Physics 2D, Winter 2005

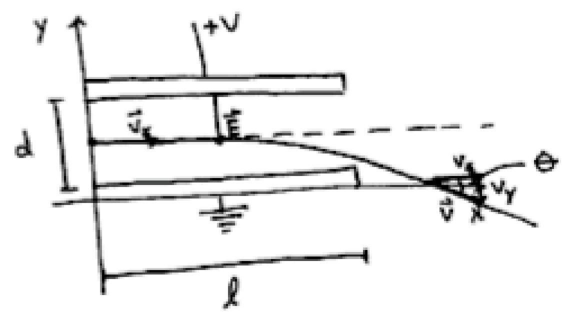
Week 5 Exercise Solutions

#3.) A mystery particle enters the region between the plates of a Thomson apparatus as shown in Fig. 3.5. The deflection angle θ is measured to be 0.20 radians (downwards) for this particle when $V = 2000$ V, $\ell = 10.0$ cm, and $d = 2.00$ cm. If a perpendicular magnetic field of magnitude 4.57×10^{-2} T is applied simultaneously with the electric field, the particle passes through the plates without deflection.

a.) Find q/m for this particle

Oh my god, that's a lot of words

Figure 3.5 looks like



First of all, note that

$$\vec{F}_E = q\vec{E} = -\frac{qV}{d}\hat{j}$$

And since the particle is deflected downward, q must be positive! (Otherwise, $\vec{F} = -\frac{(-|q|)V}{d}\hat{j} = \frac{|q|V}{d}\hat{j}$ and the particle would deflect upward)

#3.) continued

Initially,

$$v_y = 0$$

and

$$\vec{F} = -\frac{qV}{d}\hat{j} = m\vec{a}$$

$$\therefore \vec{a} = a_y\hat{j} = -\frac{qV}{md}\hat{j}$$

$$a_y = \frac{dv_y}{dt} = -\frac{qV}{md}$$

$$v_y = -\frac{qV}{md}t + c$$

at $t=0$, $v_y = 0$

$$\therefore \boxed{v_y = -\frac{qV}{md}t} \qquad (1)$$

The particle's initial velocity is

$$\vec{v} = v_x\hat{i}$$

and since the acceleration is in the $\hat{j}$ direction, this remains unchanged throughout the problem.

The particle must travel a distance l through the plates

$$\Rightarrow v_x = \frac{l}{t}$$

$$t = \frac{l}{v_x}$$

#3.) continued

Substitute into (1) to get the velocity of the particle in the y direction as it leaves the plates

$$v_y = -\frac{qVl}{mdv_x}$$

Using trig,

$$\tan\theta = \frac{v_y}{v_x} = -\frac{qVl}{mdv_x^2}$$

You might remember that for small θ (Taylor expand that mutha!)

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \dots \simeq \theta$$

(if θ is small, θ^3 is even smaller, and we can forget it!)

$$\Rightarrow \tan\theta \simeq \theta = -\frac{qVl}{mdv_x^2}$$

$$\boxed{\frac{q}{m} = -\frac{dv_x^2}{Vl}\theta} \qquad (2)$$

Now turn on the $\vec{B}$ field. They tell us we get no deflection, which means the Electric and magnetic forces cancel.

#3.) continued

$$F_E = qE$$

$$F_B = qv_x B$$

$$qE = qv_x B$$

$$\Longrightarrow |v_x| = \frac{|E|}{|B|} = \frac{V}{Bd} \qquad (3)$$

Substitute into (2) to get

$$\frac{q}{m} = -\frac{V\theta}{B^2 \ell d}$$

Now,

$V = 2000\,V$

$\ell = 0.10\,m$

$d = 0.02\,m$

$B = 4.57 \times 10^{-2}\,T$

$\theta = -0.20$ (- cuz it's deflected downwards)

$$\boxed{\frac{q}{m} = 95.8 \times 10^{6}\ C/kg}$$ Score!

b.) Name that particle!

At this point, you guys don't know too many particles except

#3.) continued

proton: $q_p = +e$ $\quad m_p = 1.673\times10^{-27}$ kg

neutron: $q_n = 0$ $\quad m_n = 1.675\times10^{-27}$ kg

electron: $q_e = -e$ $\quad m_e = 9.109\times10^{-31}$ kg

alpha: $q_\alpha = +2e$ $\quad m_\alpha \simeq 4m_p$

photon: $q_\gamma = 0$ $\quad m_\gamma = 0$

It stands to reason that it's probably one of these.

We know our particle is positively charged, so it's either an alpha or proton. Check to see that

$$\frac{q_p}{m_p} = 95.8\times10^{6}\ \text{C/kg}$$

$$\boxed{\text{That particle is a proton!}}$$

c.) Find the horizontal speed with which the particle leaves the plates

Easiest thing to do is use (3)

$$\boxed{v_x = \frac{V}{Bd} = 2.19\times10^{6}\ \text{m/s}}$$

#3.) continued

d.) Must we use relativistic dynamics for this particle?

Well,

$v_x = 2.19\times10^6 \, m/s = 0.0073 \, c$

So $v \ll c$

$\Longrightarrow \boxed{\text{Nope}}$

#5.) A Thomson type experiment with relativistic electrons

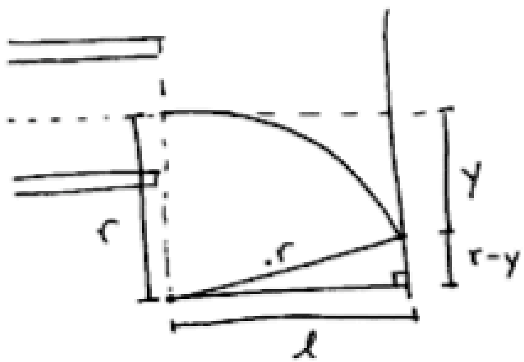
One of the earliest experiments to show that $p = \gamma m v$ (rather than $p = mv$) was that of Neumann. The apparatus shown in figure P3.5 is identical to Thomson's except that the source of high speed electrons is a radioactive radium source, and the magnetic field $\vec{B}$ is arranged to act on the electron over its entire trajectory from source to detector. The combined electric and magnetic fields act as a velocity selector, only passing electrons with speed $v = \frac{V}{Bd}$, while in the region where there is only a magnetic field the electron moves in a circle of radius r, with r given by $p = Bre$. This latter region ($\vec{E} = 0$, $\vec{B} = \text{const.}$) acts as a momentum selector because electrons with larger momenta have paths with larger radii.

#5.) continued

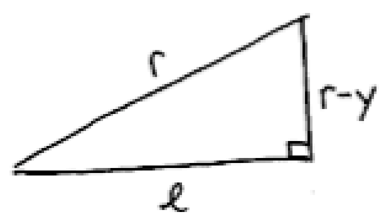
a.) Show that the radius of the circle described by the electron is given by

$$r = \frac{(\ell^2 + y^2)}{2y}$$

As soon as the electron leaves the plates, it starts traveling in a circular path.



So we have a right triangle in there that looks like



Pythagoras tells us

$$r^2 = \ell^2 + (r-y)^2 = \ell^2 + r^2 + y^2 - 2ry$$

$$2ry = \ell^2 + y^2$$

$$\therefore \boxed{r = \frac{(\ell^2 + y^2)}{2y}}$$ ∎

#5.) continued

b.) Typical values for the Neumann experiment were

$d = 2.51 \times 10^{-4}$ m

$B = 0.0177$ T

$\ell = 0.0247$ m

For $V = 1060$ V, y, the most critical value, was measured to be

$y = 0.0024 \pm 0.0005$ m

Show that these values disagree with the y value calculated from $p = mv$, but agree with the y value calculated from $p = \gamma mv$ within experimental error

From eq. 3.6,

$$v_x = \frac{V}{Bd}$$

<u>for non relativistic</u>

$$p = mv = Bre$$

$$\frac{mV}{Bd} = Bre$$

$$\Rightarrow r = \frac{mV}{B^2 de}$$

from part a.),

$$r = \frac{\ell^2 + y^2}{2y}$$

#5.) continued

$$\frac{mV}{B^2 de} = \frac{\ell^2+y^2}{2y}$$

$$y^2 - \frac{2mV}{B^2 de} y + \ell^2 = 0$$

$$\Longrightarrow y = \frac{\frac{2mV}{B^2 de} \pm \sqrt{\left(\frac{2mV}{B^2 de}\right)^2 - 4\ell^2}}{2}$$

Plug in numbers to get

$$\boxed{y = 0.00408 \text{ m}}$$

(other solution has $r > y$, which won't work)

<u>For Relativistic</u>

$$p = \gamma m v_x = Bre$$

$$r = \frac{\gamma m v}{Be}$$

$$r = \frac{\ell^2+y^2}{2y} \Longrightarrow y^2 - 2ry + \ell^2 = 0$$

$$y = \frac{2r \pm \sqrt{(2r)^2 - 4\ell^2}}{2} = \frac{\frac{2\gamma mV}{B^2 de} \pm \sqrt{\left(\frac{2\gamma mV}{B^2 de}\right) - 4\ell^2}}{2}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{V^2}{B^2 d^2 c^2}}}$$

#5.) continued

Plug in numbers

$$\boxed{y = 0.00243\,m}$$

So we get

$$\boxed{\begin{array}{l} \text{nonrelativistic}: y = 0.00408\,m \\ \text{relativistic}: y = 0.00243\,m \\ \text{measured} \\ \quad y = 0.0024 \pm 0.0005\,m \end{array}}$$

So relativistic calculation agrees with experiment.

#8.) A parallel beam of α particles with fixed kinetic energy is incident normally on a piece of gold foil.

a.) If 100 α particles per minute are detected at 20°, how many will be counted at 40°, 60°, 80°, and 100°?

This is Rutherford scattering. Use eq. 3.16

$$\Delta n = \frac{k^2 Z^2 e^4 N n A}{4R^2 (\frac{1}{2} m_\alpha v_\alpha^2)^2 \sin^4(\phi/2)}$$

#8.) continued

So if we count rate

$$\Delta n_1 = \frac{\text{Bunch of crap}}{\sin^4(20°/2)}$$

and rate

$$\Delta n_2 = \frac{\text{Bunch of crap}}{\sin^4(\phi/2)}$$

$$\Longrightarrow \frac{\Delta n_2}{\Delta n_1} = \frac{\sin^4(20°/2)}{\sin^4(\phi/2)}$$

$$\Delta n_2 = \frac{\Delta n_1 \sin^4(20°/2)}{\sin^4(\phi/2)}$$

for $\phi = 40°$, $\Delta n_2 = 6.64$ cpm

$60°$, $\Delta n_2 = 1.45$ cpm

$80°$, $\Delta n_2 = 0.533$ cpm

$100°$, $\Delta n_2 = 0.264$ cpm

(cpm = counts per minute)

b.) If the kinetic energy of the incident α particles is doubled, how many scattered α particles will be observed at $20°$?

Note that

$$\Delta n \propto \frac{1}{K^2}$$

$\Longrightarrow$ if $K' = 2K$

$$\boxed{\Delta n' = \frac{\Delta n}{4} = 25 \text{ cpm}}$$

#5.) If the original α particles were incident on a copper foil of the same thickness, how many scattered α particles would be detected at 20°? Note that $\rho_{Cu} = 8.9 g/cm^3$ and $\rho_{Au} = 19.3 g/cm^3$

From eq. 3.16,

$$\frac{\Delta n_{Cu}}{\Delta n_{Au}} = \frac{(Z_{Cu}^2 N_{Cu})}{(Z_{Au}^2 N_{Au})}$$

$Z_{Cu} = 29$

$Z_{Au} = 79$

N = # of target nuclei per unit area

= (# of nuclei per unit volume)(thickness)

= (ρ (Molar mass) $\cdot N_A$) (thickness)

$\Rightarrow N_{Cu} = 8.43 \times 10^{22}$ (thickness)

$N_{Au} = 5.90 \times 10^{22}$ (thickness)

So

$$\boxed{\Delta n_{Cu} = 19.3 \text{ cpm}}$$

#9.1 It is observed that α particles with kinetic energies of 13.9 MeV and higher, incident on Cu foils, do not obey Rutherford's $(\sin \phi/2)^{-4}$ law. Estimate the nuclear size of copper from this observation, assuming that the Cu nucleus remains fixed in a head-on collision with an α particle.

The initial energy of the system of the α and Copper nucleus is 13.9 MeV, which is just the kinetic energy of the α when far from the nucleus. The final energy can be evaluated at the distance of closest approach, where $K=0$ and

$$U = k\frac{q_\alpha q_{Cu}}{r} = \frac{k(2e)(29e)}{r}$$

where we figure r is close to the radius of the Cu nucleus. So conserve energy

$$E_f = \frac{k(2e)(29e)}{r} = 13.9\,\text{MeV}$$

$$\boxed{r = \frac{k(2e)(29e)}{13.9\,\text{MeV}} = 6.00 \times 10^{-15}\,\text{m}}$$

#14.(a) Use eq. 3.35 to calculate the radii of the first, second, and third Bohr orbits of Hydrogen.

Eq. 3.35 is

$$r_n = n^2 \frac{a_0}{Z} \quad , a_0 = 0.0529 \text{ nm} \quad (\text{Bohr radius})$$

$$Z_H = 1$$

#14.) continued

Plug and chug

$$\boxed{\begin{aligned} r_1 &= \frac{a_0}{1} = a_0 = 0.0529 \text{ nm} \\ r_2 &= 4a_0 = 0.2116 \text{ nm} \\ r_3 &= 9a_0 = 0.4761 \text{ nm} \end{aligned}}$$

b.) Find the electron's speed in the same three orbits.

Eq. 3.26 tells us

$$v_n = \sqrt{\frac{ke^2}{m_e r_n}}$$

$$\Rightarrow \boxed{\begin{aligned} v_1 &= \sqrt{\frac{ke^2}{m_e r_1}} = 2.19 \times 10^6 \text{ m/s} \\ v_2 &= 1.09 \times 10^6 \text{ m/s} \\ v_3 &= 7.28 \times 10^5 \text{ m/s} \end{aligned}}$$

c.) Is a relativistic correction necessary?

The highest speed is v_1, and

$$\frac{v_1}{c} = 0.007$$

$\therefore v_1 \ll c \Rightarrow \boxed{\text{Nope}}$

#23.) A hydrogen atom is in its ground state ($n=1$). Using the Bohr theory of the atom, calculate

a.) the radius of the orbit

Well, that's already done for us! Eq. 3.35

$$\boxed{r_n = n^2 \frac{a_0}{Z} \; ; \; a_0 = 0.0529\,\text{nm}}$$

b.) The linear momentum of the electron

Again, eq. 3.26 gives

$$p_n = m_e v_n = m_e \sqrt{\frac{ke^2}{m_e r_n}}$$

$$\Rightarrow \boxed{p_1 = 1.99 \times 10^{-24}\ \text{kg m/s}}$$

c.) The angular momentum of the electron

$$\boxed{\begin{aligned} L = m_e v r = p_1 a_0 &= 1.05 \times 10^{-34}\ \text{kg m}^2/\text{s} \\ &= \hbar \end{aligned}}$$

#23.) continued

d.) The kinetic energy

Note from eq. 3.26 and 3.27 that

$$\boxed{K = |E| = 13.6\ eV}$$

e.) the potential energy

Looking at eq. 3.25 and 3.26,

$$\boxed{U = -2K = -27.2\ eV}$$

f.) the total energy

$$\boxed{E = K + U = -13.6\ eV}$$

#26.) Calculate the longest and shortest wavelengths in the Lyman series for hydrogen, indicating the underlying electronic transition that gives rise to each. Are any of the Lyman spectral lines in the visible spectrum? Explain.

The longest wavelength will have the smallest energy, and is the jump from $E_2 \rightarrow E_1$

$$E_n = -\frac{13.6}{n^2}$$

$$\Delta E = E_2 - E_1 = -\frac{13.6}{4} + 13.6 = 10.2\ eV$$

#26.) continued

which corresponds to a wavelength of

$$\Delta E = \frac{hc}{\lambda_{max}} \Rightarrow \boxed{\lambda_{max} = \frac{hc}{\Delta E} = 121.5\,nm}$$

The shortest wavelength will have the most energy, and corresponds to a free electron falling straight to E_1.

For a free electron,

$$n_i = \infty$$

$$\Longrightarrow \Delta E = E_\infty - E_1 = 0 + 13.6 = 13.6\text{ eV}$$

$$\therefore \boxed{\lambda_{min} = \frac{hc}{\Delta E} = 91.16\,nm}$$

The visible spectrum starts at $\lambda = 350$ nm, so all the Lyman lines are in the UV range.

#31.) a.) Find the frequency of the electron's orbital motion, f_e, around a fixed nucleus of charge $+Ze$ by using eq. 3.24 and $f_e = (v/2\pi r)$ to obtain

$$f_e = \frac{m_e k^2 Z^2 e^4}{2\pi \hbar^3}\left(\frac{1}{n^3}\right)$$

Equation 3.24 is the angular momentum quantization condition

$$m_e v r = n\hbar$$

#31.) continued

$$\Rightarrow v = \frac{n\hbar}{m_e r}$$

$$\therefore f_e = \frac{v}{2\pi r} = \frac{n\hbar}{2\pi m_e r^2}$$

Eq. 3.35 says

$$r_n = \frac{n^2 a_0}{Z} \Rightarrow r^2 = \frac{n^4 a_0^2}{Z^2}$$

$$\therefore f_e = \frac{n\hbar}{2\pi m_e}\left(\frac{Z^2}{n^4 a_0^2}\right) = \frac{Z^2\hbar}{2\pi m_e n^3 a_0^2}$$

and

$$a_0 = \frac{\hbar^2}{m_e k e^2} \qquad \text{(eq. 3.29)}$$

$$\therefore f_e = \frac{Z^2\hbar}{2\pi m_e n^3}\left(\frac{m_e^2 k^2 e^4}{\hbar^4}\right)$$

$$\boxed{f_e = \frac{m_e k^2 Z^2 e^4}{2\pi \hbar^3}\left(\frac{1}{n^3}\right)} \quad \blacksquare$$

#31.) continued

b.) Show that the frequency of the photon emitted when an electron jumps from an outer to inner orbit can be written

$$f_{photon} = \frac{k Z^2 e^2}{2a_0 h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$= \frac{m_e k^2 e^4 Z^2}{2\pi \hbar^3}\left(\frac{n_i + n_f}{2n_i^2 n_f^2}\right)(n_i - n_f)$$

For an electronic transition between adjacent orbits, $n_i - n_f = 1$ and

$$f_{photon} = \frac{m_e k^2 e^4 Z^2}{2\pi \hbar^3}\left(\frac{n_i + n_f}{2n_i^2 n_f^2}\right)$$

Now examine the factor

$$\left(\frac{n_i + n_f}{2n_i^2 n_f^2}\right)$$

By substitution of integers show that

$$\frac{1}{n_i^3} < \frac{n_i + n_f}{2n_i^2 n_f^2} < \frac{1}{n_f^3}$$

Umm... ok. So, first of all, the emitted photon must obey

$$\Delta E = E_{n_i} - E_{n_f} = h f_{photon}$$

$$\Rightarrow f_{photon} = \frac{E_{n_i} - E_{n_f}}{h}$$

#31.) continued

Eq. 3.36 says

$$E_n = -\frac{ke^2Z^2}{2a_0n^2}$$

$$\Rightarrow \Delta E = \frac{ke^2Z^2}{2a_0}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\therefore \boxed{f_{photon} = \frac{ke^2Z^2}{2a_0h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

$$a_0 = \frac{\hbar^2}{m_eke^2}$$

$$\Rightarrow f_{photon} = \frac{ke^2Z^2}{4\pi\hbar a_0}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$= \frac{m_ek^2e^4Z^2}{4\pi\hbar^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$= \frac{m_ek^2e^4Z^2}{2\pi\hbar^3}\left(\frac{n_i^2 - n_f^2}{2n_i^2n_f^2}\right)$$

$$\boxed{f_{photon} = \frac{m_ek^2e^4Z^2}{2\pi\hbar^3}\left(\frac{n_i+n_f}{2n_i^2n_f^2}\right)(n_i-n_f)}$$

To show when $n_i - n_f = 1$

$$\frac{1}{n_i^3} < \frac{n_i+n_f}{2n_i^2n_f^2} < \frac{1}{n_f^3}$$

Just plug in numbers like $n_i = 2, n_f = 1$; $n_i = 3, n_f = 2$; etc.

#31.) continued

c.) What do you conclude about the frequency of emitted radiation compared with the frequencies of orbital revolution in the initial and final states? What happen as $n_i \to \infty$

Since

$$f_e \propto \frac{1}{n^3}$$

We can write

$$\frac{1}{n_i^3} < \frac{n_i + n_f}{2n_i^2 n_f^2} < \frac{1}{n_f^3}$$

as

$$\boxed{f_{e_i} < f_{photon} < f_{e_f}}$$

i.e, the photon frequency is between the initial and final orbital frequencies.

If $n_f = n_i - 1$, we can write

$$f_{photon} = \frac{m_e k^2 Z^2 e^4}{2\pi \hbar^3}\left(\frac{n_i + n_i - 1}{2n_i^2(n_i-1)^2}\right)$$

$$= \frac{m_e k^2 Z^2 e^4}{2\pi \hbar^3}\left(\frac{2n_i - 1}{2(n_i-1)^2 n_i^2}\right)$$

as $n_i \longrightarrow \infty$

$$\longrightarrow \frac{m_e k^2 Z^2 e^4}{2\pi \hbar^3}\left(\frac{2n_i}{2n_i^4}\right) = \frac{m_e k^2 Z^2 e^4}{2\pi \hbar^3}\left(\frac{1}{n_i^3}\right)$$

#31) continued

i.e. $f_{photon} \propto \frac{1}{n_i^3}$

So that the emitted photon frequency is the same as the initial electron orbital frequency, as expected classically (The correspondence principle right here, kids)

#33) A muon is a particle with a charge equal to that of an electron and a mass equal to to 207 times the mass of an electron. Muonic lead is formed when ²⁰⁸Pb captures a muon to replace an electron. Assume that the muon moves in such a small orbit that it "sees" a nuclear charge of Z=82. According to the Bohr theory, what are the radius and energy of the ground state of muonic lead? Use the concept of reduced mass introduced in problem 32.

The idea of reduced mass is just to replace m_e in and equation with μ, where

$$\mu = \frac{m_e M}{m_e + M}$$

where M is the nuclear mass

To find the radius,

$$\boxed{r_n = n^2 \frac{a_0}{Z} = \frac{n^2 \hbar^2}{\mu k e^2 Z} = 3.1\times 10^{-15}\,m}$$

33.) continued

For energy,

$$E_n = -\frac{ke^2}{2a_0}\frac{Z^2}{n^2}$$

$$= -\frac{ke^2Z^2(\mu ke^2)}{2\hbar^2 n^2}$$

$$\Rightarrow \boxed{E_1 = -18.9 \text{ MeV}}$$

4-36 (a) $f_{\text{revolution}} = \frac{v}{2\pi r} = \frac{\frac{n\hbar}{m_e r}}{2\pi r} = \frac{n\hbar}{2\pi m_e r^2}$ as $r^2 = n^4(a_0)^2$,

$$f_{\text{revolution}} = \frac{\hbar}{2\pi m_e a_0^2 n^3} = \frac{6.58\times10^{15}}{n^3\text{ Hz}}.$$

Thus:

for $n = 100$, $f_{\text{revolution}} = 6.62\times10^{9}$ Hz

for $n = 1\,000$, $f_{\text{revolution}} = 6.62\times10^{6}$ Hz

for $n = 10\,000$, $f_{\text{revolution}} = 6.62\times10^{3}$ Hz

using $r_n = n^2 a_0 = n^2\left(0.529\times10^{-10}\text{ m}\right)$,

$$r_{100} = 0.529\times10^{-6}\text{ m}$$

$$r_{1\,000} = 0.529\times10^{-4}\text{ m}$$

$$r_{10\,000} = 0.529\times10^{-2}\text{ m} \cong \frac{1}{2}\text{ cm}$$

(b) $$f_{\text{photon}} = \frac{\Delta E}{h} = \frac{13.6Z^2}{h}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) = \left(\frac{(13.6)(1)^2\text{ eV}}{4.14\times10^{-15}\text{ eV s}}\right)\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$$

$$= 3.285\times10^{15}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)\text{Hz}$$

$$\text{for } n = 100,\ f_{\text{photon}} = 6.579\,8 \times 10^{9}\ \text{Hz}$$
$$\text{for } n = 1\,000,\ f_{\text{photon}} = 6.579\,8 \times 10^{6}\ \text{Hz}$$
$$\text{for } n = 10\,000,\ f_{\text{photon}} = 6.579\,8 \times 10^{3}\ \text{Hz}$$

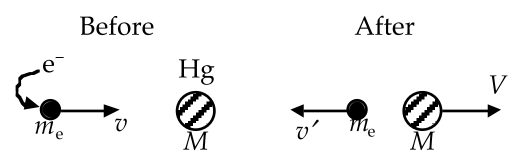
Thus, the difference in frequency, $\Delta f = f_{\text{revolution}} - f_{\text{photon}}$, are

$$n = 100,\ \Delta f = -0.05 \times 10^{9}\ \text{Hz}$$
$$n = 1\,000,\ \Delta f = -0.04 \times 10^{6}\ \text{Hz}$$
$$n = 10\,000,\ \Delta f = -0.05 \times 10^{3}\ \text{Hz}$$

(c) These results show that f_{photon} tends to $f_{\text{revolution}}$ in the limit of large quantum numbers $(n = 10\,000)$ and macroscopic sizes $\left(r \sim \frac{1}{2}\ \text{cm}\right)$.

4-37 $hf = \Delta E = \dfrac{4\pi^2 m_e k^2 e^4}{2h^2}\left(\dfrac{1}{(n-1)^2} - \dfrac{1}{n^2}\right)$, $f = \left(\dfrac{2\pi^2 m_e k^2 e^4}{h^3}\right)\left(\dfrac{2n-1}{(n-1)^2(n^2)}\right)$ as $n \to \infty$, $f \to \left(\dfrac{2\pi^2 m_e k^2 e^4}{h^3}\right)\left(\dfrac{2}{n^3}\right)$. The revolution frequency is $f = \dfrac{v}{2\pi r} = \left(\dfrac{1}{2\pi}\right)\left(\dfrac{ke^2}{m_e}\right)^{1/2}\left(\dfrac{1}{r^{3/2}}\right)$ where $r = \dfrac{n^2 h^2}{4\pi^2 m_e k e^2}$ substituting for r, $f = \left(\dfrac{1}{2\pi}\right)\left(\dfrac{ke^2}{m_e}\right)^{1/2}\left(\dfrac{8\pi^3 m_e k e^3 (m_e k)^{1/2}}{n^3 h^3}\right) = \dfrac{4\pi^2 m_e k^2 e^4}{h^3 n^3}$.

4-38 (a)



Conservation of momentum: $m_e v = -m_e v' + MV$ or $v = -v' + \alpha V$ where $\alpha = \dfrac{M}{m_e}$. Conservation of energy: $v^2 = v'^2 + \alpha V^2$. The fraction of kinetic energy lost by the electron $= \dfrac{K_{\text{Hg}}}{K_e} = \dfrac{\frac{1}{2}MV^2}{\frac{1}{2}m_e v^2} = \dfrac{\alpha V^2}{v^2}$. So we need to find v^2 in terms of V. Substituting $v = \alpha V - v'$ into $v^2 = v'^2 + \alpha V^2$: $(\alpha V - v')^2 = v'^2 + \alpha V^2 (\alpha V)(\alpha V - V - 2v') = 0$. This equation has solutions $V = 0$ and $v' = (\alpha - 1)\dfrac{V}{2}$. Substituting $v' = (\alpha - 1)\dfrac{V}{2}$ into

$v^2 = v'^2 + \alpha V^2$ yields $v^2 = \left(\alpha^2 - 2\alpha + 1\right)\dfrac{V^2}{4 + \alpha V^2} = V^2\,\dfrac{(1+\alpha)^2}{4v}$. Thus
$\dfrac{\Delta K}{K} = \dfrac{K_{\mathrm{Hg}}}{K_{\mathrm{e}^-}} = \dfrac{\alpha V^2}{v^2} = \dfrac{4\alpha V^2}{V^2(1+\alpha)^2} = \dfrac{\frac{4M}{m_{\mathrm{e}}}}{\left(\frac{1+M}{m_{\mathrm{e}}}\right)^2}$.

(b) $\dfrac{M}{m_{\mathrm{e}}} = \dfrac{200.6\ \mathrm{u}}{5.49\times 10^{-4}\ \mathrm{u}} = 365{,}400 >> 1$ so $\dfrac{1+M}{m_{\mathrm{e}}} \approx \dfrac{M}{m_{\mathrm{e}}}$ and
$\dfrac{\Delta K}{K} = \dfrac{4m_{\mathrm{e}}}{M} = 1.09\times 10^{-5}$.