Physics 2D, Winter 2005

Week 3 Exercise Solutions

See last week's solutions for problems 2-4 through 2-16.

2-15 (a) $K = \gamma mc^2 - mc^2 = Vq$ and so, $\gamma^2 = \left(1 + \frac{Vq}{mc^2}\right)^2$ and $\frac{v}{c} = \left\{1 - \left(1 + \frac{Vq}{mc^2}\right)^{-2}\right\}^{1/2}$

$$\frac{v}{c} = \left\{1 - \frac{1}{1 + \left(5.0 \times 10^4 \text{ eV}/0.511 \text{ MeV}\right)^2}\right\}^{1/2} = 0.412\,7$$

or $v = 0.413c$.

(b) $K = \frac{1}{2} mv^2 = Vq$

$$v = \left(\frac{2Vq}{m}\right)^{1/2} = \left\{\frac{2\left(5.0 \times 10^4 \text{ eV}\right)}{0.511 \text{ MeV}/c^2}\right\}^{1/2} = 0.442c$$

(c) The error in using the classical expression is approximately $\frac{3}{40} \times 100\%$ or about 7.5% in speed.

2-18 (a) The mass difference of the two nuclei is

$$\Delta m = 54.927\,9 \text{ u} - 54.924\,4 \text{ u} = 0.003\,5 \text{ u}$$
$$\Delta E = (931 \text{ MeV/u})(0.003\,5 \text{ u}) = 3.26 \text{ MeV}.$$

(b) The rest energy for an electron is 0.511 MeV. Therefore,

$$K = 3.26 \text{ MeV} - 0.511 \text{ MeV} = 2.75 \text{ MeV}.$$

2-20 $\Delta m = m - m_p - m_e = 1.008\,665 \text{ u} - 1.007\,276 \text{ u} - 0.000\,548\,5 \text{ u} = 8.404 \times 10^{-4} \text{ u}$

$E = c^2\left(8.404 \times 10^{-4} \text{ u}\right) = \left(8.404 \times 10^{-4} \text{ u}\right)(931.5 \text{ MeV/u}) = 0.783 \text{ MeV}$.

2-22 (a) Using Equation 2.4

$p = e^- BR = \left(1.60 \times 10^{-19} \text{ C}\right) BR \text{ kg} \cdot \text{m/C} \cdot \text{s} = 1.60 \times 10^{-19} BR \text{ kg} \cdot \text{m/s}$.

To convert $\text{kg} \cdot \text{m/s}$ to MeV/c, use

$$1 \text{ MeV}/c = \frac{\left(10^6\right)\left(1.60 \times 10^{-19} \text{ C}\right)(1 \text{ J/C})}{3.00 \times 10^8 \text{ m/s}} = 5.34 \times 10^{-22} \text{ kg} \cdot \text{m/s},$$ so that

$$p=\frac{\left(1.60\times10^{-19}BR\ \text{kg}\cdot\text{m/s}\right)\left(1\ \text{MeV}/c\right)}{5.34\times10^{-22}\ \text{kg}\cdot\text{m/s}}=300BR\ \text{MeV}/c\,.$$

Substituting numerical values

$$p(\text{in MeV}/c)=300BR=(300)(2.00\ \text{T})(0.343\ \text{m})=206\ \text{MeV}/c\,.$$

Since the momentum of the K^0 is zero before the decay, conservation of momentum requires the pion momenta to be equal in magnitude and opposite in direction. The pion's speed u may be found by noting that $\frac{p}{E}=\frac{mu/\sqrt{1-u^2/c^2}}{mc^2/\sqrt{1-u^2/c^2}}=\frac{u}{c^2}$ or $\frac{u}{c}=\frac{pc}{E}$ where p is the pion momentum and E is the pion's total energy. Thus for either pion, $\frac{u}{c}=\frac{pc}{E}=\frac{pc}{\left[p^2c^2+\left(mc^2\right)^2\right]^{1/2}}$ where m is the pion's mass. Finally,

$$\frac{u}{c}=\frac{206\ \text{MeV}}{\sqrt{(206\ \text{MeV})^2+(104\ \text{MeV})^2}}=0.827\,.$$

(b) Conservation of mass-energy requires that $E_{K^0}=2E$ where E_{K^0} is the total energy of a pion. As the K^0 pion decays at rest,

$$E_{K^0}=m_{K^0}c^2=2\sqrt{p^2c^2+\left(mc^2\right)^2}=2\sqrt{(206)^2+(140)^2}\ \text{MeV}=498\ \text{MeV}\,,$$

or $m_{K^0}=498\ \text{MeV}/c^2$.

2-23 In this problem, M is the mass of the initial particle, m_l is the mass of the lighter fragment, v_l is the speed of the lighter fragment, m_h is the mass of the heavier fragment, and v_h is the speed of the heavier fragment. Conservation of mass-energy leads to

$$Mc^2=\frac{m_lc^2}{\sqrt{1-v_l^2/c^2}}+\frac{m_hc^2}{\sqrt{1-v_h^2/c^2}}$$

From the conservation of momentum one obtains

$$(m_l)(0.987\,c)(6.22)=(m_h)(0.868\,c)(2.01)$$

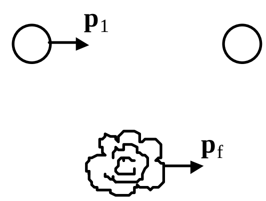
$$m_l=\frac{(m_h)(0.868\,c)(2.01)}{(0.987)(6.22)}=0.284m_h$$

Substituting in this value and numerical quantities in the mass-energy conservation equation, one obtains $3.34\times10^{-27}\ \text{kg}=6.22m_l+2.01m_h$ which in turn gives $3.34\times10^{-27}\ \text{kg}=(6.22)(0.284)m_l+2.01m_h$ or $m_h=\frac{3.34\times10^{-27}\ \text{kg}}{3.78}=8.84\times10^{-28}\ \text{kg}$ and $m_l=(0.284)m_h=2.51\times10^{-28}\ \text{kg}$.

2-24 The moving observer sees the charge as stationary, so she says it feels no magnetic force.

$$q(\mathbf{E}+\mathbf{v}\times\mathbf{B}) = q(\mathbf{E}' + \text{zero}), \ \mathbf{E}' = \mathbf{E}+\mathbf{v}\times\mathbf{B}.$$

2-30 Take the two colliding protons as the system

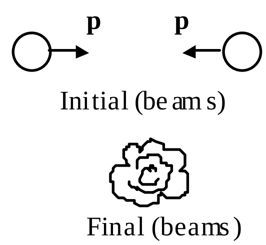


$E_1 = K + mc^2$
$E_2 = mc^2$

$E_1^2 = p_1^2c^2 + m^2c^4$
$p_2 = 0.$

In the final state, $E_f = K_f + Mc^2$: $E_f^2 = p_f^2c^2 + M^2c^4$.

By energy conservation, $E_1 + E_2 = E_f$, so



$$E_1^2 + 2E_1E_2 + E_2^2 = E_f^2$$
$$p_1^2c^2 + m^2c^4 + 2(K+mc^2)mc^2 + m^2c^4 = p_f^2c^2 + M^2c^4$$

By conservation of momentum, $p_1 = p_f$.

Then $M^2c^4 = 2Kmc^2 + 4m^2c^4 = \dfrac{4Km^2c^4}{2mc^2} + 4m^2c^4$

$$Mc^2 = 2mc^2\sqrt{1+\frac{K}{2mc^2}}.$$

By contrast, for colliding beams in the original state, we have $E_1 = K + mc^2$ and $E_2 = K + mc^2$. In the final state, $E_f = Mc^2$

$$E_1 + E_2 = E_f: \ K + mc^2 + K + mc^2 = Mc^2$$

$Mc^2 + 2mc^2\left(1+\dfrac{K}{2mc^2}\right).$

2-32 (a) $\rho = \dfrac{\text{energy}}{\Delta t} = \dfrac{2\text{ J}}{100\times10^{-15}\text{ s}} = 2.00\times10^{13}\text{ W}$

(b) The kinetic energy of one electron with $v = 0.999\,9c$ is

$$(\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-0.999\,9^2}}\right)(9.11\times10^{-31}\text{ kg})(3\times10^8\text{ m/s})^2 = 69.7(8.20\times10^{-14}\text{ J})$$
$$= 5.72\times10^{-12}\text{ J}$$

Then we require $\frac{0.01}{100}(2\ \text{J}) = N\left(5.72\times10^{-12}\ \text{J}\right)$

$$N = \frac{2\times10^{-4}\ \text{J}}{5.72\times10^{-12}\ \text{J}} = 3.50\times10^{7}\ .$$