Physics 2D, Winter 2005

Week 4 Exercise Solutions

4] a] $e_{TOT} = \sigma T^4 = 5.67\times10^{-8}\frac{W}{m^2K^4}(3000K)^4$

$$= \boxed{4.6\times10^6 \, W/m^2}$$

b] $75W = e_{TOT}\cdot \text{Area}$

$\Rightarrow \text{Area} = 1.63\times10^{-5}\,m^2 = \boxed{16.3\,mm^2}$

10] 1 photon has $E = hf = (6.626\times10^{-34}\,J\cdot s)(94MHz)$

$$= 6.23\times10^{-26}\,J$$

$100kW = 10^5\frac{J}{s}$, so this means $\frac{10^5}{6.23\times10^{-26}} = \boxed{1.6\times10^{30}\text{ photons/sec}}$

13] So $K_{max} = 2.92eV$.

Thus $2.92eV = hf - \phi$

But $hf = \frac{hc}{\lambda} = \frac{1240\,eV\cdot nm}{250nm} = 4.96\,eV$

So $\phi = (4.96 - 2.92\,eV) = \boxed{2.04eV}$

17 a) $E_{light} = \frac{hc}{\lambda} = 4.13\,eV$

So $\boxed{\text{lithium \& beryllium}}$ will eject electrons, since

$\phi < E_{light}$

b) $K_{max} = hf - \phi = \begin{cases} 1.83\,eV & \text{for lithium} \\ 0.23\,eV & \text{for beryllium.} \end{cases}$

20 $K_{max} = \frac{hc}{\lambda} - \phi$, so for the first wavelength,

$1\,eV = \frac{hc}{\lambda} - \phi$. The second wavelength is $\frac{\lambda}{2}$, and so

$4\,eV = \frac{2hc}{\lambda} - \phi$. We want to solve for ϕ, so just

divide the 2^{nd} eqn by 2 and subtract:

$$1\,eV = \frac{hc}{\lambda} - \phi$$

$$-\left(2\,eV = \frac{hc}{\lambda} - \frac{\phi}{2}\right)$$

$$-1\,eV = -\frac{\phi}{2} \Rightarrow \boxed{\phi = 2\,eV}$$

22 Need to find K_{max}, so need v for the electrons.

For e^-'s in a magnetic field, we know $F = \boxed{qvB = \frac{mv^2}{r}}$

(it's safe to use non-relativistic mechanics here, since these are not very energetic electrons).

So $v = \frac{qrB}{m}$, and $K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{q^2r^2B^2}{m}$

again, nonrelativistic is OK.

25 a) $\lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)$

$$= \frac{h}{m_e c}(1-\cos 30^\circ)$$

$$= 3.25\times10^{-13}\,m = \boxed{3.25\times10^{-4}\,nm}$$

b) $\lambda' = \lambda_0 + \frac{h}{m_e c}(1-\cos\theta)$.

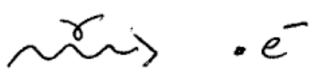
And $\lambda_0 = \frac{hc}{E} = \frac{1240\,eV\cdot nm}{300\,keV} = 4.13\times10^{-3}\,nm$.

So $\lambda' = 4.46\times10^{-3}\,nm \Rightarrow E' = \frac{hc}{\lambda} = \frac{1240\,eV\,nm}{4.46\times10^{-3}\,nm} = \boxed{278\,keV}$

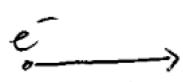
c) $KE_{e^-} = E' - E = \boxed{22\,keV}$ (that's just the kinetic energy — you could include rest energy too if you wanted).

27 The setup:

Before



After



Energy: $E_\gamma + m_e c^2 = E_e$

Momentum: $p_\gamma = p_e$

So since $E_\gamma = p_\gamma c$ and $E_e = \sqrt{p_e^2 c^2 + m^2 c^4}$

we can combine these to say

$$\boxed{pc + m_e c^2 = \sqrt{p^2 c^2 + m_e^2 c^4}} \qquad (p = p_e = p_\gamma)$$

Square both sides: $p^2c^2 + 2pm_e c^3 + m_e^2 c^4 = p^2 c^2 + m_e^2 c^4$

$$\Rightarrow \boxed{2pm_e c^3 = 0}$$

This means $p = 0$, which says there's $\boxed{\text{no photon}}$ to begin with. So the only solution is to have a free electron just hanging out.

30| We know $\lambda' = \lambda_0 + \frac{h}{m_e c}(1-\cos\theta)$.

For max energy transfer, λ should increase (i.e. the energy of the photon should decrease) as much as possible

So $\theta = 180° \Rightarrow \boxed{\lambda' = \lambda_0 + \frac{2h}{m_e c}}$

Now, $E = \frac{hc}{\lambda}$ for a photon.

Let's call $E' = \frac{hc}{\lambda'}$, $E_0 = \frac{hc}{\lambda_0}$. So then

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\left(\lambda_0 + \frac{2h}{m_e c}\right)}.$$

Let's assume $\frac{2h}{m_e c} \ll \lambda_0$ and check this later.

Then $\frac{1}{\lambda_0 + \frac{2h}{m_e c}} = \frac{1}{\lambda_0\left(1 + \frac{2h}{m_e c \lambda_0}\right)} \approx \frac{1}{\lambda_0}\left(1 - \frac{2h}{m_e c \lambda_0}\right)$

So $\boxed{E' = E_0 - \frac{2h^2c^2}{m_e c^2 \lambda_0^2}}$ (I multiplied top & bottom of the fraction by c)

This makes sense: $\Delta E < 0$ for the photon.

So $|\Delta E| = \frac{2h^2c^2}{m_e c^2 \lambda_0^2}$, and we know $|\Delta E| = 30\text{keV}$.

So $\lambda_0 = \left(\frac{2(hc)^2}{m_ec^2 |\Delta E|}\right)^{1/2}$

$$= \left(\frac{2(1240\ \text{eV}\cdot\text{nm})^2}{511\ \text{keV}\cdot 30\ \text{keV}}\right)^{1/2} = \boxed{1.42\times 10^{-2}\ \text{nm}}$$

Is this consistent with $\frac{2h}{m_e c} \ll \lambda_0$?

Well, $\frac{2h}{m_e c} = \frac{2hc}{m_e c^2} = \frac{2\cdot 1240}{511 \times 10^3}\ \text{nm} = 4.85\times 10^{-3}\ \text{nm}$

so $\frac{\left(\frac{2h}{m_e c}\right)}{\lambda_0} = 0.34$ which is kind of small.

Not a great approximation, but a decent one.

34 From #30, we know $\lambda_0 = \left(\frac{2(hc)^2}{m_e c^2 |\Delta E|}\right)^{1/2}$

So $\lambda_0 = \left(\frac{2(1240\ \text{eV}\cdot\text{nm})^2}{511\ \text{keV}\cdot 50\ \text{keV}}\right)^{1/2} = 1.1\times 10^{-2}\ \text{nm}$.

$\Rightarrow E = \frac{hc}{\lambda_0} = \frac{1240\ \text{eV nm}}{1.1\times 10^{-2}\ \text{nm}} = \boxed{113\ \text{keV}}$

36] ~~[illegible]~~ $E_{final,photon} = 80$ keV

$E_e = 25$ keV

So by ewsy conservation, $E_{i,photon} = 105$ keV

$\Rightarrow \lambda = \frac{hc}{E} = \boxed{1.18\times10^{-2}\text{ nm}}$

3-43 (a) A 4000 Å wavelength photon is backscattered, $\theta = \pi$ by an electron. The energy transferred to the electron is determined by using the Compton scattering formula $\lambda' - \lambda_0 = \left(\frac{hc}{E_e}\right)(1-\cos\theta)$ where we take $E_e = m_e c^2$ for the rest energy of the electron

and so $E_e \approx 0.511$ MeV. Upon substitution, one obtains

$$\Delta\lambda = 2(0.002\ 43\text{ nm}) = 0.004\ 86\text{ nm}\ .$$

The energy of a photon is related to its wavelength by the relation $E = \frac{hc}{\lambda}$, so the change in energy associated with a corresponding change in wavelength is given by $\Delta E = -\left(\frac{hc}{\lambda^2}\right)\Delta\lambda$. Upon making substitutions one obtains the magnitude $\Delta E = 6.037\ 9\times10^{-24}$ J and using the conversion factor 1 Joule of energy is equivalent to 1.602×10^{-19} eV. The result is $\Delta E = 3.77\times10^{-5}$ eV.

(b) This may be compared to the energy that would be acquired by an electron in the photoelectric effect process. Here again the energy of a photon of wavelength λ is given by $E = \frac{hc}{\lambda}$. With $\lambda = 400$ nm, one obtains

$$E = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.0\times10^{8}\text{ m/s})}{400\times10^{-9}\text{ m}} = 4.97\times10^{-19}\text{ J}$$

and upon converting to electron volts, $E = 3.10\ \text{eV}$. $\dfrac{\Delta E}{E_{\text{photon}}} \approx 10^{-5}$.

The maximum energy transfer is about five orders of magnitude smaller than the energy necessary for the photoelectric effect.

(c) Could "a violet photon" eject an electron from a metal by Compton scattering? The answer is no, because the maximum energy transfer occurring at $\theta = \pi$ is not sufficient.