Physics 2D, Winter 2005

Week 6 Exercise Solutions

4-40 The collision between the electron and a hydrogen atom will take a little bit more energy from the electron than merely the energy level separation because the hydrogen atom must gain some kinetic energy as well, in order for momentum to be conserved.

Energy conservation, assuming the speeds involved are much less than c, gives

$$K_{e,i} = K_{e,f} + \Delta E + K_H$$
$$i.e.,\ \frac{p_{e,i}^{\ 2}}{2m_e} = \frac{p_{e,f}^{\ 2}}{2m_e} + \Delta E + \frac{p_H^{\ 2}}{2m_H}$$

while momentum conservation gives

$$p_{e,i} = p_{e,f} + p_H$$

Now, the for the lowest $p_{e,I}$, we will have $p_{e,f} = 0$, leaving

$$p_H = p_{e,i}$$
$$and$$
$$\frac{p_{e,i}^{\ 2}}{2m_e} - \frac{p_{e,i}^{\ 2}}{2m_H} = \Delta E$$

Solving this for the initial kinetic of the electron gives

$$\frac{p_{e,i}^{\ 2}}{2m_e} = \frac{\Delta E}{1 - m_e / m_H} \approx \frac{10.2\text{eV}}{1 - 0.511/940} \approx 10.206\text{eV}$$

As should be expected, this is very close to just ΔE

Chapter 5

#4.) The "seeing" ability, or resolution, of radiation is determined by its wavelength. If the size of an atom is of the order of 0.1 nm, how fast must an electron travel to have a wavelength small enough to "see" an atom.

We require

$$\lambda = 0.1\,nm = \frac{h}{p}$$

$$\Rightarrow p = \frac{h}{0.1\,nm} = mv \qquad (v \ll c\text{, so this is ok})$$

$$\therefore \boxed{v = \frac{h}{m_e(0.1\,nm)} = 7.28 \times 10^6\,m/s}$$

#6.) An electron and a proton each have kinetic energy equal to 50 keV. What are their deBroglie wavelengths?

Since $K = 50\,keV \Rightarrow v \ll c$

$\therefore$ we can use

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\Rightarrow p = \sqrt{2mK} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{\sqrt{2mK}}$$

for e^-,

$$\boxed{\lambda_e = \frac{h}{\sqrt{2m_eK}} = 5.36 \times 10^{-3}\,nm}$$

for proton,

$$\boxed{\lambda_p = \frac{h}{\sqrt{2m_pK}} = 1.28 \times 10^{-4}\,nm}$$

#11.) For an electron to be confined to a nucleus, its de Broglie wavelength would have to be less than 10^{-14} m.

a.) What would be the kinetic energy of an electron confined to this region?

That is one small number, so to be safe we had better use relativity.

We need

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{h}{1\times10^{-14}\,\text{m}} = 124 \text{ MeV}/c$$

Now,

$$E^2 = p^2c^2 + m^2c^4$$

$$\rightarrow E = 0.124 \text{ GeV} \qquad \text{(Whoa momma!)}$$

$$\boxed{K = E - mc^2 = 124 \text{ MeV}}$$ That's a spicy meatball!

b.) Heck no. An e^- with that much kinetic energy would be able to bust out of a nucleus.

#13.) Figure P4.13 shows the top three planes of a crystal with planar spacing d. If $2d\sin\theta = 1.01\lambda$ for the two waves shown, and high energy electrons of wavelength λ penetrate many planes deep into the crystal, which atomic plane produces a wave that cancels the surface reflection? Blah Blah Blah ...

To get destructive interference, we need

$$2d\sin\theta = \underbrace{(m+\tfrac{1}{2})\lambda}_{\text{path difference}} \quad \text{where } m = 0, 1, \ldots$$

In other words, we need to get the path difference to look like

somenumber.5

The problem tells us that between two planes, the path difference is 1.01λ. If we multiply this by 50, we get

$$50.5\lambda$$

$\rightarrow$ $\boxed{\text{the } 50^{th} \text{ plane produces the required wave}}$

5-19

First of all,

$$\frac{K}{m_p c^2} = \frac{1.0\times10^6\,eV}{938.3\times10^6\,eV} = 0.11\% \rightarrow \underline{\text{nonrelativistic}}$$

$$\Rightarrow K = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2mK} = 2.312\times10^{-20}\,kg\frac{m}{s}$$

$$\Delta p = 0.05\,p = 1.160\times10^{-21}\,kg\frac{m}{s}$$

$$\Delta p\,\Delta x_{min} = \frac{\hbar}{2}$$

$$\rightarrow \boxed{\Delta x_{min} = \frac{\hbar}{2\Delta p} = 4.56\times10^{-14}\,m}$$

5-22

We're talking kinetic energies that are on the order of the rest mass energy, so we should use relativity to be safe.

(a)

We know

$$K = mc^2(\gamma - 1)$$

$$\Rightarrow \gamma = \frac{K}{mc^2} + 1 = 1.02$$

$$= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

which gives $v = 5.91 \times 10^7$ m/s

Now,

$$p = \gamma m v = 5.49 \times 10^{-23}\ \frac{\text{kg m}}{\text{s}}$$

They tells us the uncertainty in momentum is

$$\Delta p = 0.01 p$$

And

$$\Delta x_{min}\, \Delta p = \frac{\hbar}{2}$$

$$\Delta x_{min} = \frac{\hbar}{2\Delta p} = \frac{\hbar}{(0.02p)}$$

$$\boxed{\Delta x_{min} = 0.0961 \text{ nm}}$$

(b) using the same method as in (a) gives

$\gamma = 2.96$

$p = 7.60 \times 10^{-22}\ \frac{kg\,m}{s}$

$\Rightarrow \boxed{\Delta x_{min} = 0.00694\ nm}$

c.) 100 MeV

$\gamma = 197$

$p = 5.38 \times 10^{-20}\ kg\,m/s$

$\Rightarrow \boxed{\Delta x_{min} = 9.80 \times 10^{-14}\ m}$

5-24

a.) Estimate the uncertainty in the electron's momentum in terms of r

We're told that

$$\Delta x = r$$

$$\Rightarrow \boxed{\Delta p \sim \frac{\hbar}{\Delta x} = \frac{\hbar}{r}}$$

(factors of $\frac{1}{2}$ or whatever don't really matter)

b.) Estimate the electron's kinetic, potential, and total energies in terms of r.

$$K = \frac{p^2}{2m}$$

$$p = p_{avg} + \Delta p = \Delta p$$

$$= \frac{(\Delta p)^2}{2m}$$

$$\boxed{K = \frac{\hbar^2}{2m_e r^2}}$$

$$\boxed{U = -\frac{ke^2}{r}}$$

$$\boxed{E = K + U = \frac{\hbar^2}{2m_e r^2} - \frac{ke^2}{r}}$$

c.) The actual value of r is the one that minimizes the total energy, resulting in a stable atom. Find that value of r and the resulting total energy. Compare your answer with the predictions of the Bohr theory.

$$E = \frac{\hbar^2}{2m_e r^2} - \frac{ke^2}{r}$$

To minimize, take $\frac{dE}{dr}$ and set equal to zero

$$\frac{dE}{dr} = -\frac{\hbar^2}{m_e r^3} + \frac{ke^2}{r^2} = 0$$

$$\Rightarrow \boxed{r = \frac{\hbar^2}{m_e k e^2} = a_0 !}$$

$$\therefore \boxed{E_{min} = +\frac{\hbar^2}{2m_e a_0^2} - \frac{ke^2}{a_0} = -13.6 \text{ eV}}$$

Keen! That's the same as the Bohr theory. However, It's kind of just dumb luck (actually it's cuz I've seen this before) that we got the same numbers. When I said

$$\Delta p = \frac{\hbar}{r}$$

I could've thrown in any constants, like $\Delta p = \frac{\hbar}{2r}$ or somethin'!

5-25

The lifetime of the particle is its time uncertainty, i.e.,

$\Delta t = 0.100 \times 10^{-9}\,s$

Using the energy-time form of the uncertainty principle,

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta E \geq \frac{\hbar}{2\Delta t} = 3.29 \times 10^{-6}\,eV$$

So the width of the line is much less than $\pm 5\,eV$, and the detectors cannot measure it.

5-26

The max is 30 counts, which means half max is 15 counts

So $FWHM = 110\ MeV/c^2$

$$\Rightarrow \Delta E = \frac{110\,MeV}{2} = 55\,MeV$$

And

$$\Delta E \Delta t_{min} = \frac{\hbar}{2}$$

$$\Rightarrow \boxed{\Delta t_{min} = \frac{\hbar}{2\Delta E} = 6.0 \times 10^{-24}\,s}$$

5-28

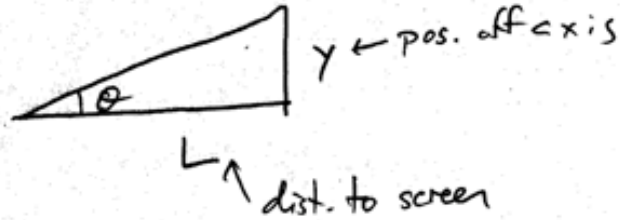
a] $\lambda = \frac{h}{p} = \frac{h}{mv} = \boxed{989\,\text{nm}}$

b] $d \sin\theta = \frac{\lambda}{2}$ for 1^{st} min.

$(1\,\text{mm}) \sin\theta = \frac{989\,\text{nm}}{2}$

$\Rightarrow \sin\theta = 4.95 \times 10^{-4}$

But $\theta = \frac{y}{L}$



$\Rightarrow y = (10\,\text{m})(4.95 \times 10^{-4})$

$\boxed{y = 4.94\,\text{mm}}$

c] No! This diffraction pattern arises b/c of the wave-like properties of the neutron beam. If we knew which slit it went through, we'd never see a diffraction pattern.