Department of Physics University of California San Diego	Modern Physics (4E) Prof. V. Sharma Final Exam (June 11, 2004)

Problem 1: Mystery Metal:[20 pts]

Photons of wavelength 450nm are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular arc of radius 20cm in a magnetic field whose strength is equal to 2.0×10^{-5}T. What is the work function of the metal?

Problem 2: Quantum Pool:[20 pts]

An x-ray photon of wavelength 0.02480nm strikes a free stationary electron. The photon scatters off at 90^o with respect to the direction of incidence. Determine (a) the momentum of the incident photon (b) the momentum of the scattered photon (c) the kinetic energy of the scattered electron, and (d) the wavelength of the scattered photon.

Problem 3: Across The Universe! :[20 pts]

An air gun is used to shoot 1.0g particles at 100m/s through a bore (hole) of diameter 2.0mm. How far from the rifle must an observer be to see the beam spread by 1.0cm because of the uncertainty principle?

Problem 4: Harmonic Oscillator [30 pts]

Consider a 3D harmonic oscillator, described by the potential $V(x,y,z)=\frac{1}{2}m\omega^2(x^2+y^2+z^2)$. (a) What is the energy of the ground state of this system? What is the degeneracy of this energy? (b) Write down the wavefunction of the ground state. (c) Find the expectation value $\langle xy \rangle$ for the ground state. (d) What is the energy of the first excited state? What is the degeneracy of this energy?

Problem 5: Step On It: [30 pts]

A free particle of mass m with wave number k_1 is traveling to the right. At x=0, the potential jumps from zero to V_0 and remains at this value for positive x. (a) If the total energy is $E=\frac{\hbar^2k_1^2}{2m}=2V_0$, what is the wave number k_2 in the region x>0 ? (b) Calculate the transmission (T) coefficient at the potential step. (c) If 10^6 particles with wave number k_1 are incident upon the potential step, how many particles are expected to continue along the positive x direction ? (d) How does your answer to part (c) compare with the classical predictions?

Problem 6: Déjà vu: [30 pts]

Consider a particle in a 1-D Harmonic oscillator potential ($U(x) = \frac{1}{2}m\omega^2 x^2$) described initially by a wave that is a superposition of the ground state and the first excited states of the oscillator: $\Psi(x,0) = C[\psi_0(x) + \psi_1(x)]$ (a) show that the value $1/\sqrt{2}$ normalizes this wavefunction assuming ψ_1 and ψ_2 are themselves normalized. (b) Find the expression for $\Psi(x,t)$ at any later time t. (c) Show that the average energy in this state is the arithmetic mean $\left(E_1 + E_2\right)/2$ of the ground state and the first excited state energies E_1 and E_2 . (d) Show that the average particle position oscillates with time as $< x >= x_0 + A\cos(\Omega t)$ where $x_0 = \frac{1}{2}\left(\int x\,|\psi_0|^2\,dx + \int x\,|\psi_1|^2\,dx\right)$ and $A = \int x\psi_0^*\psi_1 dx$ and $\Omega = \left(E_2 - E_1\right)/\hbar$.

Problem 7: Lithium: The Musical! [30 pts]

For the ground state of the Li^{++} ion, calculate the average (a) potential energy and (b) kinetic energy.

Problem 8: The Incredible Shrinking Atom:[20 pts]

A collection of hydrogen atoms is placed in a 3.5 T magnetic field. Ignoring the effects of spin, what are the possible wavelengths of photons emitted when an atom transitions from a 3*s* state to a 2*p* state?

Some Useful Numbers, Equations and Identities

$$\text{Speed of Light, } c = 2.998 \times 10^8 \text{m/s}; \ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$p = \frac{mu}{\sqrt{1 - u^2/c^2}}; \ E = \frac{mc^2}{\sqrt{1 - u^2/c^2}} = K + mc^2$$

$$E^2 = p^2c^2 + m^2c^4$$

$$For\ x < 1; (1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{2}x^2 \pm \frac{n(n-1)(n-2)}{3}x^3 + ..$$

$$\text{Planck's Constant, } h = 6.626 \times 10^{-34} \text{J} \times \text{S} = 4.136 \times 10^{-15} \text{eV} \times \text{S}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{J}; \quad 1 \text{ MeV/c} = 5.344 \times 10^{-22} \text{Kg.m/s}$$

$$\text{Coulomb's Constant, } k = 8.99 \times 10^9 \text{N} \times \text{m}^2/\text{C}^2$$

$$\text{Gravitational Constant, } G = 6.67 \times 10^{-11} \text{N} \times \text{m}^2/\text{kg}^2$$

$$\text{Stefan} - \text{Boltzmann's Constant, } \sigma = 5.670399 \times 10^{-8} \text{W/m}^2 \times \text{K}^4$$

$$\text{Wien's Wavelength Displacement Constant} = 2.898 \times 10^{-3} \text{m} \times \text{K}$$

$$\text{Boltzmann's Constant, } k = 1.381 \times 10^{-23} \text{J/K}$$

$$\text{Electron Mass} = 9.11 \times 10^{-31} \text{Kg}; \quad \text{Electron Charge} = 1.602 \times 10^{-19} \text{C}$$

$$\text{Atomic Mass Unit } u = 1.6606 \times 10^{-27} \text{Kg or } 931.5 \text{ MeV/c}^2$$

$$\text{Proton Mass} = 1.673 \times 10^{-27} \text{Kg or } 1.0073\text{u}$$

$$\text{Neutron Mass} = 1.675 \times 10^{-27} \text{Kg or } 1.0087\text{u}$$

$$\text{Electron Rest Energy} = 0.511 \text{ MeV/c}^2; \quad \text{Proton Rest Energy} = 938 \text{ MeV/c}^2$$

$$1\text{kW} \times \text{Hour} = 3.6 \times 10^6 \text{J}$$

$$\text{Intensity of Black Body Radiation } I = \sigma \times T^4$$

$$\text{Force on a charged particle in B field : } \overrightarrow{F} = q\overrightarrow{v} \times \overrightarrow{B}$$

$$\text{Centripetal Acc.} = v^2/R \text{ where v and R are velocity and radius of orbit}$$

$$\text{Momentum of a charged particle in B field : } p = qBR$$

$$\text{Photoelectric Effect : } K_{max} = hf - \phi$$

$$\text{Compton Wavelength } \lambda_c \text{ for scattering off electron} = 0.00243 \text{ nm}$$

$$\text{In Compton Scattering } \Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\text{Construct. interfer. when path diff. between two adjacent rays is } d\sin\phi = n\lambda$$

$$\text{TDSE}: \ -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

$$\text{TISE}: -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x) = E\psi(x)$$

$$\text{Ground state energy } E_1 \text{ for particle in 1D rigid box}(0 < x < L) \ = \frac{\pi^2\hbar^2}{2mL^2}$$

$$\text{Normalized wavefunction of particle in 1D rigid box}(0 < x < L) \ = \sqrt{\frac{2}{L}}\sin(\frac{n\pi}{L}x)$$

$$\text{Operator}: \ [\hat{p}] \ = \ \frac{\hbar}{i}\frac{\partial}{\partial x}, \quad [\hat{p^2}] \ = \ -\hbar^2\frac{\partial^2}{\partial x^2}, \ [\hat{E}] \ = \ i\hbar\frac{\partial}{\partial t}$$

Time dependent form of Wave Function: $\Psi(\vec{r},t) = \psi(\vec{r})e^{-i\omega t}\,;\omega \ = \ E/\hbar$

$$\text{Uncertainty on Observable Q}: \quad \Delta Q = \sqrt{\langle Q^2\rangle - \langle Q\rangle^2}$$

$$\text{ExpectationValue} < Q > \ = \int \psi(x)^*[Q]\psi(x)dx$$

$$< f(r) >= \int_0^\infty f(r)P(r)dr$$

$$\int u \cdot dv \ = \ uv \ - \ \int v \cdot du$$

$$\int \sin x \, dx = -cosx$$

$$\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4}\,; \quad \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4}$$

$$\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}$$

$$\int x\cos^2 x\,dx = \frac{x^2}{2} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8}$$

$$\int x^2\sin^2 x\,dx = \frac{x^3}{6} - (\frac{x^2}{4} - \frac{1}{8})\sin 2x - \frac{x\cos 2x}{4}$$

$$\sin(\theta_1)\sin(\theta_2) \;=\; (1/2)[\cos(\theta_1-\theta_2) - \cos(\theta_1+\theta_2)]$$

$$\cos(\theta_1)\cos(\theta_2) \;=\; (1/2)[\cos(\theta_1-\theta_2) - \cos(\theta_1+\theta_2)]$$

$$\sin(\theta_1)\cos(\theta_2) \;=\; (1/2)[\sin(\theta_1-\theta_2) - \sin(\theta_1+\theta_2)]$$

$$\int_{-1}^{+1} e^{-ax}\,dx = \frac{1}{a}[e^{a} - e^{-a}]$$

$$\int_{-1}^{+1} xe^{-ax}\,dx = \frac{1}{a^2}[e^{a} - e^{-a} - a(e^{a} + e^{-a})]$$

$$\int_{0}^{+\infty} e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}},\ a > 0$$

$$\int_{0}^{+\infty} x^2 e^{-ax^2}\,dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}},\ a > 0$$

$$\int_{0}^{+\infty} x^{(2n)} e^{-ax^2}\,dx = \frac{1.3..(2n-1)}{2^{(n+1)}}\sqrt{\frac{\pi}{a^{2n+1}}},\ a > 0$$

$$\int_{0}^{+\infty} x^{(2n+1)} e^{-ax^2}\,dx = \frac{n!}{2a^{(n+1)}},\ a > 0$$

$$\int_{0}^{+\infty} x^{n} e^{-x}\,dx = n!$$

$$\int_{0}^{+\infty} x^{n} e^{-x/\alpha}\,dx = n!\alpha^{n+1}$$

$$\int_{1}^{+\infty} e^{-ax}\,dx = \frac{e^{-a}}{a}$$

$$\int_{1}^{+\infty} xe^{-ax}\,dx = \frac{e^{-a}}{a^2}(1+a)$$

$$\int_{0}^{b} x^2 e^{-x}\,dx = 2 - (2 + 2b + b^2)e^{-b}$$

$$\int_0^b x^3 e^{-x}\, dx = 6 - (6 + 6b + 3b^2 + b^3)e^{-b}$$

$$\int_0^b x^4 e^{-x}\, dx = 24 - (24 + 24b + 12b^2 + 4b^3 + b^4)e^{-b}$$

$$\int_0^b x^2 e^{-x}\, dx = 2 - (2 + 2b + b^2)e^{-b}$$

The ground state wavefunction for 1D oscillator under $U(x) = \frac{1}{2}m\omega^2 x^2$ has the form:

$$\psi_0(x) \propto e^{-\frac{m\omega x^2}{2\hbar}}$$

The wavefunction for Oscillator's first excited state:

$$\psi_1(x) \propto \sqrt{\frac{m\omega}{\hbar}}\, x e^{-\frac{m\omega x^2}{2\hbar}}$$

Next excited state:

$$\psi_2(x) \propto (1 - \frac{2m\omega x^2}{\hbar})e^{-\frac{m\omega x^2}{2\hbar}}$$

The energy of the nth Oscillator state $E_n = (n + \frac{1}{2})\hbar\omega$.

Volume element in Sph. polar coordinates is $dV = r^2 dr sin\theta d\theta d\phi$

$$\text{Reduced Mass } \mu = \frac{M_1.M_2}{M_1 + M_2}$$

$$\mathrm{R}_{1,0}(\mathrm{r}) = (\frac{\mathrm{Z}}{\mathrm{a_o}})^{3/2} 2\mathrm{e}^{-\frac{\mathrm{Zr}}{\mathrm{a_o}}}$$

$$\mathrm{R}_{2,0}(\mathrm{r}) = (\frac{\mathrm{Z}}{2\mathrm{a_o}})^{3/2} (2 - \frac{\mathrm{Zr}}{\mathrm{a_0}})\mathrm{e}^{-\frac{\mathrm{Zr}}{2\mathrm{a_o}}}$$

$$\mathrm{R}_{2,1}(\mathrm{r}) = (\frac{\mathrm{Z}}{2\mathrm{a_o}})^{3/2} \frac{\mathrm{Zr}}{\sqrt{3}\mathrm{a_0}}\mathrm{e}^{-\frac{\mathrm{Zr}}{2\mathrm{a_o}}}$$

$$\text{Radial Prob. Density } \mathrm{P(r)} = \mathrm{r}^2|\mathrm{R(r)}|^2$$

$$\mathrm{Y_o^o}(\theta, \phi) = \Theta(\theta)\Phi(\phi) = \frac{1}{2\sqrt{\pi}}$$

$$\mathrm{Y}_1^{\mathrm{o}}(\theta,\phi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\theta \text{ and } \mathrm{Y}_1^{\pm 1}(\theta,\phi) = \mp\frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta \mathrm{e}^{\pm \mathrm{i}\phi}$$

$$\text{Orbit radius } \mathrm{r_n} = \frac{\mathrm{n}^2\hbar^2}{\mathrm{Zm_eke^2}}$$

$$\text{bohr radius } \mathrm{a_0} = \frac{\hbar^2}{\mathrm{m_eke^2}} = 0.0529 \text{ nm}$$

$$\text{Energy } \mathrm{E_n} = \frac{-\mathrm{kZ^2e^2}}{2\mathrm{a_o}}\frac{1}{\mathrm{n}^2} \text{ for } \mathrm{n} = 1, 2, 3, 4, \ldots$$

$$\text{Rydberg Constant } \mathrm{R} = 1.0973732 \times 10^7 \mathrm{m}^{-1}$$

The Total Magnetic Moment

$$\vec{\mu} = \vec{\mu}_o + \vec{\mu}_s = \frac{-e}{2m_e}[\vec{L} + g\vec{S}]$$

Magnetic Potential Energy Under a B Field

$$U = -\vec{\mu} \cdot \vec{B}$$

Phys 4E Final Exam Solns

1] $KE_{max} = hf - \phi$, and we know $f = \frac{c}{\lambda}$.

So we need to find KE_{max}. In a mag. field, we can write $\frac{mv^2}{r} = qvB \Rightarrow \frac{mv}{r} = qB \Rightarrow v = \frac{qBr}{m}$.

$$\therefore KE_{max} = \frac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \frac{1}{2}\frac{q^2B^2r^2}{m}.$$

$$\therefore \phi = \frac{hc}{\lambda} - \frac{q^2B^2r^2}{2m} = 2.76\,eV - 1.40\,eV = \boxed{1.36\,eV}$$

2] a] $E_{incident} = p_i c \Rightarrow \frac{hc}{\lambda} = p_i c \Rightarrow p_i = \frac{hc}{\lambda c} = \boxed{50\ keV/c}$

b] $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

Since $\theta = 90°$, $\cos\theta = 0$. $\therefore \lambda' = \lambda + \frac{h}{m_e c}$

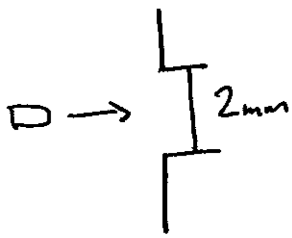
$\Rightarrow \lambda' = 0.02723\ nm$

So $\boxed{p_f = 45.5\ keV/c}$

c] Cons. of energy: $KE_\gamma + m_e c^2 = KE_{\gamma'} + m_e c^2 + KE_e$

$\therefore KE_e = KE_\gamma - KE_{\gamma'} = \boxed{4.5\ keV}$

2] Already did it: $\boxed{\lambda' = 0.02723\,nm}$

3] 

So say $\Delta x = 1\,mm$.

$$\therefore \Delta p = \frac{\hbar}{2\Delta x} = 5.27\times10^{-32}$$

So we can draw similar triangles:

$$\therefore \frac{\Delta p/m}{100\,m/s} = \frac{1/2\,cm}{L} \Rightarrow L = \frac{(\frac{1}{2}cm)(100\,\frac{m}{s})}{\Delta p/m}$$

$$\boxed{L = 9.49\times10^{27}\,m}$$

Bigger than the universe!

4a] $E_{n_x n_y n_z} = \hbar\omega\left[n_x + \frac{1}{2} + n_y + \frac{1}{2} + n_z + \frac{1}{2}\right] = \hbar\omega\left[\frac{3}{2} + n_x + n_y + n_z\right]$

So $n_x = n_y = n_z = 0 \Rightarrow \boxed{E_{000} = \frac{3}{2}\hbar\omega}$

This is not degenerate.

b] $$\boxed{\psi(x,y,z) = \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-\frac{m\omega}{2\hbar}(x^2+y^2+z^2)}}$$ (Product of all 3)

c] $$\langle xy\rangle = \left(\frac{m\omega}{\pi\hbar}\right)^{3/2} \underbrace{\int_{-\infty}^{\infty} dx\, x\, e^{-\frac{m\omega}{\hbar}x^2}}_{0} \underbrace{\int_{-\infty}^{\infty} dy\, y\, e^{-\frac{m\omega}{\hbar}y^2}}_{0} \int_{-\infty}^{\infty} dz\, e^{-\frac{m\omega}{\hbar}z^2}$$

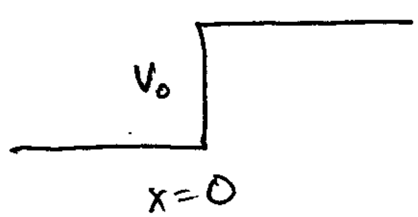
$$= \boxed{0}$$

d] $$\boxed{E_{100} = \frac{5}{2}\hbar\omega.}$$ This is degenerate,

since $E_{100} = E_{010} = E_{001}$.

$$\boxed{\text{Degeneracy} = 3}$$

5a]



a] In region II, Sch. Eqn. is $\frac{d^2\psi}{dx^2} = -\frac{2m(E-V_0)}{\hbar^2}\psi$

$$= -\frac{2mV_0}{\hbar^2}\psi$$

So $k_2^2 = \frac{2mV_0}{\hbar}$. But notice $k_1^2 = \frac{2m(2V_0)}{\hbar^2}$, so

$$K_1^2 = 2K_2^2 \Rightarrow \boxed{K_2 = \frac{K_1}{\sqrt{2}}}$$

b] $\psi_I = Ae^{ik_1x} + Be^{-ik_1x}$

$\psi_{II} = Ce^{ik_2x}$

Smooth & Cts. gives $\begin{cases} A+B=C \\ k_1(A-B) = k_2C \end{cases} \Rightarrow 2A = \left(1+\frac{k_2}{k_1}\right)C$

$$\Rightarrow C = \frac{2k_1}{k_1+k_2}A, \quad B = \frac{k_1-k_2}{k_1+k_2}A$$

So $R = \left(\frac{k_1-k_2}{k_1+k_2}\right)^2$, $T = 1-R = \frac{4k_1k_2}{(k_1+k_2)^2}$

$$T = \frac{4\left(\frac{k_1}{k_2}\right)}{\left(\frac{k_1}{k_2}+1\right)^2} = \frac{4\sqrt{2}}{(1+\sqrt{2})^2} = \boxed{0.97}$$

c] $.97 \times 10^6 = \boxed{9.7 \times 10^5}$

d] Classically, they all get transmitted.

6]

a] $\int_{-\infty}^{\infty} |\Psi(x,0)|^2 dx = \int_{-\infty}^{\infty} c^2 \Big[\underset{1}{|\psi_0|^2} + \underset{1}{|\psi_1|^2} + \underset{0}{2\psi_0\psi_1} \Big]$

$= c^2 \cdot 2$, so $\boxed{c^2 = \frac{1}{2}}$ normalizes

b] $\Psi(x,t) = \frac{1}{\sqrt{2}} \Big[\psi_0(x) e^{-i\frac{E_0}{\hbar}t} + \psi_1(x) e^{-i\frac{E_1}{\hbar}t} \Big]$

where $E_n = \hbar\omega(n + \frac{1}{2})$

c] $\langle E \rangle = \langle i\hbar \frac{\partial}{\partial t} \rangle$

$= \int dx \frac{1}{2} \Big[\psi_0 e^{+i\frac{E_0}{\hbar}t} + \psi_1 e^{+i\frac{E_1}{\hbar}t} \Big] \Big[E_0 \psi_0 e^{-i\frac{E_0}{\hbar}t} + E_1 \psi_1 e^{-i\frac{E_1}{\hbar}t} \Big]$

$= \boxed{\frac{E_0 + E_1}{2}}$

d] $\langle x \rangle = \frac{1}{2} \int_{-\infty}^{\infty} dx \Big[\psi_0 e^{i\frac{E_0}{\hbar}t} + \psi_1 e^{i\frac{E_1}{\hbar}t} \Big] x \Big[\psi_0 e^{-i\frac{E_0}{\hbar}t} + \psi_1 e^{-i\frac{E_1}{\hbar}t} \Big]$

$= \frac{1}{2} \Big(\int x|\psi_0|^2 dx + \int x|\psi_1|^2 dx \Big) + \int \psi_0 \psi_1 x \Big[\cos\Big(\frac{E_1 - E_0}{\hbar} t \Big) \Big]$

$= \boxed{x_0 + A\cos(\Omega t)}$

7a] $V(r) = -\frac{kZe^2}{r}$. $\psi_{100} = 2e^{-Zr/a_0}\left(\frac{Z}{a_0}\right)^{3/2}\cdot\frac{1}{\sqrt{4\pi}}$

$$\left\langle \frac{1}{r}\right\rangle = \int_0^\infty dr \int_0^\pi d\theta \int_0^{2\pi} d\phi\, r^2\sin\theta\, \frac{1}{r}\cdot 4e^{-2Zr/a_0}\left(\frac{Z}{a_0}\right)^3\cdot\frac{1}{4\pi}$$

$$= \int_0^\infty dr\, 4re^{-2Zr/a_0}\left(\frac{Z}{a_0}\right)^3 = 4\left(\frac{Z^3}{a_0^3}\right)\int_0^\infty dr\, r e^{-2Zr/a_0}$$

$u = \frac{2Zr}{a_0}$; $du = \frac{2Z}{a_0} r$

$$\text{So } \left\langle \frac{1}{r}\right\rangle = 4\left(\frac{Z}{a_0}\right)^3\left(\frac{a_0}{2Z}\right)^2 \underbrace{\int_0^\infty du\, u e^{-u}}_{1}$$

$$= \cancel{4}\,\frac{Z^3}{a_0^3}\,\frac{a_0^2}{\cancel{4}Z^2} = \frac{Z}{a_0}$$

So $\langle V(r)\rangle = -\frac{kZ^2e^2}{a_0} =$ [redacted] $= \boxed{-245\,eV}$ $(Z=3)$.

b] $E_{TOT} = Z^2(-13.6\,eV) =$ [redacted] $\boxed{-122.4\,eV}$

So $\langle KE\rangle =$ [redacted] $= \boxed{122.6\,eV}$

8] $U = -\vec{\mu}\cdot\vec{B}$

$$= -\mu_z B_z = +\frac{e}{2m_e} L_z B_z$$

$$= \frac{e m_\ell}{2m_e} \hbar B_z$$

$$= \frac{e\hbar}{2m_e} m_\ell B_z$$

So in the 3s state, $\ell = 0 \Rightarrow m_\ell = 0$

$$E_{3s} = \frac{-13.6\,eV}{9} = -1.51\,eV$$

<u>But</u> $E_{2p} = -\frac{13.6\,eV}{4} + \frac{e\hbar}{2m_e} B_z m_\ell$

$$= -3.4\,eV + (2.026 \times 10^{-4}\,eV)\,m_\ell$$

$$= \begin{cases} -3.4 + 2.026\times10^{-4}\,eV \\ -3.4\,eV \\ -3.4 - 2.026\times10^{-4}\,eV \end{cases}$$

So just subtract for 3 possible ΔE's

$$\lambda = \frac{hc}{\Delta E} = \boxed{656.08465\,nm,\ 656.155\,nm,\ 656.014\,nm}$$