1-2 IN THE REST FRAME:
In an elastic collision, energy and momentum are conserved.

\[ p_i = m_1v_{i1} + m_2v_{i2} = p_f = m_1v_{f1} + m_2v_{f2} \]

or \[ m_1(v_{i1} - v_{f1}) = -m_2(v_{i2} - v_{f2}) \]. The energy equation is

\[ E_i = E_f = \frac{1}{2} m_1v_{i1}^2 + \frac{1}{2} m_2v_{i2}^2 = \frac{1}{2} m_1v_{f1}^2 + \frac{1}{2} m_2v_{f2}^2 \]

or \[ m_1(v_{i1} - v_{f1})(v_{i1} + v_{f1}) = -m_2(v_{i2} - v_{f2})(v_{i2} + v_{f2}) \]. Substituting the momentum equation into the energy equation yields a very simple and general result (true even for three-dimensional collision if the velocities are replaced as vectors) \((v_{i1} + v_{i2}) = (v_{i2} + v_{i2})\) or as Newton put it originally, the final relative velocity is opposite to the initial relative velocity:

\[ (v_{i1} - v_{f1}) = - (v_{i2} - v_{f2}) \]

for elastic collision (and a fraction of the initial for general collisions). Now, putting in the numerical values, the momentum equation, and this relative velocity equation gives:

\[ m_1v_{f1} + m_2v_{f2} = 0.3v_{f1} + 0.2v_{f2} = 0.9 \text{ kg m/s}, \text{ and } (5 - (-3)) = - (v_{i1} - v_{i2}) \]. Solving the two equations, two unknowns, we find \(v_{i1} = -1.4 \text{ m/s} \) and \(v_{i2} = +6.6 \text{ m/s} \).

IN THE MOVING FRAME:
The Galilean velocity transformations hold.

\[
\begin{align*}
  v'_{1i} &= v_{1i} - v' = 20 \text{ m/s} - 10 \text{ m/s} = 10 \text{ m/s} \\
  v'_{2i} &= v_{2i} - v' = 0 \text{ m/s} - 10 \text{ m/s} = -10 \text{ m/s} \\
  p'_{i} &= m_1v'_{1i} + m_2v'_{2i} = (2 \text{ 000 kg})(10 \text{ m/s}) - (1 \text{ 500 kg})(10 \text{ m/s}) = 5 \times 10^3 \text{ kg m/s} \\
  p'_{f} &= (2 \text{ 000 kg} + 1 \text{ 500 kg})v'_{f} = (3 \text{ 500 kg})(v_{f} - 10 \text{ m/s}), \text{ and because } v_{i} = 11.4 \text{ m/s}, v_{f} = 6.6 \text{ m/s} \\
  p'_{f} &= 5 \times 10^3 \text{ kg m/s} \\
\end{align*}
\]

1-3 IN THE REST FRAME:
In an elastic collision energy and momentum are conserved.

\[ p_i = m_1v_{i1} + m_2v_{i2} = (0.3 \text{ kg})(5 \text{ m/s}) + (0.2 \text{ kg})(-3 \text{ m/s}) = 0.9 \text{ kg m/s} \]

\[ p_f = m_1v_{f1} + m_2v_{f2} \]

This equation has two unknowns, therefore, apply the conservation of kinetic energy

\[ E_i = E_f = \frac{1}{2} m_1v_{i1}^2 + \frac{1}{2} m_2v_{i2}^2 = \frac{1}{2} m_1v_{f1}^2 + \frac{1}{2} m_2v_{f2}^2 \]

and conservation of momentum one finds that \(v_{f1} = -1.31 \text{ m/s} \) and \(v_{f2} = 6.47 \text{ m/s} \) or \(v_{f1} = -1.56 \text{ m/s} \) and \(v_{f2} = 6.38 \text{ m/s} \). The difference in values is due to the rounding off errors in the numerical calculations of the mathematical quantities. If these two values are averaged the values are \(v_{f1} = -1.4 \text{ m/s} \) and \(v_{f2} = 6.6 \text{ m/s} \), \(p_f = 0.9 \text{ kg m/s} \). Thus, \(p_i = p_f \).
**IN THE MOVING FRAME:**
Make use of the Galilean velocity transformation equations. \( p'_i = m_i v'_i + m_2 v'_2 \); where \( v'_1 = v_{1i} - v' = 5 \text{ m/s} - (-2 \text{ m/s}) = 7 \text{ m/s} \). Similarly, \( v'_2 = -1 \text{ m/s} \) and \( p'_i = 1.9 \text{ kg} \cdot \text{m/s} \). To find \( p'_i \) use \( v'_1 = v_{1i} - v' \) and \( v'_2 = v_{2i} - v' \) because the prime system is now moving to the left.
Using these results give \( p'_i = 1.9 \text{ kg} \cdot \text{m/s} \).

1-4 (a) In all cases one wants the speed of the plane relative to the ground. For the upwind and downwind legs, where \( v' \) in the figure is given by \( (c^2 - v^2)^{1/2} \).

\[
t_{u+d} = \frac{L}{c - v} + \frac{L}{c + v} = \frac{2L}{c} \left( \frac{1}{1 - v^2/c^2} \right).
\]

For the crosswind case, the plane’s speed along \( L \) is \( v' = (c^2 - v^2)^{1/2} \):

\[
t_c = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c} \left( \frac{1}{\sqrt{1 - (v/c)^2}} \right).
\]

\[
t_{u+d} = \frac{2(100 \text{ mi})}{500 \text{ mi/h}} \left( \frac{1}{\sqrt{1 - (0.1)^2}} \right) = 0.4167 \text{ h}
\]

\[
t_c = \frac{2(100 \text{ mi})}{500 \text{ mi/h}} \left( \frac{1}{\sqrt{0.96}} \right) = 0.4082 \text{ h}
\]

(b) \( \Delta t = t_{u+d} - t_c = 0.0085 \text{ h} = 0.009 \text{ h} \) or \( 0.510 \text{ min} = 0.5 \text{ min} \)

1-5 This is a case of dilation. \( T = \gamma T' \) in this problem with the proper time \( T' = T_0 \)

\[
T = \left[ 1 - \left( \frac{v}{c} \right)^2 \right]^{-1/2} T_0 \Rightarrow \frac{v}{c} = \left[ 1 - \left( \frac{T_0}{T} \right)^2 \right]^{1/2}.
\]

in this case \( T = 2T_0, v = \left[ 1 - \left( \frac{L_0/2}{L_0} \right)^2 \right]^{1/2} = \left[ 1 - \left( \frac{1}{4} \right) \right]^{1/2} \) therefore \( v = 0.866c \).

1-6 This is a case of length contraction. \( L = \frac{L'}{\gamma} \) in this problem the proper length \( L' = L_0 \)

\[
L = \left[ 1 - \frac{v^2}{c^2} \right]^{-1/2} L_0 \Rightarrow v = c \left[ 1 - \left( \frac{L}{L_0} \right)^2 \right]^{1/2} \text{; in this case } L = \frac{L_0}{2}, v = \left[ 1 - \left( \frac{L_0/2}{L_0} \right)^2 \right]^{1/2} = \left[ 1 - \left( \frac{1}{4} \right) \right]^{1/2} \text{ therefore } v = 0.866c .
\]

1-7 The problem is solved by using time dilation. This is also a case of \( v << c \) so the binomial expansion is used \( \Delta t = \gamma \Delta t' = \left[ 1 + \frac{v^2}{2c^2} \right] \Delta t' \)

\[
\Delta t = \left( 24 \text{ h/day} \right) \left( 3600 \text{ s/h} \right) = 86400 \text{ s} \text{;} \quad \Delta t' = \Delta t - 1 = 86399 \text{ s} .
\]
\[ v = \left[ \frac{2(86\ 400\ \text{s} - 86\ 399\ \text{s})}{86\ 399\ \text{s}} \right]^{1/2} = 0.004\ 8c = 1.44 \times 10^6\ \text{m/s}. \]

1-8 \[ L = \frac{L'}{\gamma} \]
\[ \frac{1}{\gamma} = \frac{L}{L'} = \left[ 1 - \frac{v^2}{c^2} \right]^{1/2} \]
\[ v = c \left[ 1 - \left( \frac{L}{L'} \right)^2 \right]^{1/2} = c \left[ 1 - \left( \frac{75}{100} \right)^2 \right]^{1/2} = 0.661c \]

1-10 (a) \[ \tau = \gamma \tau' \text{ where } \beta = \frac{v}{c} \text{ and} \]
\[ \gamma = \left( 1 - \beta^2 \right)^{-1/2} = \tau \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} = \left( 2.6 \times 10^{-8}\ \text{s} \right) \left[ 1 - (0.95)^2 \right]^{1/2} = 8.33 \times 10^{-8}\ \text{s} \]

(b) \[ d = v\tau = (0.95) \left( 3 \times 10^8 \right) (8.33 \times 10^{-8}\ \text{s}) = 24\ \text{m} \]

1-12 (a) \[ 70\ \text{beats/min or } \Delta t' = \frac{1}{70}\ \text{min} \]

(b) \[ \Delta t = \gamma \Delta t' = \left[ 1 - (0.9)^2 \right]^{-1/2} \left( \frac{1}{70} \right) \text{ min} = 0.0328\ \text{min/beat or the number of beats per minute } \approx 30.5 = 31. \]

1-14 (a) Only the \( x \)-component of \( L_0 \) contracts.

\[ L_x' = L_0 \cos \theta_0 \Rightarrow L_x = L_0 \cos \theta_0 / \gamma \]
\[ L_y' = L_0 \sin \theta_0 \Rightarrow L_y = L_0 \sin \theta \]
\[ L = \sqrt{L_x^2 + L_y^2} = \sqrt{L_0^2 \cos^2 \theta / \gamma^2 + L_0^2 \sin^2 \theta_0} \]
\[ = L_0 \sqrt{\cos^2 \theta_0 \left( 1 - \frac{v^2}{c^2} \right) + \sin^2 \theta_0} = L_0 \sqrt{1 - \frac{v^2}{c^2} \cos^2 \theta_0} \]

(b) As seen by the stationary observer, \[ \tan \theta = \frac{L_y}{L_x} = \frac{L_0 \sin \theta_0}{L_0 \cos \theta_0 / \gamma} = \gamma \tan \theta_0. \]
1-16 For an observer approaching a light source, \( \lambda_{ob} = \left[ \frac{(1 - \frac{v}{c})^2}{(1 + \frac{v}{c})^2} \right] \lambda_{source} \). Setting \( \beta = \frac{v}{c} \) and after some algebra we find,

\[
\beta = \frac{\lambda_{source}^2 - \lambda_{obs}^2}{\lambda_{source}^2 + \lambda_{obs}^2} = \frac{(650 \text{ nm})^2 - (550 \text{ nm})^2}{(650 \text{ nm})^2 + (550 \text{ nm})^2} = 0.166
\]

\[
v = 0.166c = (4.98 \times 10^7 \text{ m/s}) (2.237 \text{ mi/h}) (\text{m/s})^{-1} = 1.11 \times 10^8 \text{ mi/h}.
\]

1-17 (a) Galaxy A is approaching and as a consequence it exhibits blue shifted radiation.

From Example 1.6, \( \beta = \frac{v}{c} = \frac{\lambda_{source}^2 - \lambda_{obs}^2}{\lambda_{source}^2 + \lambda_{obs}^2} \) so that \( \beta = \frac{(550 \text{ nm})^2 - (450 \text{ nm})^2}{(550 \text{ nm})^2 + (450 \text{ nm})^2} = 0.198 \).

Galaxy A is approaching at \( v = 0.198c \).

(b) For a red shift, B is receding. \( \beta = \frac{v}{c} = \frac{\lambda_{source}^2 - \lambda_{obs}^2}{\lambda_{source}^2 + \lambda_{obs}^2} \) so that

\[
\beta = \frac{(700 \text{ nm})^2 - (550 \text{ nm})^2}{(700 \text{ nm})^2 + (550 \text{ nm})^2} = 0.237 \). Galaxy B is receding at \( v = 0.237c \).

1-18 (a) Let \( f_c \) be the frequency as seen by the car. Thus, \( f_c = \frac{f_{source}}{c + v} \) and, if \( f \) is the frequency of the reflected wave, \( f = \frac{f_{source}}{c - v} \). Combining these equations gives

\[
f = f_{source} \frac{(c + v)}{(c - v)}
\]

(b) Using the above result, \( f(c - v) = f_{source}(c + v) \), which gives

\[
(f - f_{source}) c = (f + f_{source}) v = 2 f_{source} v.
\]

The beat frequency is then \( f_{beat} = f - f_{source} = \frac{2 f_{source} v}{c} = \frac{2v}{\lambda} \).

(c) \( f_{beat} = \frac{2 (30.0 \text{ m/s}) (10.0 \times 10^9 \text{ Hz})}{3.00 \times 10^8 \text{ m/s}} = \frac{2(30.0 \text{ m/s})}{0.030 \text{ m}} = 2 \text{ 000 Hz} = 2.00 \text{ kHz} \)

\[
\lambda = \frac{c}{f_{source}} = \frac{3.00 \times 10^8 \text{ m/s}}{10.0 \times 10^9 \text{ Hz}} = 3.00 \text{ cm}
\]

(d) \( v = \frac{f_{beat} \lambda}{2} \) so,

\[
\Delta v = \frac{\Delta f_{beat} \lambda}{2} = \frac{(5 \text{ Hz})(0.030 \text{ m})}{2} = 0.075 \text{ m/s} = 0.2 \text{ mi/h}
\]