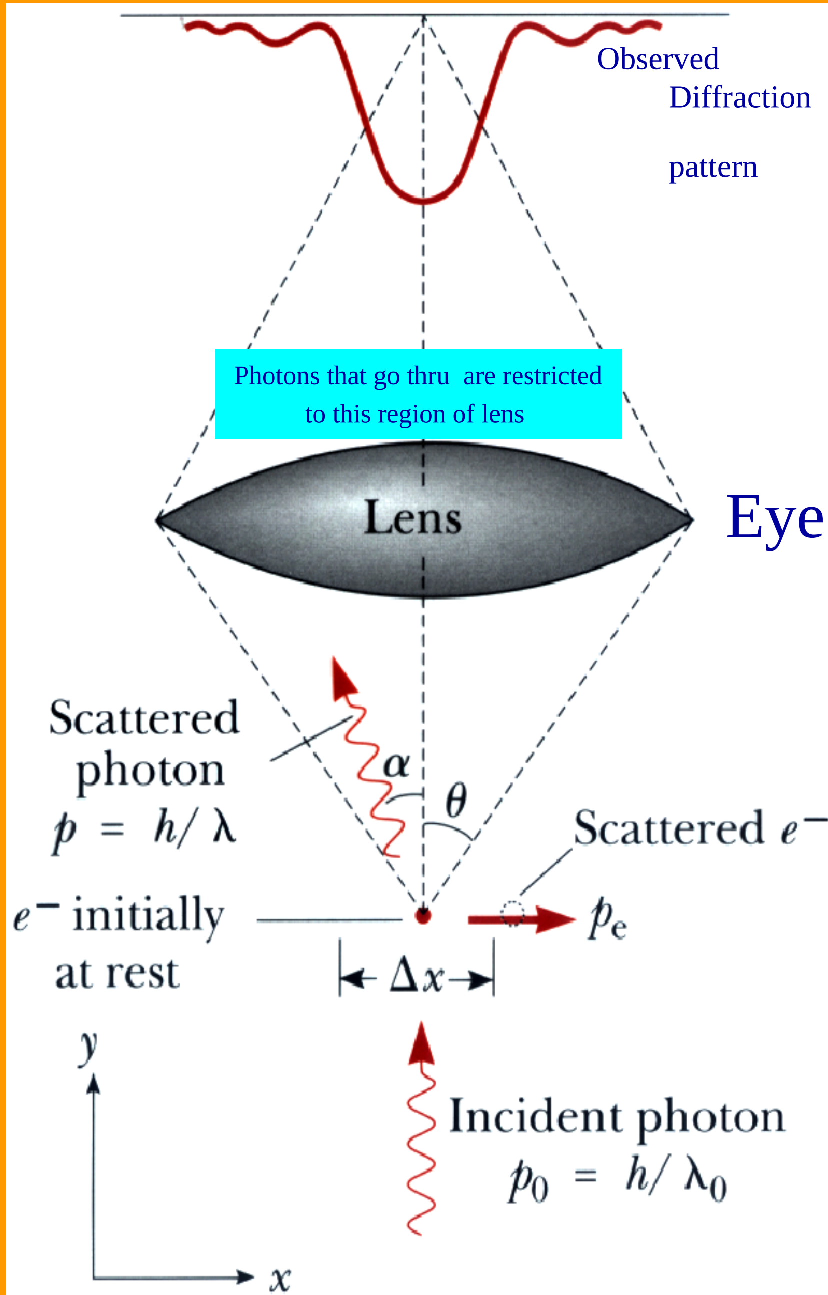


Not resolved   Barely resolved   Resolved

$$\text{Resolving power } \Delta x \simeq \frac{\lambda}{2\sin\theta}$$

θ depends on lens radius d

Putting it all together: Act of Observing an Electron



- Incident light (p,λ) scatters off electron
- To be collected by lens → γ must scatter thru angle α
  - $-\vartheta \leq \alpha \leq \vartheta$
- Due to Compton scatter, electron picks up momentum
  - P_X , P_Y

$$-\frac{h}{\lambda}\sin\theta \leq P_x \leq \frac{h}{\lambda}\sin\theta$$

electron momentum uncertainty is

$$\Delta p \cong \frac{\sim 2h}{\lambda}\sin\theta$$

- After passing thru lens, photon diffracts, lands somewhere on screen, image (of electron) is fuzzy
- How fuzzy ? Optics says shortest distance between two resolvable points is :

$$\Delta x = \frac{\lambda}{2\sin\theta}$$

- Larger the lens radius, larger the $\vartheta \Rightarrow$ better resolution

$$\Rightarrow \Delta p.\Delta x \simeq \left(\frac{2h\sin\theta}{\lambda}\right)\left(\frac{\lambda}{2\sin\theta}\right) = h$$

$$\Rightarrow \Delta p.\Delta x \geq \hbar/2$$

Aftermath of Uncertainty Principle

- Deterministic (Newtonian) physics topples over
  - Newton's laws told you all you needed to know about trajectory of a particle
    - Apply a force, watch the particle go !
      - Know every thing ! X, v, p , F, a
      - Can predict exact trajectory of particle if you had perfect device

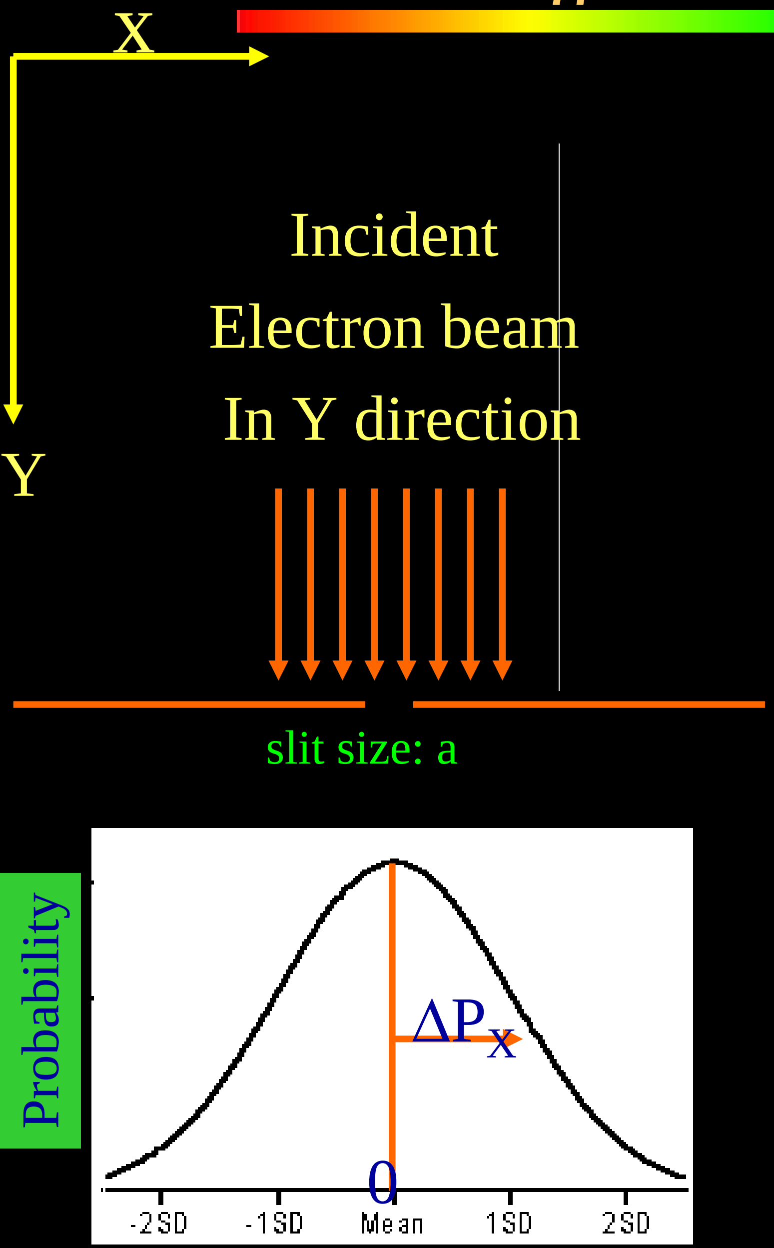
- No so in the subatomic world !
  - Of small momenta, forces, energies
  - Can't predict anything exactly
    - Can only predict probabilities
      - There is so much chance that the particle landed here or there
      - Cant be sure !....cognizant of the errors of thy observations

All Measurements Have Associated Errors

- If your measuring apparatus has an intrinsic inaccuracy (error) of amount Δp
- Then results of measurement of momentum p of an object at rest can easily yield a range of values accommodated by the measurement imprecision :
  - $-\Delta p \leq p \leq \Delta p$: you will measure any of these values for the momentum of the particle

- Similarly for all measurable quantities like X, t, Energy !

Matter Diffraction & Uncertainty Principle



X component P_X of momentum

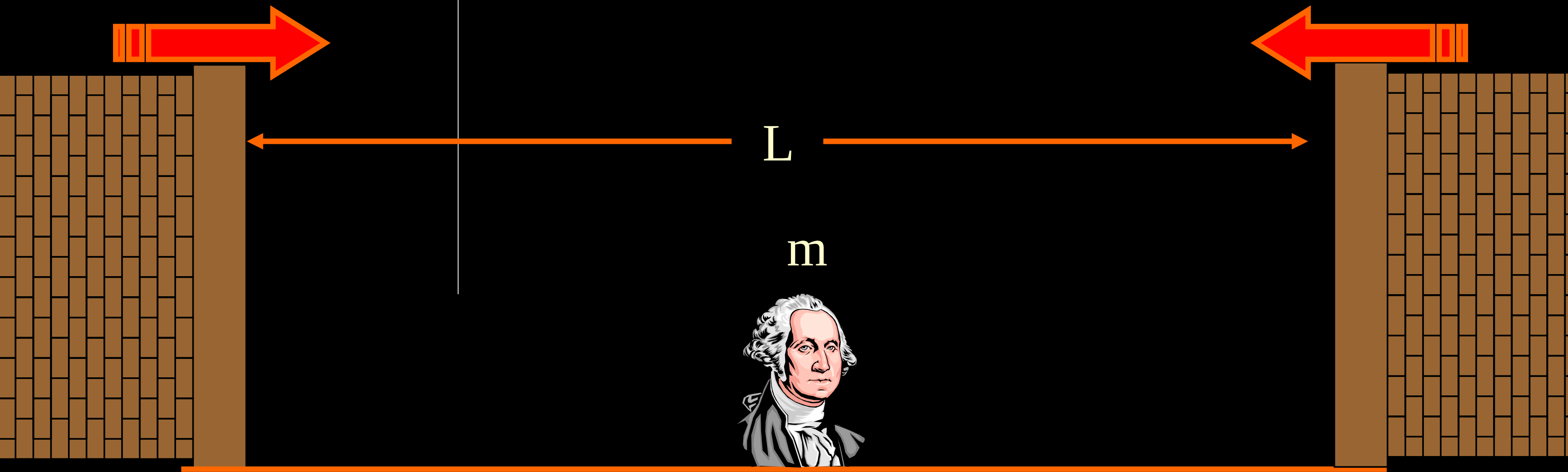
Momentum measurement beyond slit show particle not moving exactly in Y direction, develops a X component Of motion $-\Delta p_x \le p_x \le \Delta p_x$ **with**

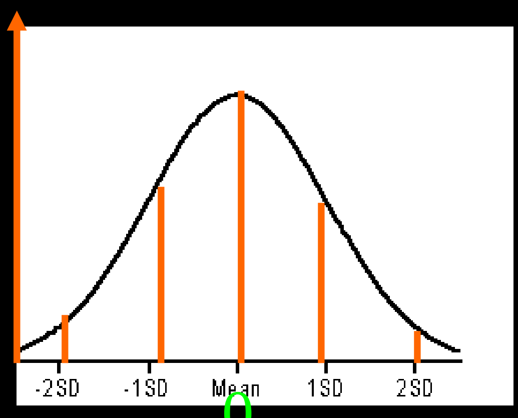
$$\Delta p_X = h/(2\pi\, a)$$

Particle at Rest Between Two Walls

Object of mass M at rest between two walls originally at infinity

What happens to our perception of George's momentum as the walls are brought in ?





George's Momentum p

On average, measure <p> = 0

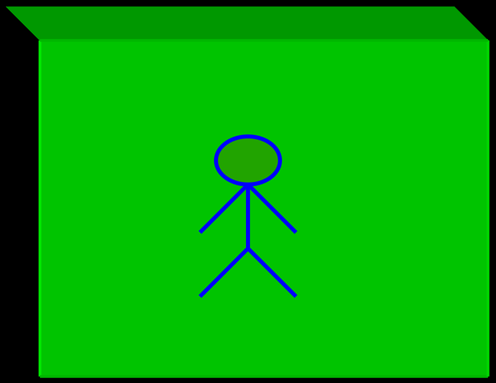
but there are quite large fluctuations!

Width of Distribution = ΔP

$$\Delta P = \sqrt{(P^2)_{ave} - (P_{ave})^2}\,; \qquad \Delta P \sim \frac{\hbar}{L}$$

Implications of Uncertainty Principles

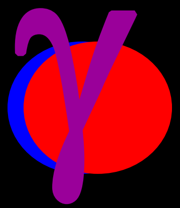
A bound "particle" is one that is confined in some finite region of space.



One of the cornerstones of Quantum mechanics is that bound particles can not be stationary – <u>even at Zero absolute temperature !</u>

There is a non-zero limit on the kinetic energy of a bound particle

Matter-Antimatter Collisions and Uncertainty Principle



Look at Rules of Energy and Momentum Conservation : Are they ?

$$E_{before} = mc^2 + mc^2 \quad \text{and} \quad E_{after} = 2mc^2$$

$P_{before} = 0$ **but since photon produced in the annihilation →** $P_{after} = 2mc$ **!**

Such violation are allowed but must be consumed instantaneously !
Hence the name “virtual” particles

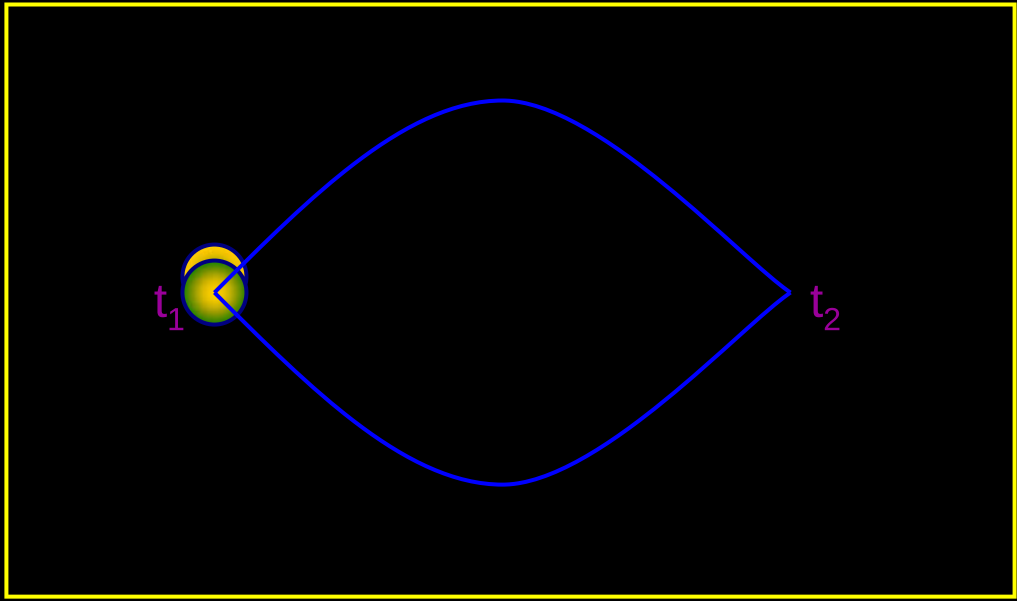
Fluctuations In The Vacuum : Breaking Energy Conservation Rules

Vacuum, at any energy, is bubbling with particle creation and annihilation

$\Delta E \cdot \Delta t \approx h/2\pi$ implies that you can (in principle) pull out an **elephant + anti-elephant** from NOTHING (Vaccum) but for a very very short time Δt !!

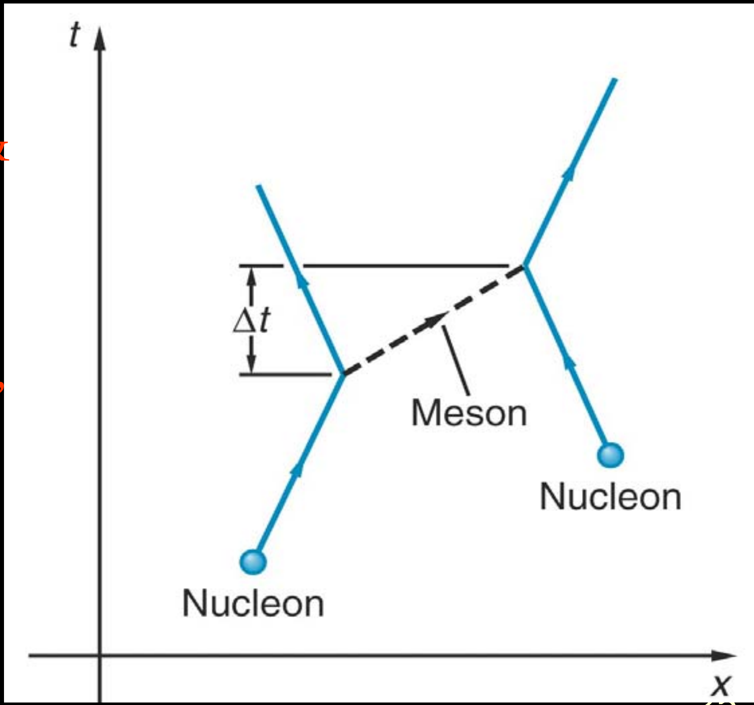
How Much Time : $\Delta t = \dfrac{\hbar}{2Mc^2}$

How cool is that !



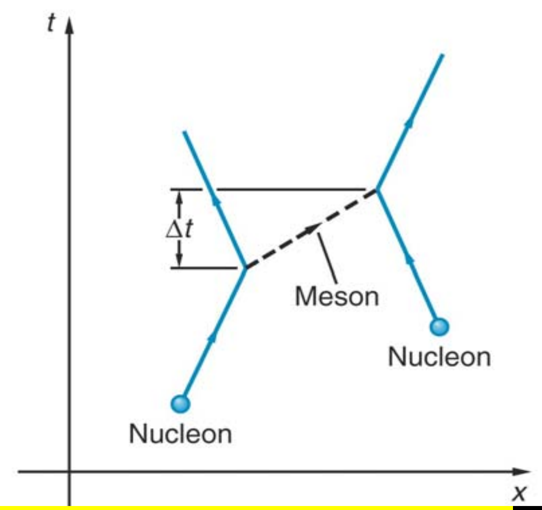
How far can the virtual particles propagate ? Depends on their mass

Repulsive force



- Strong Nuclear force can be modeled as exchange of virtual particles called $\pi^{\pm}$ mesons by nucleons (protons & neutrons)
- $\pi^{\pm}$ mesons are emitted by proton and reabsorbed by a neutron
- The short range of the Nuclear force is due to the “large” mass of the exchanged meson
- $M_{\pi} = 140$ MeV/c^2

Range of Nuclear Exchange Force



How long can the emitted virtual particle last?

$\Delta E \times \Delta t \geq \hbar$

The virtual particle has rest mass + kinetic energy

$\Rightarrow$ Its energy $\boxed{\Delta E \geq Mc^2}$

$\Rightarrow$ Particle can not live for more than $\boxed{\Delta t \leq \hbar / Mc^2}$

Range R of the meson (and thus the exchange force)

$R = c\Delta t = c\hbar / Mc^2 = \hbar / Mc$

For $M = 140\ MeV/c^2 \Rightarrow R \simeq \dfrac{1.06 \times 10^{-34}\ J.s}{(140 MeV/c^2) \times c^2 \times (1.60 \times 10^{-13}\ J/MeV)}$

$$\boxed{R \simeq 1.4 \times 10^{-15}\ m = 1.4\ fm}$$

Subatomic Cinderella Act

- Neutron emits a charged pion for a time Δt and becomes a (charged) proton
- After time Δt , the proton reabsorbs charged pion particle (π^-) to become neutron again
- But in the time Δt that the positive proton and π^- particle exist, they can interact with other charged particles
- After time Δt strikes , the Cinderella act is over !