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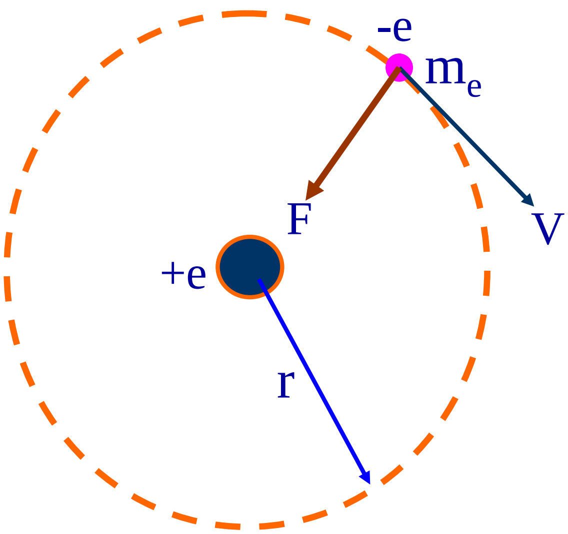
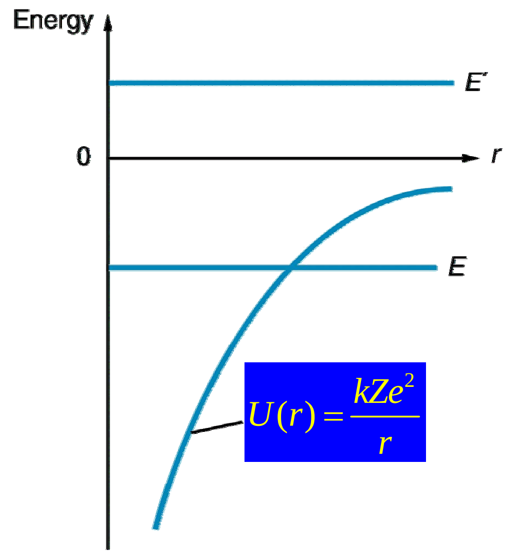
Physics 2D Lecture Slides
Lecture 28: Mar 9th

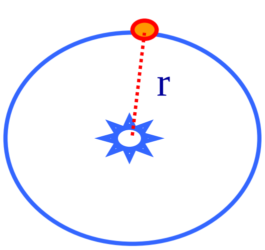
Vivek Sharma
UCSD Physics

The Coulomb Attractive Potential That Binds the electron and Nucleus (charge +Ze) into a Hydrogenic atom





The Hydrogen Atom In Its Full Quantum Mechanical Glory



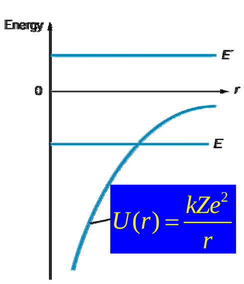
$$U(r) \propto \frac{1}{r} = \frac{1}{\sqrt{x^2+y^2+z^2}} \quad \Rightarrow \text{ x,y,z all mixed up !}$$

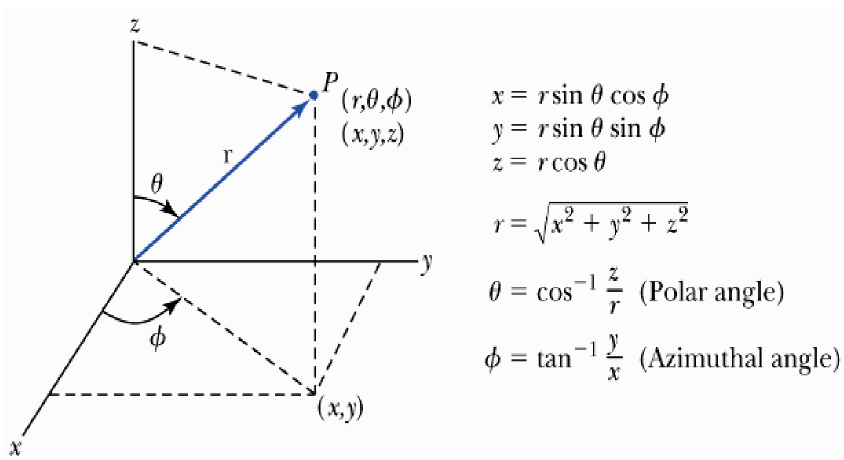
As in case of particle in 3D box, we should use seperation of variables (x,y,z ??) to derive 3 independent differential. eqns.

This approach will get very ugly since we have a "conjoined triplet"

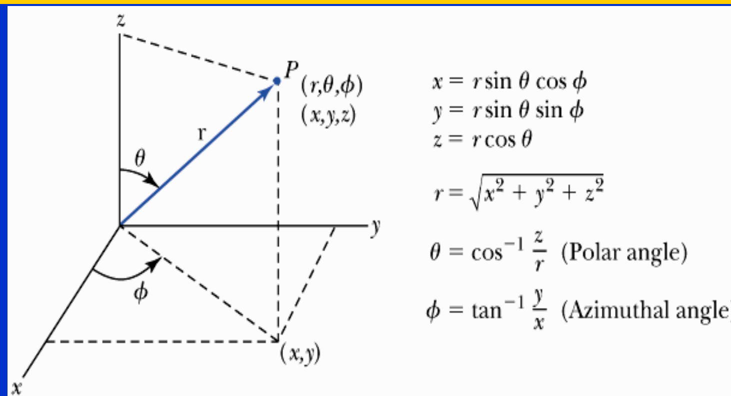
To simplify the situation, choose more appropriate variables

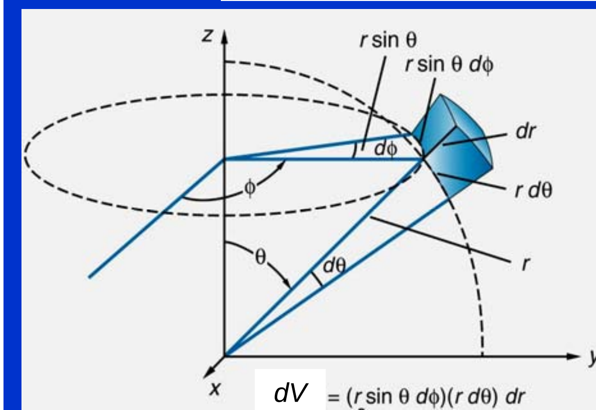
Cartesian coordinates (x,y,z) $\rightarrow$ Spherical Polar (r,θ,ϕ) coordinates





Spherical Polar Coordinate System

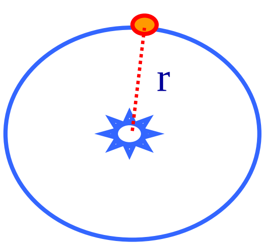


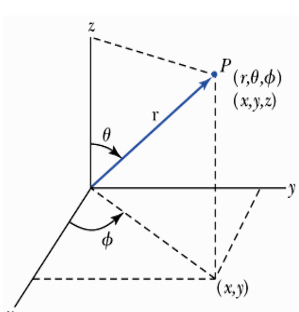


Volume Element dV

$$dV = (r\sin\theta d\phi)(rd\theta)(dr)$$
$$= r^2\sin\theta drd\theta d\phi$$

The Hydrogen Atom In Its Full Quantum Mechanical Glory





Instead of writing Laplacian $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$,

write ∇^2 for spherical polar coordinates:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial^2\phi}$$

Thus the T.I.S.Eq. for $\psi(x,y,z) = \psi(r,\theta,\phi)$ becomes

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi(r,\theta,\phi)}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi(r,\theta,\phi)}{\partial\theta}\right) +$$
$$\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi(r,\theta,\phi)}{\partial^2\phi} + \frac{2m}{\hbar^2}(E-U(r))\psi(r,\theta,\phi) = 0$$

with $U(r) \propto \frac{1}{r} = \frac{1}{\sqrt{x^2+y^2+z^2}}$

The Schrodinger Equation in Spherical Polar Coordinates (is bit of a mess!)

The TISE is :

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial^2\phi}+\frac{2m}{\hbar^2}(E-U(r))\psi(r,\theta,\phi)=0$$

Try to free up second last term from all except ϕ

This requires multiplying thruout by $\boxed{r^2\sin^2\theta}\Rightarrow$

$$\sin^2\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\sin\theta\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{\partial^2\psi}{\partial^2\phi}+\frac{2mr^2\sin^2\theta}{\hbar^2}(E+\frac{ke^2}{r})\psi=0$$

For Seperation of Variables, Write $\psi(r,\theta,\phi)=R(r).\Theta(\theta).\Phi(\phi)$

Plug it into the TISE above & divide thruout by $\psi(r,\theta,\phi)=R(r).\Theta(\theta).\Phi(\phi)$

Note that :

$$\boxed{\begin{aligned}\frac{\partial\Psi(r,\theta,\phi)}{\partial r}&=\Theta(\theta).\Phi(\phi)\frac{\partial R(r)}{\partial r}\\ \frac{\partial\Psi(r,\theta,\phi)}{\partial\theta}&=R(r)\Phi(\phi)\frac{\partial\Theta(\theta)}{\partial\theta}\\ \frac{\partial\Psi(r,\theta,\phi)}{\partial\theta}&=R(r)\Theta(\theta)\frac{\partial\Phi(\phi)}{\partial\phi}\end{aligned}}$$

$\Rightarrow$ when substituted in TISE

Don't Panic: Its simpler than you think !

$$\frac{\sin^2\theta}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right)+\frac{\sin\theta}{\Theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial^2\phi}+\frac{2mr^2\sin^2\theta}{\hbar^2}(E+\frac{ke^2}{r})=0$$

Rearrange by taking the ϕ term on RHS

$$\boxed{\frac{\sin^2\theta}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right)+\frac{\sin\theta}{\Theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{2mr^2\sin^2\theta}{\hbar^2}(E+\frac{ke^2}{r})=-\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial^2\phi}}$$

LHS is fn. of r,θ & RHS is fn of ϕ only , for equality to be true for all r,θ,ϕ

$$\Rightarrow\boxed{\text{LHS= constant = RHS = } m_l^2}$$

Deconstructing The Schrodinger Equation for Hydrogen

Now go break up LHS to seperate the r & θ terms.....

$$\text{LHS:}\frac{\sin^2\theta}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right)+\frac{\sin\theta}{\Theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{2mr^2\sin^2\theta}{\hbar^2}(E+\frac{ke^2}{r})=m_l^2$$

Divide Thruout by $\sin^2\theta$ and arrange all terms with r away from $\theta \Rightarrow$

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right)+\frac{2mr^2}{\hbar^2}(E+\frac{ke^2}{r})=\frac{m_l^2}{\sin^2\theta}-\frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)$$

Same argument : LHS is fn of r, RHS is fn of θ;

For them to be equal for all r, $\theta \Rightarrow$ LHS = const = RHS = $l(l+1)$

What is the mysterious $l(l+1)$? Just a number like 2(2+1)

So What do we have after all the shuffling!

$$\frac{d^2\Phi}{d\phi^2}+m_l^2\Phi=0..................(1)$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\left[l(l+1)-\frac{m_l^2}{\sin^2\theta}\right]\Theta(\theta)=0.....(2)$$

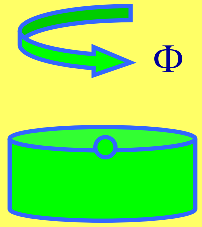
$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{\partial R}{\partial r}\right)+\left[\frac{2mr^2}{\hbar^2}(E+\frac{ke^2}{r})-\frac{l(l+1)}{r^2}\right]R(r)=0....(3)$$

These 3 "simple" diff. eqn describe the physics of the Hydrogen atom.

All we need to do now is guess the solutions of the diff. equations

Each of them, clearly, has a different functional form

And Now the Solutions of The S. Eqns for Hydrogen Atom



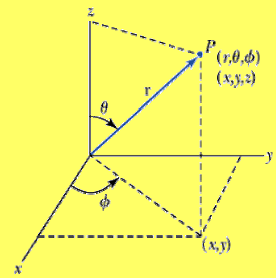
The Azimuthal Diff. Equation : $\boxed{\dfrac{d^2\Phi}{d\phi^2} + m_l^2\Phi = 0}$

Solution : $\Phi(\phi) = A\, e^{im_l\phi}$ but need to check "Good Wavefunction Condition"

Wave Function must be Single Valued for all $\phi \Rightarrow \Phi(\phi) = \Phi(\phi + 2\pi)$

$\Rightarrow \Phi(\phi) = A\, e^{im_l\phi} = A\, e^{im_l(\phi+2\pi)} \Rightarrow m_l = 0, \pm1, \pm2, \pm3....$(*Magnetic* Quantum #)

The Polar Diff. Eq: $\boxed{\dfrac{1}{\sin\theta}\dfrac{d}{d\theta}\left(\sin\theta\dfrac{d\Theta}{d\theta}\right) + \left[l(l+1) - \dfrac{m_l^2}{\sin^2\theta}\right]\Theta(\theta) = 0}$



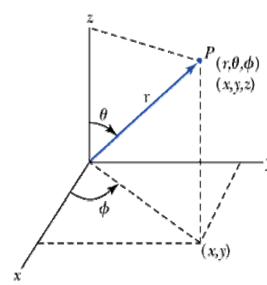
Solutions : go by the name of "Associated Legendre Functions"

only exist when the integers l and m_l are related as follows

$m_l = 0, \pm1, \pm2, \pm3.... \pm l$; l = positive number

l : Orbital Quantum Number

Wavefunction Along Azimuthal Angle ϕ and Polar Angle θ



For $l = 0, m_l = 0 \Rightarrow \boxed{\Theta(\theta) = \dfrac{1}{\sqrt{2}}}$;

For $l = 1, m_l = 0, \pm1 \Rightarrow$ Three Possibilities for the Orbital part of wavefunction

$[l = 1, m_l = 0] \Rightarrow \Theta(\theta) = \dfrac{\sqrt{6}}{2}\cos\theta$

$[l = 1, m_l = \pm1] \Rightarrow \Theta(\theta) = \dfrac{\sqrt{3}}{2}\sin\theta$

$[l = 2, m_l = 0] \Rightarrow \Theta(\theta) = \dfrac{\sqrt{10}}{4}(3\cos^2\theta - 1)$

....*and* so on and so forth (see book for more Functions)

Radial Differential Equations and Its Solutions

The Radial Diff. Eqn: $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{\partial R}{\partial r}\right)+\left[\frac{2mr^2}{\hbar^2}(E+\frac{ke^2}{r})-\frac{l(l+1)}{r^2}\right]R(r)=0$

Solutions : Associated Laguerre Functions R(r), Solutions exist only if:

1. E>0 or has negtive values given by

$$E=-\frac{ke^2}{2a_0}\left(\frac{1}{n^2}\right); \quad \text{with } a_0=\frac{\hbar^2}{mke^2}=\text{Bohr Radius}$$

2. And when n = integer such that $l=0,1,2,3,4,.......(n-1)$

n = principal Quantum # or the "big daddy" quantum #

The Hydrogen Wavefunction: ψ(r,θ,ϕ) and Ψ(r,θ,ϕ,t)

To Summarize : The hydrogen atom is brought to you by the letters:

$n = 1,2,3,4,5,....\infty$
$l=0,1,2,3,,4....(n-1)$
$m_l=0,\pm1,\pm2,\pm3,...\pm l$

Quantum # appear only in Trapped systems

The Spatial part of the Hydrogen Atom Wave Function is:

$$\psi(r,\theta,\phi)=R_{nl}(r)\ .\ \Theta_{lm_l}(\theta)\ \ .\ \Phi_{m_l}(\phi)=R_{nl}Y_l^{m_l}$$

$Y_l^{m_l}$ are known as Spherical Harmonics. They define the angular structure in the Hydrogen-like atoms.

The Full wavefunction is $\Psi(r,\theta,\varphi,t)=\ \psi(r,\theta,\phi)e^{-\frac{iE}{\hbar}t}$

Radial Wave Functions For n=1,2,3

n	l	m_l	$R(r)=$
1	0	0	$\frac{2}{a_0^{3/2}} e^{-r/a}$
2	0	0	$\frac{1}{2\sqrt{2}a_0^{3/2}}(2-\frac{r}{a_0})e^{-\frac{r}{2a_0}}$
3	0	0	$\frac{2}{81\sqrt{3}a_0^{3/2}}(27-18\frac{r}{a_0}+2\frac{r^2}{a_0^2})e^{-\frac{r}{3a_0}}$

n=1 → K shell
n=2 → L Shell
n=3 → M shell
n=4 → N Shell
……

l=0 → s(harp) sub shell
l=1 → p(rincipal) sub shell
l=2 → d(iffuse) sub shell
l=3 → f(undamental) ss
l=4 → g sub shell
……..

Symbolic Notation of Atomic States in Hydrogen

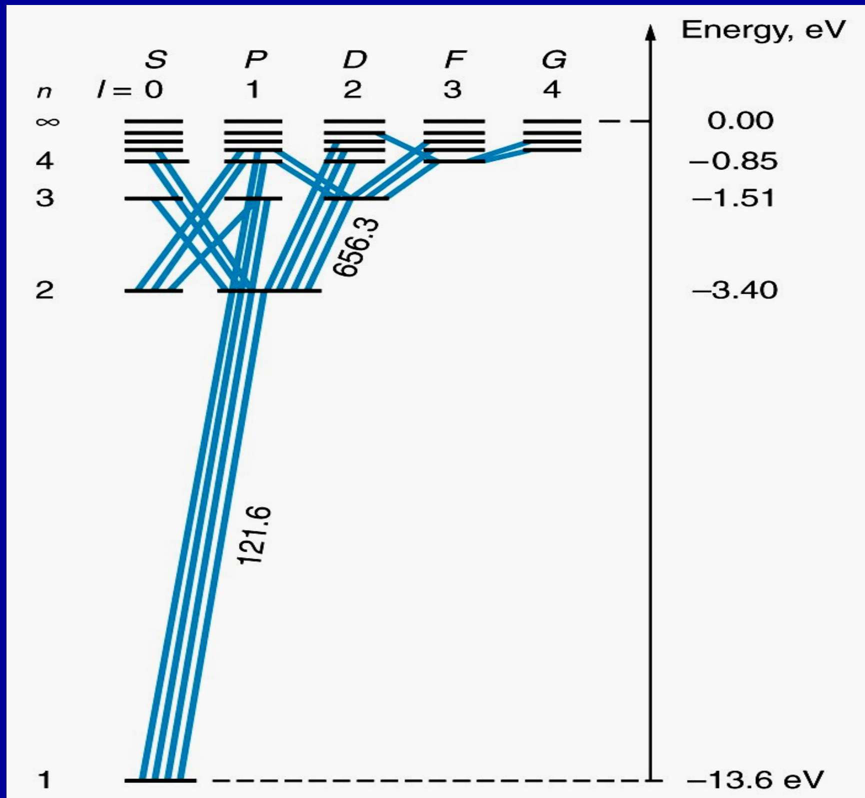
$l \rightarrow$ $n \downarrow$	$s\ (l=0)$	$p\ (l=1)$	$d\ (l=2)$	$f\ (l=3)$	$g\ (l=4)$	.....
1	$1s$					
2	$2s$	$2p$				
3	$3s$	$3p$	$3d$			
4	$4s$	$4p$	$4d$	$4f$		
5	$5s$	$5p$	$5d$	$5f$	$5g$	

Note that:

- n =1 is a non-degenerate system
- n>1 are all degenerate in l and m_l.
  All states have same energy
  But different angular configuration

$$E=-\frac{ke^2}{2a_0}\left(\frac{1}{n^2}\right)$$

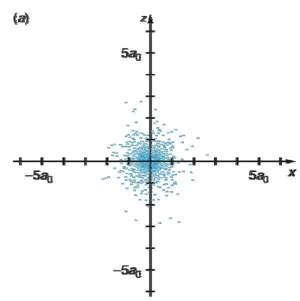
Energy States, Degeneracy & Transitions



Facts About Ground State of H Atom

$n=1,\ l=0,\ m_l=0 \Rightarrow R(r)=\frac{2}{a_0^{3/2}}\ e^{-r/a};\ \ \Theta(\theta)=\frac{1}{\sqrt{2\pi}};\ \ \Phi(\phi)=\frac{1}{\sqrt{2}}$

$$\boxed{\Psi_{100}(r,\theta,\phi)=\frac{1}{a_0\sqrt{\pi}}e^{-r/a}}$$ ......look at it carefully



1. Spherically symmetric $\Rightarrow$ no θ,ϕ dependence (structure)

2. Probability Per Unit Volume : $\left|\Psi_{100}(r,\theta,\phi)\right|^2=\frac{1}{\pi a_0^3}e^{-\frac{2r}{a}}$

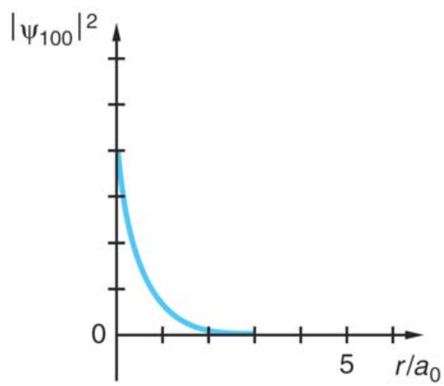
Likelihood of finding the electron is same at all θ,ϕ and depends only on the radial seperation (r) between electron & the nucleus.

3 Energy of Ground State =$-\frac{ke^2}{2a_0}=-13.6eV$

Overall The Ground state wavefunction of the hydrogen atom is quite *boring*

Not much chemistry or Biology could develop if there was only the ground state of the Hydrogen Atom!

We need structure, we need variety, we need some curves!

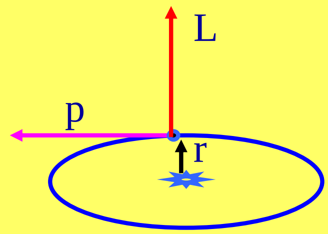


Interpreting Orbital Quantum Number *(l)*

Radial part of S.Eqn: $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\left[\frac{2m}{\hbar^2}(E+\frac{ke^2}{r})-\frac{l(l+1)}{r^2}\right]R(r)=0$

For H Atom: $E = K + U = K_{RADIAL} + K_{ORBITAL} - \frac{ke^2}{r}$; substitute this in E

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{2m}{\hbar^2}\left[K_{RADIAL}+K_{ORBITAL}-\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]R(r)=0$$



Examine the equation, if we set $\boxed{K_{ORBITAL}=\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}}$ *then*

what remains is a differential equation in r

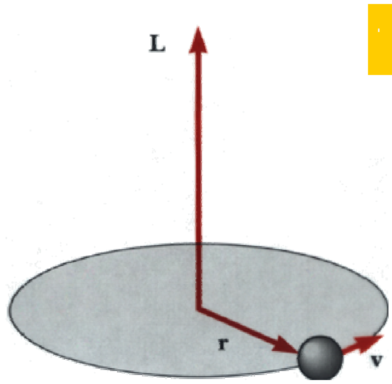
$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{2m}{\hbar^2}[K_{RADIAL}]R(r)=0$ which depends only on radius r of orbit

Further, we also know that $K_{ORBITAL}=\frac{1}{2}mv_{orbit}^2$; $\vec{L}=\vec{r}\times\vec{p}$; $|L| = mv_{orb}r \Rightarrow \boxed{K_{ORBITAL}=\frac{L^2}{2mr^2}}$

Putting it all togather: $\boxed{K_{ORBITAL}=\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}=\frac{L^2}{2mr^2}}$ $\Rightarrow$ magnitude of Ang. Mom $|L|=\sqrt{l(l+1)}\hbar$

Since l = *positive* integer=0,1,2,3...(n-1) $\Rightarrow$ angular momentum $\boxed{|L|=\sqrt{l(l+1)}\hbar = \text{discrete values}}$

$|L|=\sqrt{l(l+1)}\hbar$: QUANTIZATION OF Electron's Angular Momentum



Magnetic Quantum Number m_l

$\vec{L}=\vec{r}\times\vec{p}$ (Right Hand Rule)

Classically, direction & Magnitude of $\vec{L}$ always well defined

QM: Can/Does $\vec{L}$ have a definite direction ? Proof by Negation:

Suppose $\vec{L}$ was precisely known/defined ($\vec{L}\parallel\hat{z}$)

Since $\vec{L}=\vec{r}\times\vec{p}\Rightarrow$ Electron MUST be in x-y orbit plane

$\Rightarrow \Delta z = 0$; $\Delta p_z \Delta z \sim \hbar \Rightarrow \Delta p_z \sim \infty; E=\frac{p^2}{2m}\sim\infty$!!!

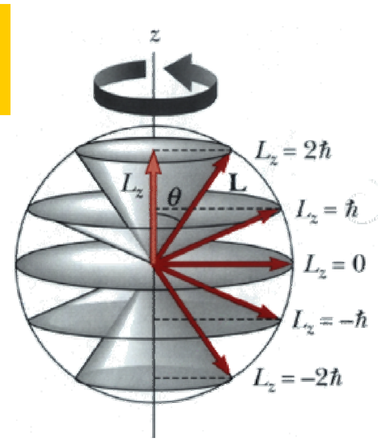
So, in Hydrogen atom, $\vec{L}$ can not have precise measurable value

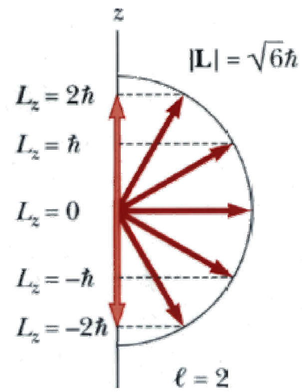
Uncertainty Principle & Angular Momentum : $\boxed{\Delta L_z \Delta\phi \sim \hbar}$

Magnetic Quantum Number m_l

Consider $\ell = 2$

$|L| = \sqrt{\ell(\ell+1)} = \sqrt{6}\hbar$





In Hydrogen atom, $\vec{L}$ can not have precise measurable value

Arbitararily picking Z axis as a reference direction:

$\vec{L}$ vector spins around Z axis (precesses).

The Z component of $\vec{L}$: $|L_Z| = m_l\hbar; \quad m_l = \pm1, \pm2, \pm3 \ldots \pm l$

Note: since $|L_Z| < |L|$ (always)

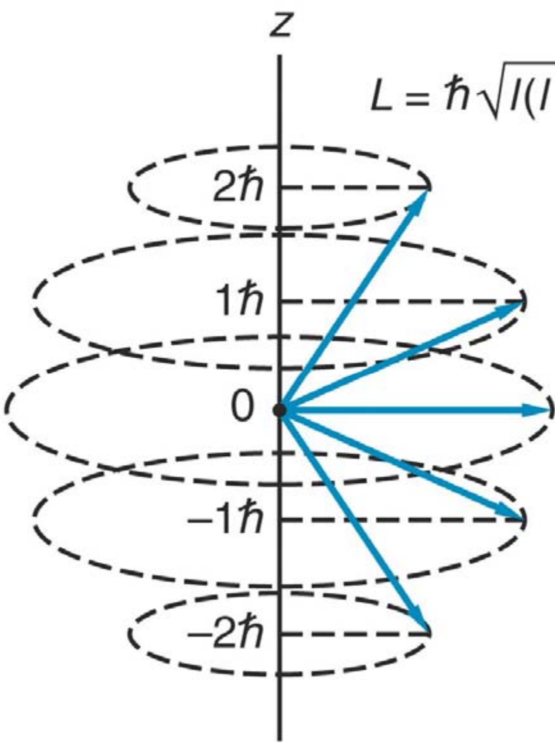
since $m_l\hbar < \sqrt{l(l+1)}\,\hbar$ It can never be that $|L_Z| = m_l\hbar = \sqrt{l(l+1)}\,\hbar$

(breaks Uncertainty Principle)

So......the Electron's dance has begun !

L=2, m_l=0,±1, ± 2 : Pictorially

$$L = \hbar\sqrt{l(l+1)} = \hbar\sqrt{2(2+1)} = \hbar\sqrt{6}$$



Electron "sweeps"

Conical paths of different ϑ:

$\cos\vartheta = L_Z/L$

On average, the angular momentum

Component in x and y cancel out

$\langle L_X \rangle = 0$

$\langle L_Y \rangle = 0$

Where is it likely to be ? → Radial Probability Densities

$\Psi(r,\theta,\phi) = R_{nl}(r)\,.\,\Theta_{lm_l}(\theta)\;\;.\;\Phi_{m_l}(\phi) = R_{nl}Y_l^{m_l}$

Probability Density Function in 3D:

$$\boxed{P(r,\theta,\phi) = \Psi^*\Psi = |\Psi(r,\theta,\phi)|^2 = \;|R_{nl}|^2 . |Y_l^{m_l}|^2}$$

Note: 3D Volume element $\boxed{dV = r^2.\sin\theta.dr.d\theta.d\phi}$

Prob. of finding particle in a tiny volume dV is

$P.dV = \;|R_{nl}|^2 . |Y_l^{m_l}|^2 .r^2.\sin\theta.dr.d\theta.d\phi$

The Radial part of Prob. distribution: P(r)dr

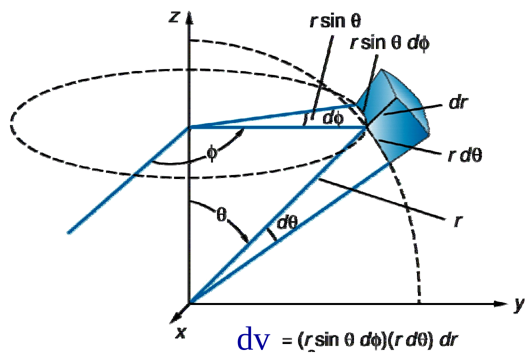
$$P(r)dr = |R_{nl}|^2 .r^2 dr \int_0^{\pi} |\Theta_{lm_l}(\theta)|^2 d\theta \int_0^{2\pi} |\Phi_{m_l}(\phi)|^2 \,d\phi$$

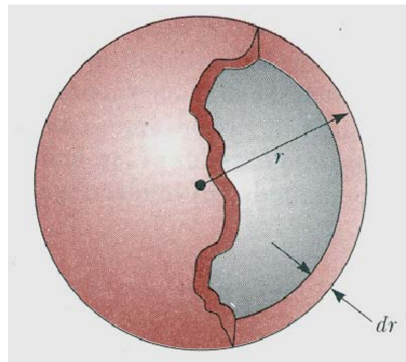
When $\Theta_{lm_l}(\theta)$ & $\Phi_{m_l}(\phi)$ are auto-normalized then

$P(r)dr = |R_{nl}|^2 .r^2 dr$; in other words $\boxed{P(r) = r^2\,|R_{nl}|^2}$

Normalization Condition: $\boxed{1 = \int_0^{\infty} r^2 |R_{nl}|^2 dr}$

Expectation Values $\boxed{\langle f(r)\rangle = \int_0^{\infty} f(r).P(r)dr}$





Ground State: Radial Probability Density

$P(r)dr = |\psi(r)|^2 .4\pi r^2 dr$

$$\Rightarrow \boxed{P(r)dr = \frac{4}{a_0^3} r^2 e^{-2\frac{r}{a_0}}}$$

Probability of finding Electron for $r > a_0$

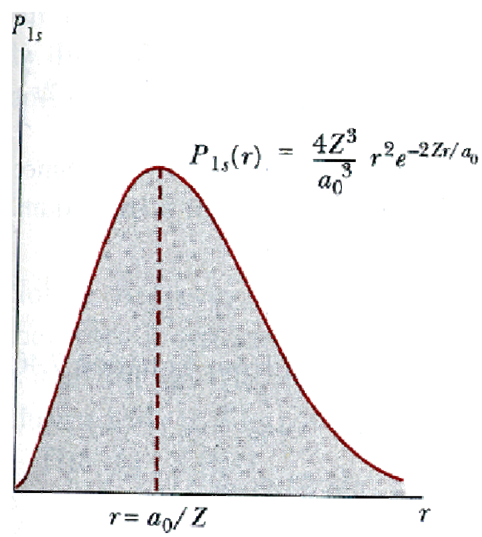
$$P_{r>a_0} = \int_a^{\infty} \frac{4}{a_0^3} r^2 e^{-2\frac{r}{a_0}} dr$$

To solve, employ change of variable

Define $z = \left[\frac{2r}{a_0}\right]$; *change* limits of integration

$$P_{r>a_0} = \frac{1}{2}\int_2^{\infty} z^2 e^{-z} dz \quad \text{(such integrals called Error. Fn)}$$

$$= -\frac{1}{2}[z^2 + 2z + 2]e^{-z}\,|_2^{\infty} = 5e^2 = 0.667 \Rightarrow 66.7\%\;\;!!$$



Most Probable & Average Distance of Electron from Nucleus

Most Probable Distance:

In the ground state $(n=1, l=0, m_l=0)$ $\boxed{P(r)dr = \frac{4}{a_0^3} r^2 e^{-2\frac{r}{a_0}}}$

Most probable distance r from Nucleus $\Rightarrow$ What value of r is P(r) max?

$$\Rightarrow \frac{dP}{dr}=0 \quad \Rightarrow \quad \frac{4}{a_0^3}\cdot\frac{d}{dr}\left[r^2 e^{-2\frac{r}{a_0}}\right]=0 \Rightarrow \left[\frac{-2r^2}{a_0}+2r\right]e^{-2\frac{r}{a_0}}=0$$

$\Rightarrow \frac{2r^2}{a_0}+2r=0 \Rightarrow \boxed{r=0 \ or \ r=a_0}$... which solution is correct?

(see past quiz) : Can the electron BE at the center of Nucleus (r=0)?

$P(r=0)=\frac{4}{a_0^3}0^2 e^{-2\frac{0}{a_0}}=0! \Rightarrow$ $\boxed{\text{Most Probable distance } r=a_0}$ (Bohr guessed right)

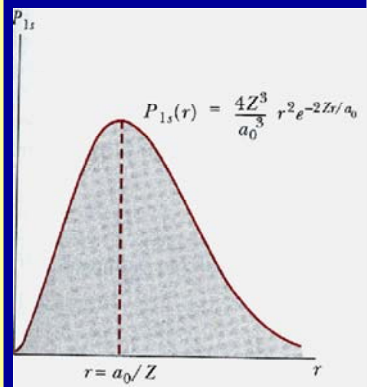
What about the AVERAGE location <r> of the electron in Ground state?

$<r>=\int_{r=0}^{\infty} rP(r)dr = \frac{4}{a_0^3}\int_0^{\infty} r.r^2 e^{-2\frac{r}{a_0}}dr$ ...change of variable $z=\frac{2r}{a_0}$

$\Rightarrow <r> = \frac{a_0}{4}\int_{z=0}^{\infty} z^3 e^{-z}dz$....... $\boxed{\text{Use general form } \int_0^{\infty} z^n e^{-z}dz = n! = n(n-1)(n-2)...(1)}$

$\Rightarrow$ $\boxed{<r> = \frac{a_0}{4}3! = \frac{3a_0}{2} \neq a_0}$! Average & most likely distance is not same. Why?

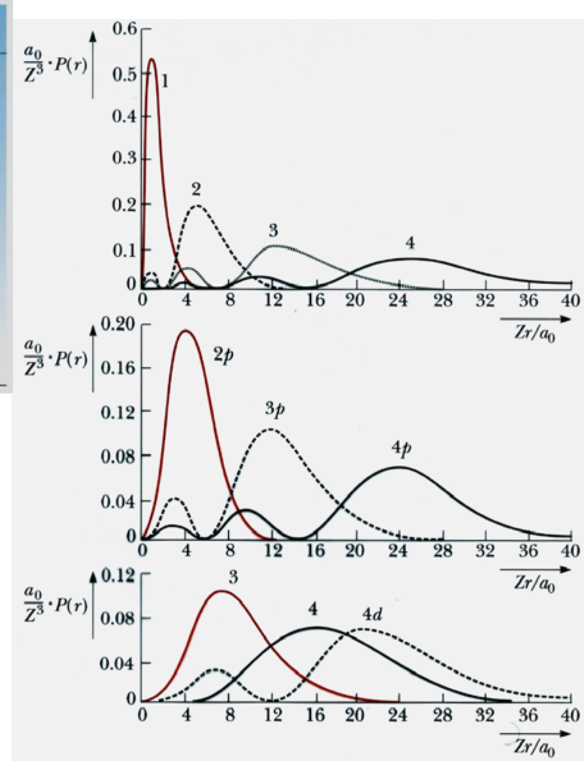
Asnwer is in the form of the radial Prob. Density: Not symmetric



Radial Probability Distribution P(r)= r^2R(r)

TABLE 7-2 Radial functions for hydrogen

$n=1$	$l=0$	$R_{10} = \frac{2}{\sqrt{a_0^3}} e^{-r/a_0}$
$n=2$	$l=0$	$R_{20} = \frac{1}{\sqrt{2a_0^3}}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}$
	$l=1$	$R_{21} = \frac{1}{2\sqrt{6a_0^3}}\frac{r}{a_0}e^{-r/2a_0}$
$n=3$	$l=0$	$R_{30} = \frac{2}{3\sqrt{3a_0^3}}\left(1-\frac{2r}{3a_0}+\frac{2r^2}{27a_0^2}\right)e^{-r/3a_0}$
	$l=1$	$R_{31} = \frac{8}{27\sqrt{6a_0^3}}\frac{r}{a_0}\left(1-\frac{r}{6a_0}\right)e^{-r/3a_0}$
	$l=2$	$R_{32} = \frac{4}{8\sqrt{30a_0^3}}\frac{r^2}{a_0^2}e^{-r/3a_0}$



Because P(r)=r^2R(r)

No matter what R(r) is for some n

The prob. Of finding electron

inside nucleus =0