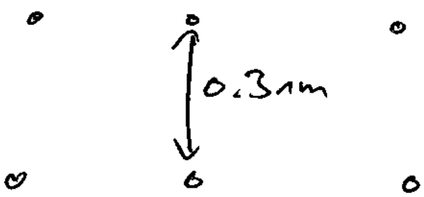
Phys 4E Week 3 HW

S: 13, 19, 22, 26, 30, 31, 38, 43, 48, 49, 50, 52

13) It's ambiguous, but say the distance between the horizontal planes is $D = 0.3$ nm.

Then $n\lambda = D\sin\phi$



$$\Rightarrow \lambda = .3\sin(42^\circ) = \boxed{.2\text{ nm}}$$

So $$p = \frac{h}{\lambda} = \frac{hc}{\lambda c} = \frac{1240\text{ eV}\cdot\text{nm}}{(.2\text{nm})\cdot c}$$

$$= 6.2\text{ keV}/c$$

This is small for neutrons, so use non-relativistic

$$KE = \frac{p^2}{2m} \Rightarrow \frac{(pc)^2}{2mc^2} = \frac{(6.2\text{keV})^2}{2(939\text{MeV})} = \boxed{2\times10^{-2}\text{ eV}}$$

19

a) The wave travels out at speed c, for .25 μs

$\therefore$ Length of packet $= (.25\,\mu s)\times c = \boxed{75\text{ m}}$

b) $f = \frac{c}{\lambda} = \boxed{1.5\times 10^{10}\text{ Hz}}$ This is the peak wavelength in the packet.

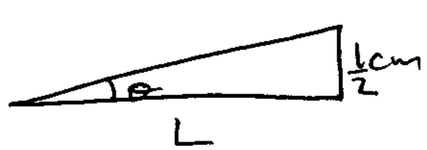
c) $\Delta\omega\Delta t \simeq 1$, and since $\Delta t \simeq .25\,\mu s$

$$\Rightarrow \Delta f \simeq \frac{1}{2\pi(.25\,\mu s)} = \boxed{637\text{ kHz}}$$

22) $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{hc}{\sqrt{2mc^2E}} = \frac{1240\text{ eV}\cdot\text{nm}}{\sqrt{2(511\text{ keV})(5\text{ eV})}}$$

$$= 0.549\text{ nm}$$

So $d\sin\theta = \frac{\lambda}{2} \Rightarrow \boxed{d = 3.15\text{ nm}}$

b) 

$\Rightarrow \tan\theta = \frac{\frac{1}{2}\text{ cm}}{L}$ for $\theta = 5^\circ$

$\Rightarrow \boxed{L = 5.7\text{ cm}}$

26

a) Don't really know when to stop counting - how small does it have to get?

$$f = \frac{1}{T} \Rightarrow T = \frac{\Delta t}{N}, \text{ so } f = \frac{N}{\Delta t}.$$

$$\therefore \Delta f = \frac{\Delta N}{\Delta t} \approx \boxed{\frac{1}{\Delta t}}$$

b) $$\lambda = \frac{\Delta x}{N} \Rightarrow k = \frac{2\pi N}{\Delta x} \Rightarrow \Delta k = \frac{2\pi \Delta N}{\Delta x} = \boxed{\frac{2\pi}{\Delta x}}$$

30) $$\Delta x \Delta p \approx \frac{\hbar}{2} \Rightarrow \lambda \Delta p \approx \frac{\hbar}{2} \Rightarrow \Delta p \approx \left(\frac{h}{\lambda}\right)\frac{1}{4\pi}$$

$$\Rightarrow \boxed{\Delta p \approx \frac{p}{4\pi}}$$

31) Set $\Delta p \approx p = 900 \frac{kg\, m}{s}$

So $$\Delta x = \frac{\hbar}{2(\Delta p)} = \frac{50}{2(900)} = \boxed{2.8 \text{ cm}}$$

You could "see" the uncertainty - it would be fuzzy!

38] Set $\Delta x = 1\ \text{fm}$

$$p \simeq \Delta p \approx \frac{\hbar}{2(1\,\text{fm})} = 98.5\ \frac{\text{MeV}}{c}$$

for neutron, ~~not~~ relativistic $\Rightarrow E = \frac{p^2}{2m} = \boxed{5\ \text{MeV}}$

Electron is relativistic: $E = \sqrt{p^2c^2 + m^2c^4}$

So $KE = \sqrt{p^2c^2+m^2c^4} - mc^2 = \boxed{98.0\ \text{MeV}}$

42] $\lambda = \frac{2L}{n}$, so $p = \frac{h}{\lambda} = \frac{hn}{2L}$

Now, $E = \frac{p^2}{2m} = \frac{h^2n^2}{8mL^2} = \boxed{n^2\left(\frac{h^2}{8mL^2}\right) \equiv n^2 E_1}$

b] $E_1 = 37.6\ \text{eV}$, so $\boxed{E_n = n^2(37.6\ \text{eV})}$

c] $f = 3(37.6\text{eV})/h \Rightarrow \lambda = \frac{hc}{3(37.6\text{eV})} = \boxed{11\ \text{nm}}$

d] $\lambda = hc/5(37.6\ \text{eV}) = \boxed{6.6\ \text{nm}}$

e] $\lambda = hc/24(37.6\text{eV}) = \boxed{1.4\ \text{nm}}$

48] a] $m_{\pi^+} + m_n - m_p = 141.3 \text{ MeV}/c^2$

$\Rightarrow \boxed{\Delta E = 141.3 \text{ MeV}}$

b] $\Delta E \Delta t \approx \frac{\hbar}{2} \Rightarrow \boxed{\Delta t = 2.3 \times 10^{-24} s}$

c] $c\Delta t = \boxed{7 \times 10^{-16} m}$, about 1 fm

49] $hf = \gamma mc^2 \Rightarrow \gamma = \frac{hf}{mc^2} \Rightarrow \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{hf}{mc^2}$

So $1 - v^2/c^2 = \left(\frac{mc^2}{hf}\right)^2 \Rightarrow \frac{v^2}{c^2} = 1 - \left(\frac{mc^2}{hf}\right)^2$

$$\Rightarrow \frac{v}{c} = \left(1 - \left(\frac{mc^2}{hf}\right)^2\right)^{1/2} \approx 1 - \frac{1}{2}\left(\frac{mc^2}{hf}\right)^2$$

So using $v > .99c$, $v/c \geq 0.99$

$\Rightarrow 1 - \frac{1}{2}\left(\frac{mc^2}{hf}\right)^2 \geq 0.99 \Rightarrow 0.01 \geq \frac{1}{2}\left(\frac{mc^2}{hf}\right)^2$

So use $\lambda = 30\text{ m}$ to get

$$1.08 \times 10^{-88} \geq m^2 \Rightarrow \boxed{m < 10^{-44}}$$

5c] $\Delta v_x \approx \frac{\hbar}{2m\Delta x}$. And $y_0 = \frac{1}{2}gt^2 \Rightarrow t = \left(\frac{2y_0}{g}\right)^{1/2}$

$$\therefore \Delta X = \Delta v_x t = \boxed{\left(\frac{2y_0}{g}\right)^{1/2}\left(\frac{\hbar}{2m\Delta x}\right)}$$

6] Now say $\Delta y \Delta p_y \approx \frac{\hbar}{2} \Rightarrow \Delta v_y \approx \frac{\hbar}{2m\Delta y}$

So now $y_0 = \frac{1}{2}gt^2 + \Delta v_y t$

$$\Rightarrow t = \frac{-\Delta v_y \pm \sqrt{(\Delta v_y)^2 + 2gy_0}}{g}$$

Take the + root b/c $\Delta v_y = 0$ should give $\left(\frac{2y_0}{g}\right)^{1/2}$

$$\therefore \boxed{\Delta X = \left(\frac{\hbar}{2m\Delta x}\right)\left[\frac{-\Delta v_y + \sqrt{(\Delta v_y)^2 + 2gy_0}}{g}\right]}$$

where $\Delta v_y \approx \frac{\hbar}{2m\Delta y}$.

52

a) $p_\gamma = \frac{E_\gamma}{c} = \frac{1\,eV}{c}$

So $\frac{p^2}{2m} = \frac{(1\,eV)^2}{2mc^2}$. Use $m_{Fe} = 56\,u$

$= 56\left(931.5\,\frac{MeV}{c^2}\right)$

$\Rightarrow E = \frac{p^2}{2m} = 9.6\times10^{-12}$

The line width is $h\Delta f \approx 10^{-8}$, so this is $\boxed{10^{-3}}$ times that.

b) Now use $p = 1\,MeV$ to get $\boxed{E = 9.6\,eV}$

This is $10^8\times$ the line width.