

**Confirmed: 2D Final Exam:
Thursday 18th March 11:30-2:30 PM WLH 2005**

Course Review 14th March 10am WLH 2005 (TBC)

Date	Time	Read	Topic	HW problems for the week	Location
Monday	11:00 am	Ch 5	Quantum Mechanics in 1 Dimension		WLH 2005
Tuesday	8:00 pm	Ch 5	Quantum Mechanics in 1 Dimension	Ch 5 :20, 21, 25, 30, 33, 34, 35, 39, 40	WLH 2005
Wednesday	11:00 am	Ch 6	Tunneling Phenomenan	Read Section 6.1	WLH 2005
Wednesday	3:00 pm	Ch 5	Discussion	-	WLH 2005
Thursday	5:00-6:20 pm	-	Problem Session		Peterson 108
Friday	11:00am	-	Quiz	Ch 5	WLH 2005



**Physics 2D Lecture Slides
Lecture 24: Mar 1st**

**Vivek Sharma
UCSD Physics**

Finite Potential Well

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + U\psi(x) = E \psi(x)$$

$$\Rightarrow \frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2} (U - E)\psi(x)$$

$$= \alpha^2 \psi(x); \quad \alpha = \sqrt{\frac{2m(U-E)}{\hbar^2}}$$

\Rightarrow General Solutions : $\psi(x) = Ae^{+\alpha x} + Be^{-\alpha x}$

Require finiteness of $\psi(x)$

$\Rightarrow \psi(x) = Ae^{+\alpha x} \dots x < 0$ (region I)
 $\psi(x) = Ae^{-\alpha x} \dots x > L$ (region III)

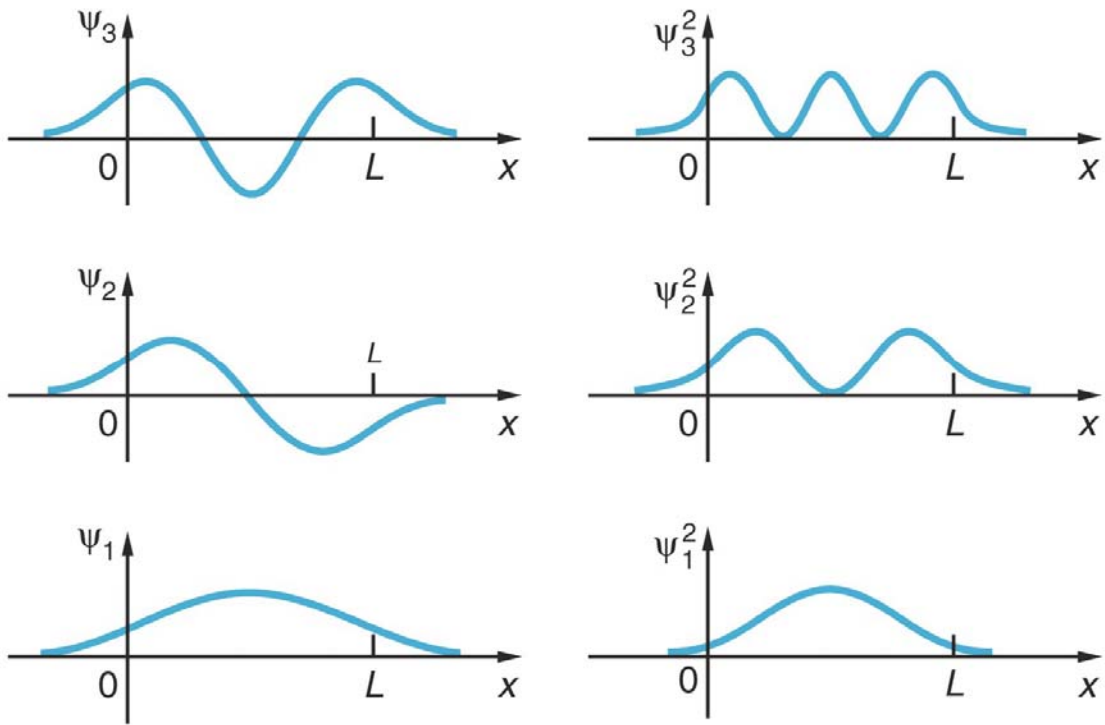
Again, coefficients A & B come from matching conditions at the edge of the walls ($x=0, L$)

But note that wave fn at $\psi(x)$ at ($x=0, L$) $\neq 0$!! (why?)

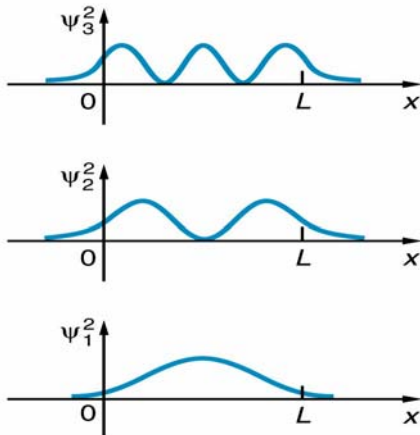
Further require Continuity of $\psi(x)$ and $\frac{d\psi(x)}{dx}$

These lead to rather different wave functions

Finite Potential Well: Particle can Burrow Outside Box



Finite Potential Well: Particle can Burrow Outside Box



Particle can be outside the box but only for a time $\Delta t \approx \hbar / \Delta E$

$\Delta E =$ Energy particle needs to borrow to

Get outside $\Delta E = U - E + KE$

The Cinderella act (of violating E

Conservation cant last very long

Particle must hurry back (cant be

caught with its hand inside the cookie-jar)

$$\text{Penetration Length } \delta = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m(U-E)}}$$

If $U \gg E \Rightarrow$ Tiny penetration

If $U \rightarrow \infty \Rightarrow \delta \rightarrow 0$

Finite Potential Well: Particle can Burrow Outside Box

$$\text{Penetration Length } \delta = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m(U-E)}}$$

If $U \gg E \Rightarrow$ Tiny penetration

If $U \rightarrow \infty \Rightarrow \delta \rightarrow 0$

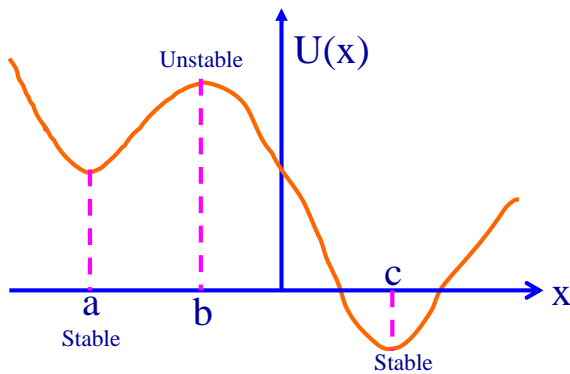
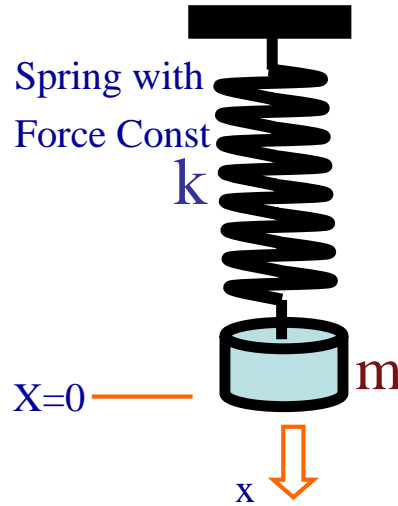
$$E_n = \frac{n^2 \pi^2 \hbar^2}{2m(L + 2\delta)^2}, n = 1, 2, 3, 4, \dots$$

When $E = U$ then solutions blow up

\Rightarrow Limits to number of bound states ($E_n < U$)

When $E > U$, particle is not bound and can get either reflected or transmitted across the potential "barrier"

Simple Harmonic Oscillator: Quantum and Classical



Particle of mass m within a potential $U(x)$

$$\vec{F}(x) = - \frac{dU(x)}{dx}$$

$$\vec{F}(x=a) = - \left. \frac{dU(x)}{dx} \right|_{x=a} = 0,$$

$$\vec{F}(x=b) = 0, \vec{F}(x=c) = 0 \dots \text{But...}$$

look at the Curvature:

$$\frac{\partial^2 U}{\partial x^2} > 0 \text{ (stable)}, \frac{\partial^2 U}{\partial x^2} < 0 \text{ (unstable)}$$

Stable Equilibrium: General Form :

$$U(x) = U(a) + \frac{1}{2} k(x-a)^2$$

$$\text{Rescale} \Rightarrow U(x) = \frac{1}{2} k(x-a)^2$$

Motion of a Classical Oscillator (ideal)

Ball originally displaced from its equilibrium position, motion confined between $x=0$ & $x=A$

$$U(x) = \frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2; \omega = \sqrt{\frac{k}{m}} = \text{Ang. Freq}$$

$$E = \frac{1}{2} kA^2 \Rightarrow \text{Changing } A \text{ changes } E$$

E can take any value & if $A \rightarrow 0$, $E \rightarrow 0$

Max. KE at $x=0$, KE=0 at $x=\pm A$

Quantum Picture: Harmonic Oscillator

Find the Ground state Wave Function $\psi(x)$

Find the Ground state Energy E when $U(x) = \frac{1}{2}m\omega^2 x^2$

Time Dependent Schrodinger Eqn:
$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + \frac{1}{2}m\omega^2 x^2 \psi(x) = E \psi(x)$$

$\Rightarrow \frac{d^2 \psi(x)}{dx^2} = \frac{2m}{\hbar^2} (E - \frac{1}{2}m\omega^2 x^2) \psi(x) = 0$ What $\psi(x)$ solves this?

Two guesses about the simplest Wavefunction:

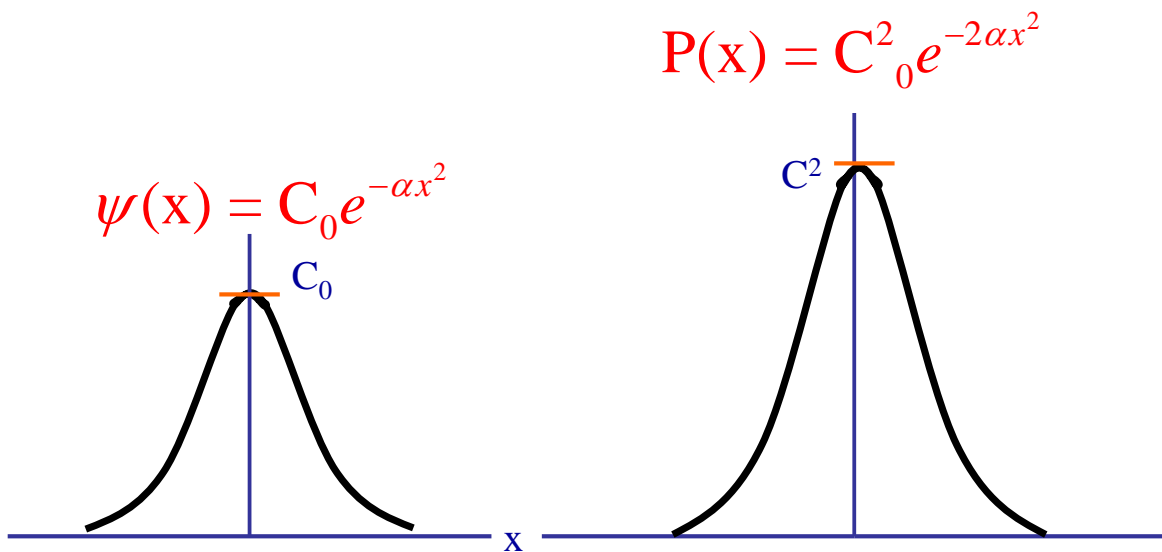
1. $\psi(x)$ should be symmetric about x 2. $\psi(x) \rightarrow 0$ as $x \rightarrow \infty$

+ $\psi(x)$ should be continuous & $\frac{d\psi(x)}{dx}$ = continuous

My guess: $\psi(x) = C_0 e^{-\alpha x^2}$; Need to find C_0 & α :

What does this wavefunction & PDF look like?

Quantum Picture: Harmonic Oscillator



How to Get C_0 & α ?? ... Try plugging in the wave-function into the time-independent Schr. Eqn.

Time Independent Sch. Eqn & The Harmonic Oscillator

Master Equation is :
$$\frac{\partial^2 \psi(x)}{\partial x^2} = \frac{2m}{\hbar^2} \left[\frac{1}{2} m \omega^2 x^2 - E \right] \psi(x)$$

Since $\psi(x) = C_0 e^{-\alpha x^2}$,
$$\frac{d\psi(x)}{dx} = C_0 (-2\alpha x) e^{-\alpha x^2},$$

$$\frac{d^2 \psi(x)}{dx^2} = C_0 \frac{d(-2\alpha x)}{dx} e^{-\alpha x^2} + C_0 (-2\alpha x)^2 e^{-\alpha x^2} = C_0 [4\alpha^2 x^2 - 2\alpha] e^{-\alpha x^2}$$

$$\Rightarrow C_0 [4\alpha^2 x^2 - 2\alpha] e^{-\alpha x^2} = \frac{2m}{\hbar^2} \left[\frac{1}{2} m \omega^2 x^2 - E \right] C_0 e^{-\alpha x^2}$$

Match the coeff of x^2 and the Constant terms on LHS & RHS

$$\Rightarrow 4\alpha^2 = \frac{2m}{\hbar^2} \frac{1}{2} m \omega^2 \quad \text{or} \quad \alpha = \frac{m\omega}{2\hbar}$$

& the other match gives $2\alpha = \frac{2m}{\hbar^2} E$, substituing $\alpha \Rightarrow$

$$E = \frac{1}{2} \hbar \omega = hf \quad \text{!!!!.....(Planck's Oscillators)}$$

What about C_0 ? We learn about that from the Normalization cond.

SHO: Normalization Condition

$$\int_{-\infty}^{+\infty} |\psi_0(x)|^2 dx = 1 = \int_{-\infty}^{+\infty} C_0^2 e^{-\frac{m\omega x^2}{\hbar}} dx$$

Since $\int_{-\infty}^{+\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$ (dont memorize this)

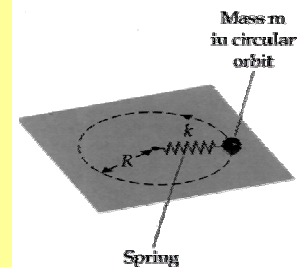
Identifying $a = \frac{m\omega}{\hbar}$ and using the identity above

$$\Rightarrow C_0 = \left[\frac{m\omega}{\pi \hbar} \right]^{\frac{1}{4}}$$

Hence the Complete NORMALIZED wave function is :

$$\psi_0(x) = \left[\frac{m\omega}{\pi \hbar} \right]^{\frac{1}{4}} e^{-\frac{m\omega x^2}{2\hbar}} \quad \text{Ground State Wavefunction}$$

has energy $E = hf$

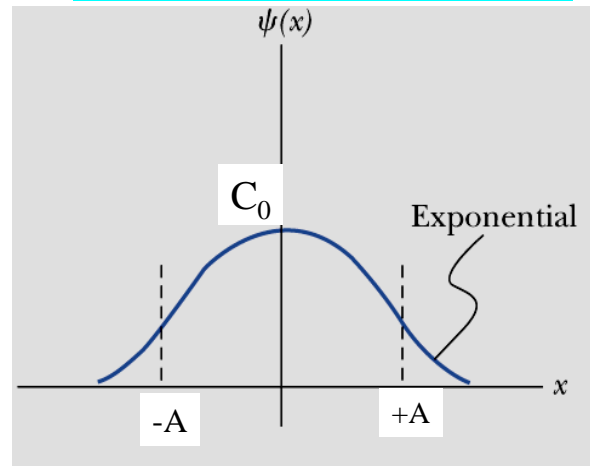
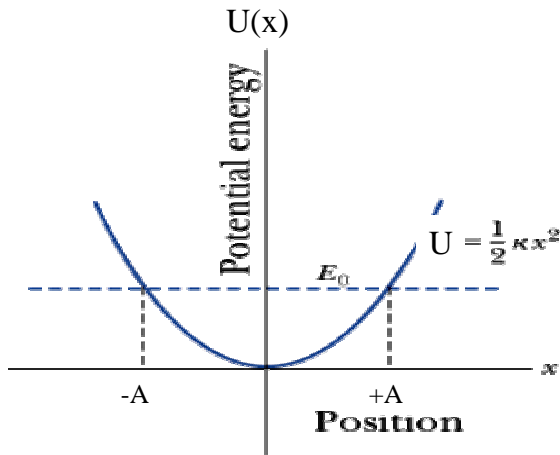


Planck's Oscillators were electrons tied by the "spring" of the mutually attractive Coulomb Force

Quantum Oscillator In Pictures

$$E = KE + U(x) > 0 \text{ for } n=0$$

Quantum Mechanical Prob for particle
To live outside classical turning points
Is finite !



Classically particle most likely to be at the turning point (velocity=0)
Quantum Mechanically , particle most likely to be at $x=x_0$ for $n=0$

Classical & Quantum Pictures of SHO compared

- Limits of classical vibration : Turning Points (do on Board)
- Quantum Probability for particle outside classical turning points $P(|x|>A) = 16\% !!$
 - Do it on the board (see Example problems in book)

Excited States of The Quantum Oscillator

$$\psi_n(x) = C_n H_n(x) e^{-\frac{m\omega x^2}{2\hbar}} ;$$

$H_n(x)$ = Hermite Polynomials

with

$$H_0(x) = 1$$

$$H_1(x) = 2x$$

$$H_2(x) = 4x^2 - 2$$

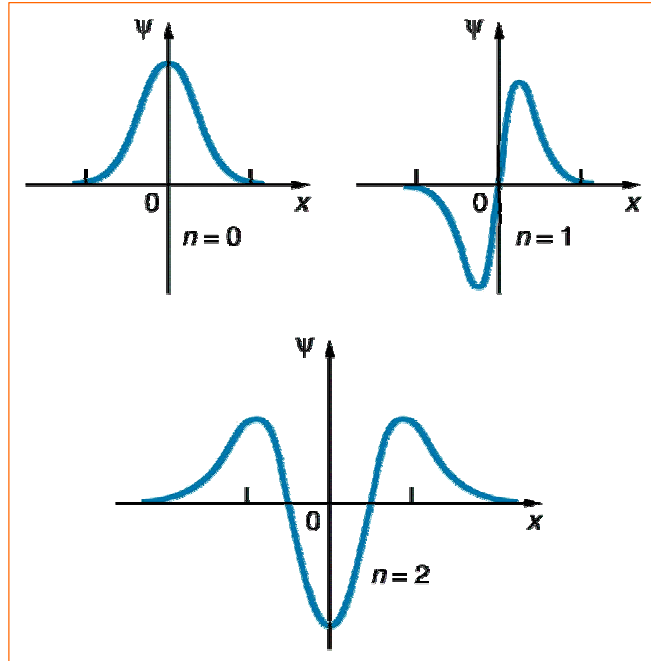
$$H_3(x) = 8x^3 - 12x$$

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n e^{-x^2}}{dx^n}$$

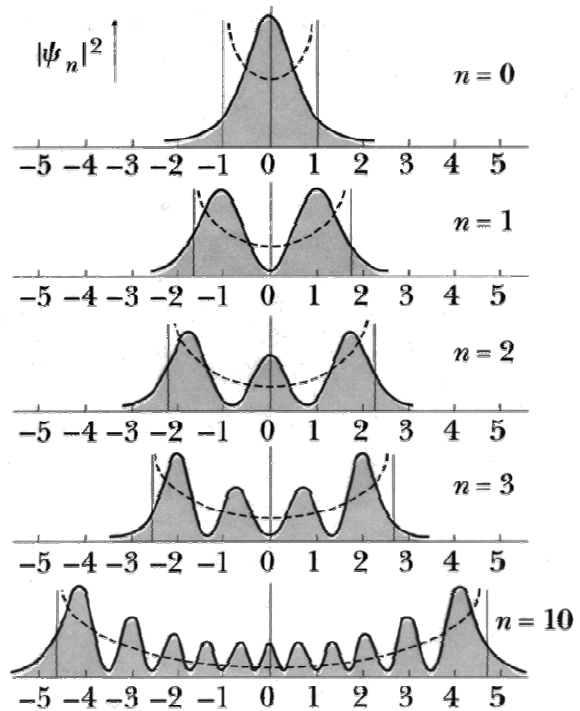
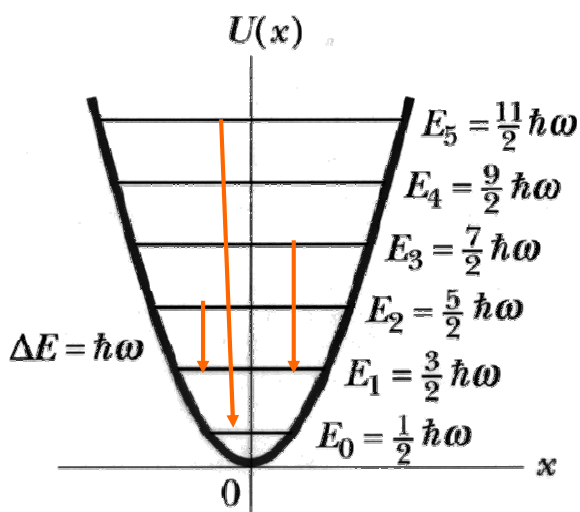
and

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega = \left(n + \frac{1}{2}\right) hf$$

Again $n=0,1,2,3,\dots,\infty$ Quantum #



Excited States of The Quantum Oscillator



Ground State Energy > 0 always

Measurement Expectation: Statistics Lesson

- Ensemble & probable outcome of a single measurement or the average outcome of a large # of measurements

$$\langle x \rangle = \frac{n_1 x_1 + n_2 x_2 + n_3 x_3 + \dots + n_i x_i}{n_1 + n_2 + n_3 + \dots + n_i} = \frac{\sum_{i=1}^n n_i x_i}{N} = \frac{\int_{-\infty}^{\infty} x P(x) dx}{\int_{-\infty}^{\infty} P(x) dx}$$

For a general Fn f(x)

$$\langle f(x) \rangle = \frac{\sum_{i=1}^n n_i f(x_i)}{N} = \frac{\int_{-\infty}^{\infty} \psi^*(x) f(x) \psi(x) dx}{\int_{-\infty}^{\infty} P(x) dx}$$

Sharpness of A Distr:

Scatter around average

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

$$\sigma = \sqrt{\langle x^2 \rangle - (\bar{x})^2}$$

$\sigma = \text{small} \rightarrow \text{Sharp distr.}$

Uncertainty $\Delta X = \sigma$

Particle in the Box, n=1, $\langle x \rangle$ & Δx ?

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L} x\right)$$

$$\begin{aligned} \langle x \rangle &= \int_{-\infty}^{\infty} \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L} x\right) x \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L} x\right) dx \\ &= \frac{2}{L} \int_0^L x \sin^2\left(\frac{\pi}{L} x\right) dx, \text{ change variable } \theta = \left(\frac{\pi}{L} x\right) \end{aligned}$$

$$\Rightarrow \langle x \rangle = \frac{2}{L\pi^2} \int_0^{\pi} \theta \sin^2 \theta, \text{ use } \sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

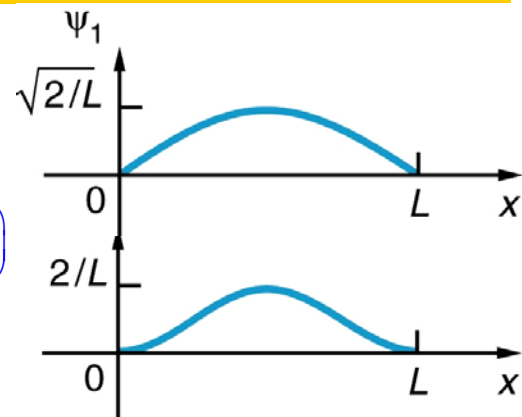
$$\Rightarrow \langle x \rangle = \frac{2L}{2\pi^2} \left[\int_0^{\pi} \theta d\theta - \int_0^{\pi} \theta \cos 2\theta d\theta \right] \text{ use } \int u dv = uv - \int v du$$

$$\Rightarrow \langle x \rangle = \frac{L}{\pi^2} \left(\frac{\pi^2}{2} \right) = \frac{L}{2} \text{ (same result as from graphing } \psi^2(x) \text{)}$$

$$\text{Similarly } \langle x^2 \rangle = \int_0^L x^2 \sin^2\left(\frac{\pi}{L} x\right) dx = \frac{L^2}{3} - \frac{L^2}{2\pi^2}$$

$$\text{and } \Delta X = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{L^2}{3} - \frac{L^2}{2\pi^2} - \frac{L^2}{4}} = 0.18L$$

$\Delta X = 20\%$ of L, Particle not sharply confined in Box



Expectation Values & Operators: More Formally

- **Observable:** Any particle property that can be measured
 - X, P, KE, E or some combination of them, e.g: x^2
 - How to calculate the probable value of these quantities for a QM state ?
- **Operator:** Associates an **operator** with each observable
 - Using these Operators, one calculates the average value of that Observable
 - The Operator acts on the Wavefunction (Operand) & extracts info about the Observable in a straightforward way → gets Expectation value for that observable

$$\langle Q \rangle = \int_{-\infty}^{+\infty} \Psi^*(x,t) [\hat{Q}] \Psi(x,t) dx$$

Q is the observable, $[\hat{Q}]$ is the operator

& $\langle Q \rangle$ is the Expectation value

Examples: $[X] = x$, $[P] = \frac{\hbar}{i} \frac{d}{dx}$

$$[K] = \frac{[P]^2}{2m} = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$$

$$[E] = i\hbar \frac{\partial}{\partial t}$$

Table 5.2 Common Observables and Associated Operators

Observable	Symbol	Associated Operator
position	x	x
momentum	p	$\frac{\hbar}{i} \frac{\partial}{\partial x}$
potential energy	U	$U(x)$
kinetic energy	K	$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$
hamiltonian	H	$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U(x)$
total energy	E	$i\hbar \frac{\partial}{\partial t}$

Operators → Information Extractors

Plug & play form

$[p]$ or $\hat{p} = \frac{\hbar}{i} \frac{d}{dx}$ Momentum Operator

gives the value of average momentum in the following way:

$$\langle p \rangle = \int_{-\infty}^{+\infty} \psi^*(x) [p] \psi(x) dx = \int_{-\infty}^{+\infty} \psi^*(x) \left(\frac{\hbar}{i} \right) \frac{d\psi}{dx} dx$$

Similarly :

$[K]$ or $\hat{K} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2}$ gives the value of average KE

$$\langle K \rangle = \int_{-\infty}^{+\infty} \psi^*(x) [K] \psi(x) dx = \int_{-\infty}^{+\infty} \psi^*(x) \left(-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} \right) dx$$

Similarly

$\langle U \rangle = \int_{-\infty}^{+\infty} \psi^*(x) [U(x)] \psi(x) dx$: plug in the $U(x)$ fn for that case

$$\text{and } \langle E \rangle = \int_{-\infty}^{+\infty} \psi^*(x) [K + U(x)] \psi(x) dx = \int_{-\infty}^{+\infty} \psi^*(x) \left(-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + U(x) \right) dx$$

Hamiltonian Operator $[H] = [K] + [U]$

The Energy Operator $[E] = i\hbar \frac{\partial}{\partial t}$ informs you of the average energy

[H] & [E] Operators

- $[H]$ is a function of x
- $[E]$ is a function of t they are really different operators
- But they produce identical results when applied to any solution of the time-dependent Schrodinger Eq.

$$\bullet [H]\Psi(x,t) = [E] \Psi(x,t)$$

$$\left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U(x,t) \right] \Psi(x,t) = \left[i\hbar \frac{\partial}{\partial t} \right] \Psi(x,t)$$

- Think of S. Eq as an expression for Energy conservation for a Quantum system

Where do Operators come from ? A touchy-feely answer

Example : [p] The momentum Extractor (operator):

Consider as an example: Free Particle Wavefunction

$$\Psi(x,t) = Ae^{i(kx-wt)} \quad ; \quad k = \frac{2\pi}{\lambda}, \lambda = \frac{h}{p} \Rightarrow k = \frac{p}{\hbar}$$

rewrite $\Psi(x,t) = Ae^{i(\frac{p}{\hbar}x-wt)}$; $\frac{\partial \Psi(x,t)}{\partial x} = i\frac{p}{\hbar} Ae^{i(\frac{p}{\hbar}x-wt)} = i\frac{p}{\hbar} \Psi(x,t)$

$$\Rightarrow \left[\frac{\hbar}{i} \frac{\partial}{\partial x} \right] \Psi(x,t) = p \Psi(x,t)$$

So it is not unreasonable to associate $[p] = \left[\frac{\hbar}{i} \frac{\partial}{\partial x} \right]$ with observable p

Example : Average Momentum of particle in box

- Given the symmetry of the 1D box, we argued last time that $\langle p \rangle = 0$: now some inglorious math to prove it !

- Be lazy, when you can get away with a symmetry argument to solve a problem..do it & avoid the evil integration & algebra....but be sure!

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right) \quad \& \quad \psi_n^*(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$$

$$\langle p \rangle = \int_{-\infty}^{+\infty} \psi^* [p] \psi dx = \int_{-\infty}^{\infty} \psi^* \left[\frac{\hbar}{i} \frac{d}{dx} \right] \psi dx$$

$$\langle p \rangle = \frac{\hbar}{i} \frac{2}{L} \frac{n\pi}{L} \int_{-\infty}^{\infty} \sin\left(\frac{n\pi}{L}x\right) \cos\left(\frac{n\pi}{L}x\right) dx$$

Since $\int \sin ax \cos ax dx = \frac{1}{2a} \sin^2 ax$...here $a = \frac{n\pi}{L}$

$$\Rightarrow \langle p \rangle = \frac{\hbar}{iL} \left[\sin^2\left(\frac{n\pi}{L}x\right) \right]_{x=0}^{x=L} = 0 \text{ since } \sin^2(0) = \sin^2(n\pi) = 0$$

We knew THAT before doing any math !

Quiz 1: What is the $\langle p \rangle$ for the Quantum Oscillator in its symmetric ground state

Quiz 2: What is the $\langle p \rangle$ for the Quantum Oscillator in its asymmetric first excited state

But what about the $\langle KE \rangle$ of the Particle in Box ?

$\langle p \rangle = 0$ so what about expectation value of $K = \frac{p^2}{2m}$?

$\langle K \rangle = 0$ because $\langle p \rangle = 0$; clearly not, since we showed $E = KE \neq 0$

Why ? What gives ?

Because $p_n = \pm \sqrt{2mE_n} = \pm \frac{n\pi\hbar}{L}$; "±" is the key!

AVERAGE $p = 0$, particle is moving back & forth

$\langle KE \rangle = \langle \frac{p^2}{2m} \rangle \neq 0$ not $\frac{\langle p^2 \rangle}{2m}$!

Be careful when being "lazy"

Quiz: what about $\langle KE \rangle$ of a quantum Oscillator?

Does similar logic apply??

Measurement Expectation: Statistics Lesson

- Ensemble & probable outcome of a single measurement or the average outcome of a large # of measurements

$$\langle x \rangle = \frac{n_1 x_1 + n_2 x_2 + n_3 x_3 + \dots + n_i x_i}{n_1 + n_2 + n_3 + \dots + n_i} = \frac{\sum_{i=1}^n n_i x_i}{N} = \frac{\int_{-\infty}^{\infty} x P(x) dx}{\int_{-\infty}^{\infty} P(x) dx}$$

For a general Fn f(x)

$$\langle f(x) \rangle = \frac{\sum_{i=1}^n n_i f(x_i)}{N} = \frac{\int_{-\infty}^{\infty} \psi^*(x) f(x) \psi(x) dx}{\int_{-\infty}^{\infty} P(x) dx}$$

Sharpness of A Distr:

Scatter around average

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

$$\sigma = \sqrt{\langle x^2 \rangle - (\bar{x})^2}$$

$\sigma = \text{small} \rightarrow \text{Sharp distr.}$

Uncertainty $\Delta X = \sigma$

Particle in the Box, $n=1$, $\langle x \rangle$ & Δx ?

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}x\right)$$

$$\langle x \rangle = \int_{-\infty}^{\infty} \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}x\right) x \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}x\right) dx$$

$$= \frac{2}{L} \int_0^L x \sin^2\left(\frac{\pi}{L}x\right) dx \quad , \text{ change variable } \theta = \left(\frac{\pi}{L}x\right)$$

$$\Rightarrow \langle x \rangle = \frac{2}{L\pi^2} \int_0^\pi \theta \sin^2\theta \quad , \text{ use } \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

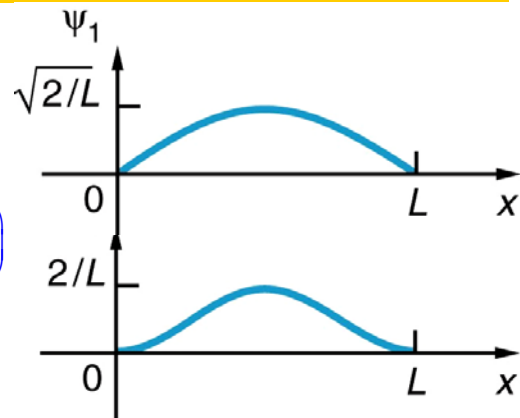
$$\Rightarrow \langle x \rangle = \frac{2L}{2\pi^2} \left[\int_0^\pi \theta d\theta - \int_0^\pi \theta \cos 2\theta d\theta \right] \quad \text{use } \int u dv = uv - \int v du$$

$$\Rightarrow \langle x \rangle = \frac{L}{\pi^2} \left(\frac{\pi^2}{2} \right) = \frac{L}{2} \quad (\text{same result as from graphing } \psi^2(x))$$

$$\text{Similarly } \langle x^2 \rangle = \int_0^L x^2 \sin^2\left(\frac{\pi}{L}x\right) dx = \frac{L^2}{3} - \frac{L^2}{2\pi^2}$$

$$\text{and } \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{L^2}{3} - \frac{L^2}{2\pi^2} - \frac{L^2}{4}} = 0.18L$$

$\Delta x = 20\%$ of L , Particle not sharply confined in Box



Schrodinger Eqn: Stationary State Form

- Recall \rightarrow when potential does not depend on time explicitly $U(x,t) = U(x)$ only... we used separation of x, t variables to simplify $\Psi(x,t) = \psi(x) \phi(t)$ & broke S. Eq. into two: one with x only and another with t only

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x) = E \psi(x)$$

$$i\hbar \frac{\partial \phi(t)}{\partial t} = E\phi(t)$$

$$\Psi(x,t) = \psi(x)\phi(t)$$

How to put **Humpty-Dumpty** back together ? e.g to say how to go from an expression of $\psi(x) \rightarrow \Psi(x,t)$ which describes time-evolution of the overall wave function

Schrodinger Eqn: Stationary State Form

$$\text{Since } \frac{d}{dt} [\ln f(t)] = \frac{1}{f(t)} \frac{df(t)}{dt}$$

$$\text{In } i\hbar \frac{\partial \phi(t)}{\partial t} = E\phi(t), \text{ rewrite as } \frac{1}{\phi(t)} \frac{\partial \phi(t)}{\partial t} = \frac{E}{i\hbar} = -\frac{iE}{\hbar}$$

and integrate both sides w.r.t. time

$$\int_{t=0}^{t=t} \frac{1}{\phi(t)} \frac{\partial \phi(t)}{\partial t} dt = \int_0^t -\frac{iE}{\hbar} dt \Rightarrow \int_0^t \frac{1}{\phi(t)} \frac{d\phi(t)}{dt} dt = -\frac{iE}{\hbar}$$

$$\therefore \ln \phi(t) - \ln \phi(0) = -\frac{iE}{\hbar} t, \text{ now exponentiate both sides}$$

$$\Rightarrow \phi(t) = \phi(0) e^{-\frac{iE}{\hbar} t}; \phi(0) = \text{constant} = \text{initial condition} = 1 \text{ (e.g)}$$

$$\Rightarrow \phi(t) = e^{-\frac{iE}{\hbar} t} \text{ \& Thus } \Psi(x,t) = \psi(x) e^{-\frac{iE}{\hbar} t} \text{ where } E = \text{energy of system}$$

Schrodinger Eqn: Stationary State Form

$$P(x,t) = \Psi^* \Psi = \psi^*(x) e^{+\frac{iE}{\hbar} t} \psi(x) e^{-\frac{iE}{\hbar} t} = \psi^*(x) \psi(x) e^{\frac{iE}{\hbar} t - \frac{iE}{\hbar} t} = |\psi(x)|^2$$

In such cases, P(x,t) is INDEPENDENT of time.

These are called "stationary" states because Prob is independent of time

Examples : Particle in a box (why?)

: Quantum Oscillator (why?)

Total energy of the system depends on the spatial orientation of the system : characteristic of the potential situation !