Department of Physics
University of California San Diego

Quantum Universe (4E)
Prof. V. Sharma
Quiz # 5 (June 4, 2004)

Problem 1 : "Spin"ing Yarn [10 pts]:

Consider a right circular cylinder of radius R with mass M uniformly distributed throughout the cylinder volume. The cylinder is set into rotation with angular speed ω about its longitudinal axis. (a) Obtain the expression for the angular momentum **L** of the rotating cylinder. (b) If charge Q is distributed uniformly over the *curved surface only*, find the magnetic moment μ of the rotating cylinder. (c) Compare your expression for μ and **L** and deduce the g-factor for this object.

Problem 2: **Reversed Perspective [10 pts]:**

A moving charge sets up a magnetic field $B = \frac{\mu_0 I}{2R}$ at the center of a circular loop of radius R carrying current I. $\mu_0 = 4\pi\times10^{-7}$ N/A^2 is the magnetic permeability of the vaccum. Suppose that an atomic electron is in a state with angular momentum quantum number $\ell = 1$ (a) Use the classical expression to calculate the value of the magnetic field B produced by this electron at the nucleus. (b) The "spinning" proton that forms the nucleus has magnetic moment $\mu_P = \frac{g_P e\hbar}{2m_p}$. The proton's gyromagnetic ratio $g_P = 2.79$. ***Estimate*** the energy of magnetic interaction of the proton with the circulating electron.

Some Formulae and Constants

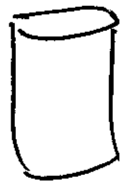
$$m_P = 1.7\times10^{-27}\,kg; m_e = 9.1\times10^{-31}\,kg$$

$$\hbar = 1.055\times10^{-34}\,J.s$$

$$\vec{L} = I\vec{\omega}$$

4E Quiz 5 Sols

1) a) $L = I\omega$.



$$I = \int r^2\, dm = \int r^2\, r\, dr\, d\theta\, h \cdot \rho = \frac{Mh}{V}\int_0^{2\pi} d\theta \int_0^R r^3\, dr$$

$$= \frac{M h \cdot 2\pi}{\pi R^2 h} \cdot \frac{R^4}{4} = \frac{1}{2} M R^2$$

So $\boxed{L = \frac{1}{2} M R^2 \omega}$

b) $\mu = IA = Q f \pi R^2 = \boxed{\frac{1}{2} Q \omega R^2}$

c) $\mu = \frac{Q}{2M} \cdot g \cdot L \Rightarrow \frac{1}{2} Q \omega R^2 = \frac{Q}{2M} \cdot g \cdot \frac{1}{2} M R^2 \omega$

$\Rightarrow \boxed{g = 2}$

2a] $B = \frac{\mu_0 I}{2R}$. $R = 4a_0$, b/c $n=2$

$$I = \frac{Q}{T} = \frac{Qv}{2\pi R} = \frac{Q(mvR)}{2\pi m R^2} = \frac{Q}{2\pi m R^2} L$$

But $L = \sqrt{l(l+1)}\,\hbar = \sqrt{2}\,\hbar$

So $I = \frac{Q\sqrt{2}\,\hbar}{2\pi m R^2}$

$$\therefore B = \frac{\mu_0 e \hbar \sqrt{2}}{4\pi m R^3} = \frac{10^{-7} e \hbar \sqrt{2}}{m R^3} = \boxed{0.28\,T}$$

b] $\mu_p = \frac{(2.79) e \hbar}{2 m_p} = 1.38 \times 10^{-26} \frac{J}{T}$

So $|U| = |\mu \cdot B| = \mu_p B = \boxed{2.4 \times 10^{-8}\,eV}$