

The Final Exam is on [Mar 18th](#) , Time and Location TBA

NOT on Monday Mar 15th as previously announced in
the Handout etc!!

Pl. make a note of this change !!

This date change is also posted in the ANNOUCEMENT section of class web page



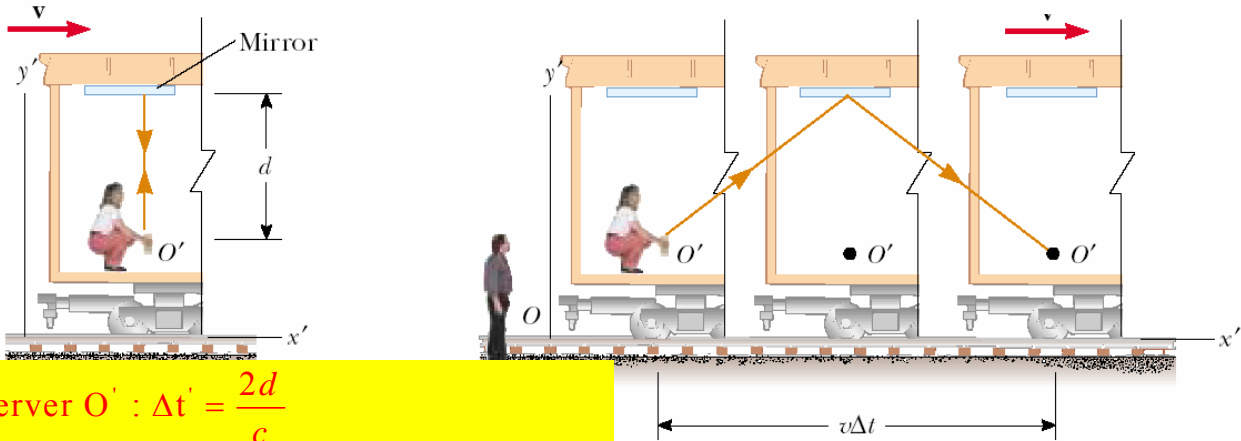
Physics 2D Lecture Slides

Lecture 4: Jan 9th 2004

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UCSD Physics

Time Dilation and Proper Time

Watching a time interval (between 2 events) with a simple clock



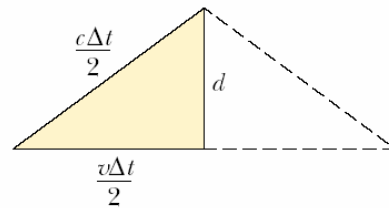
Observer O' : $\Delta t' = \frac{2d}{c}$

Observer O : Apply Pythagoras Theorem

$$\left(\frac{c\Delta t}{2}\right)^2 = (d)^2 + \left(\frac{v\Delta t}{2}\right)^2, \text{ but } d = \left(\frac{c\Delta t'}{2}\right)$$

$$\therefore c^2 (\Delta t)^2 = c^2 (\Delta t')^2 + v^2 (\Delta t)^2$$

$$\therefore \Delta t = \frac{\Delta t'}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma \Delta t', \Delta t > \Delta t'$$

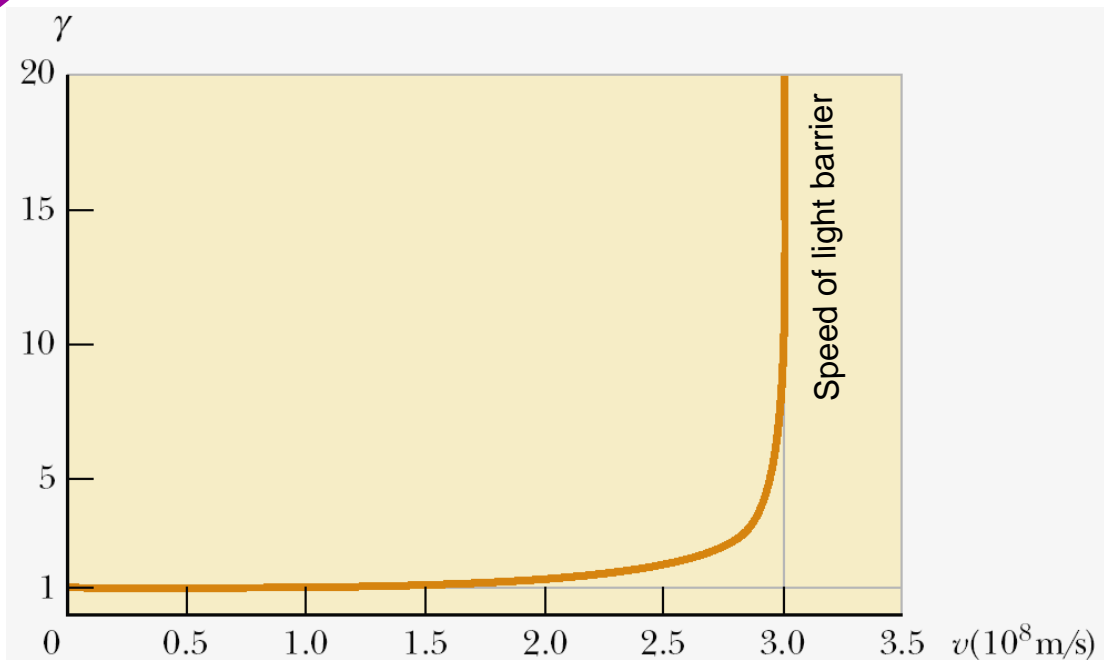


The γ factor

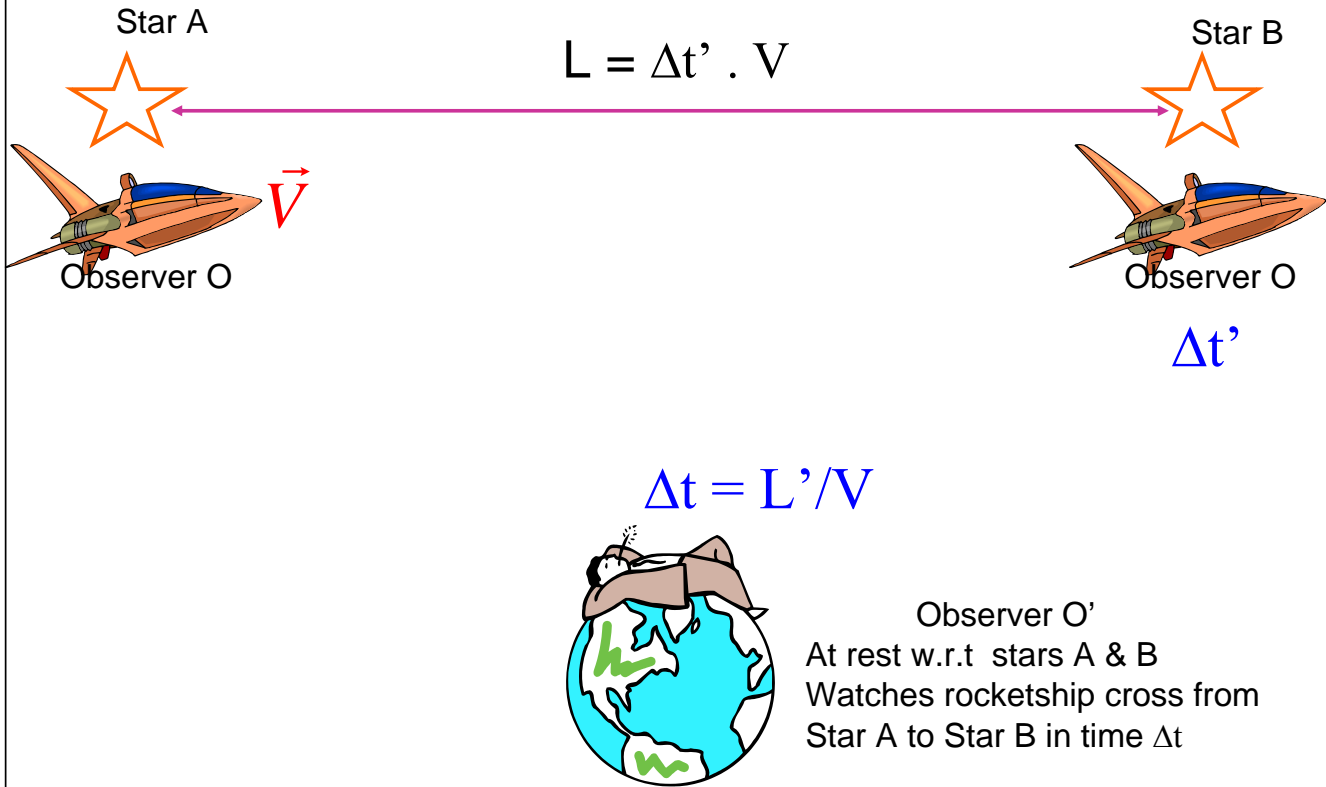
$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$

as $v \rightarrow 0, \gamma \rightarrow 1$

as $v \rightarrow c, \gamma \rightarrow \infty$



Offsetting Penalty : Length Contraction



Rocketman Vs The Earthling

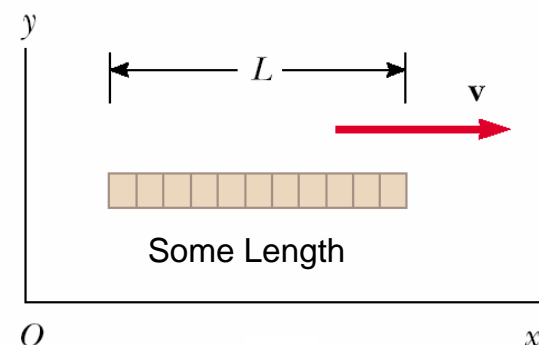
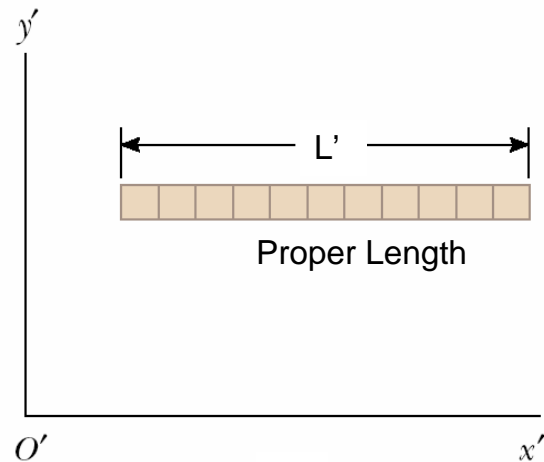
- Earth Observer saw rocketman take time $\Delta t = (L'/V)$
- Rocketman says he is at rest, Star B moving towards him with speed V from right passed him by in time $\Delta t'$, so

- $L = \Delta t' \cdot V$
- But $\Delta t' = \Delta t / \gamma$ (time dilation)
- $\Rightarrow L = V \cdot (\Delta t / \gamma)$
 $= L' / \gamma$

$$L = L' \cdot \sqrt{1 - \frac{V^2}{c^2}}$$

$$L \leq L'$$

Moving Rods Contract in direction
Of relative motion



Immediate Consequences of Einstein's Postulates: Recap

- Events that are simultaneous for one Observer are **not simultaneous** for another Observer in relative motion
- **Time Dilation** : Clocks in motion relative to an Observer appear to slow down by factor γ
- **Length Contraction** : Lengths of Objects in motion appear to be contracted in the direction of motion by factor γ^{-1}
- **New Definitions** :
 - Proper Time (who measures this ?)
 - Proper Length (who measures this ?)
 - Different clocks for different folks !
- ...But Which observer is "Right"? Every one if they are always in an inertial frame of reference (constant velocity)

Contrived Paradoxes of Relativity

A paradox is an apparently self-contradictory statement, the underlying meaning of which is revealed only by careful scrutiny. The purpose of a paradox is to arrest attention and provoke fresh thought

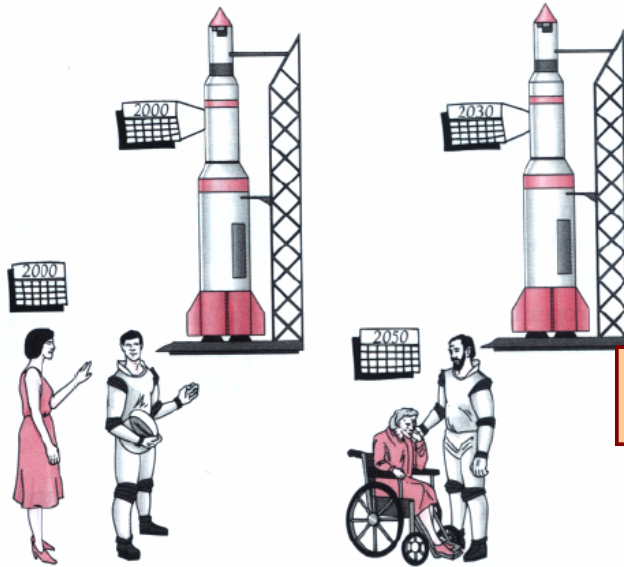
``A paradox is not a conflict within reality. It is a conflict between reality and your feeling of what reality should be like." - Richard Feynman

Construct a few paradoxes in Relativity & analyze them

Jack and Jill's Excellent Adventure: Twin Paradox

Jack & Jill are 20 yr old twins, with same heartbeat
 Jack takes off with $V = 0.8c = (4/5)c$ to a star
 20 light years Away.

Jill stays behind, watches Jack by telescope. They
 Eventually compare notes



Jill sees Jack's heart slow down
 Compared to her by the factor :

$$\sqrt{1 - (v/c)^2}$$

$$= \sqrt{1 - (0.8c/c)^2} = 0.6$$

For every 5 beats of her heart
 She sees Jack's beat only 3 !

Jack has only 3 thoughts for 5 that
 Jill has !Everything slows!

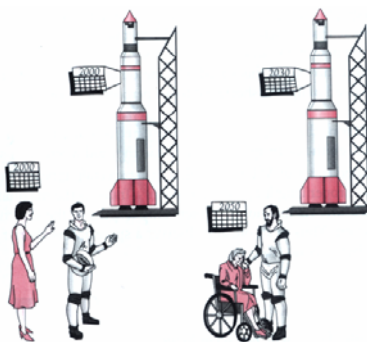
Finally Jack returns after 50 yrs
 gone by according to Jill's calendar

Only 30 years have gone by Jack's calendar
 SO Jack is 50 years old but Jane is 70 !

Where is the paradox ??

Twin Paradox ?

- Paradox : Turn argument around, motion is relative. Look at Jack's point of view !
- Jack claims he at rest, Jill is moving $v=0.8c$
- Should not Jill be 50 years old when 70 year old Jack returns from space Odyssey?



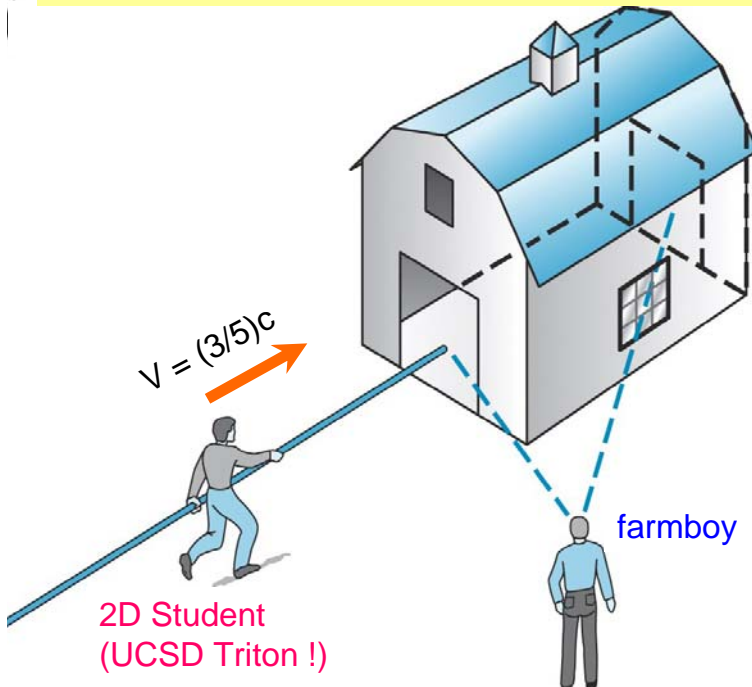
No ! ...because Jack is not always
 traveling in a inertial frame of reference
**TO GET BACK TO EARTH HE HAS TO
 TURN AROUND =>**
decelerate/accelerate
 But Jill always remained in Inertial frame
 Time dilation formula valid for Jill's
 observation of Jack but not to Jack's
observation of Jill !!....remember this always

Non-symmetric aging verified with atomic clocks taken on airplane trip around world and compared with identical clock left behind. Observer who departs from an inertial system will always find its clock slow compared with clocks that stayed in the system

Fitting a 5m pole in a 4m Barnhouse ?

Student attends 2D lecture (but does no HW) ...banished to a farm in Iowa !
 Meets a farmboy who is watching 2D lecture videos online. He does not do HW either!

There is a Barn with 2 doors 4m apart ; There is a pole with proper length = 5m
 Farm boy goads the student to run fast and fit the 5m pole within 4m barn
 The student tells the farmboy: "Dude you are nuts!" ...who is right and why ?

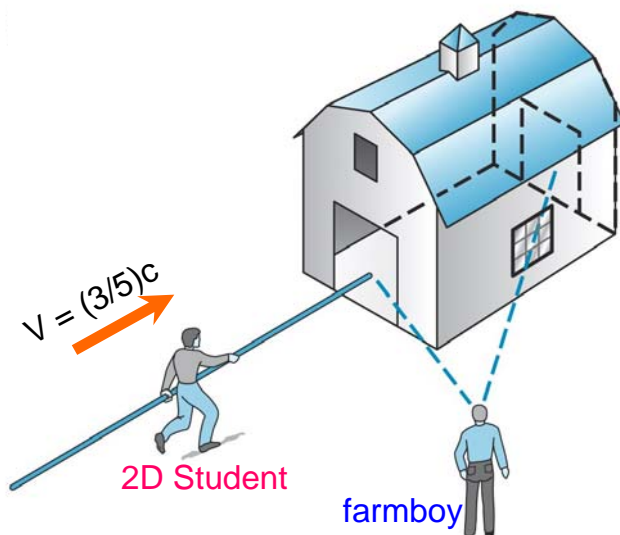


Sequence of Events

- A: Arrival of right end of pole at left end of barn
- B: Arrival of left end of pole at left end of barn
- C: Arrival of right end of pole at right end of barn

Think Simultaneity !

Fitting a 5m pole in a 4m Barnhouse ?!!



Farmboy says "You can do it"
 Student says "Dude, you are nuts"

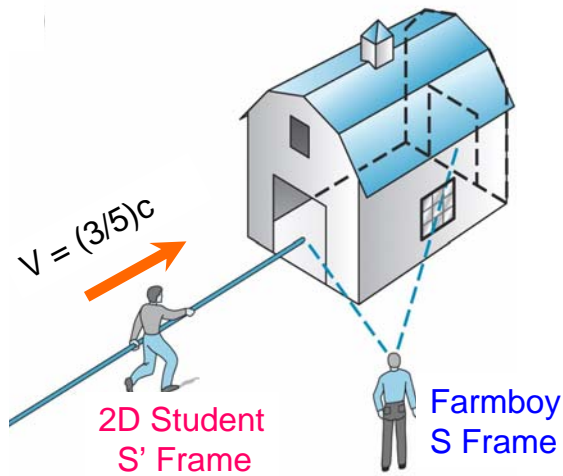
Student with pole runs with $v=(3/5)c$
 farmboy sees pole contraction factor
 $\sqrt{1-(3c/5c)^2} = 4/5$
 says pole just fits in the barn fully!

Student with pole runs with $v=(3/5)c$
 Student sees barn contraction factor
 $\sqrt{1-(3c/5c)^2} = 4/5$
 says barn is only 3.2m long, too short
 to contain entire 5m pole !

Is there a contradiction ? Is Relativity wrong?

Homework: You figure out who is right, if any and why.
 Hint: Think in terms of observing three events

Fitting a 5m pole in a 4m Barnhouse?



Let $S = \text{Barn frame}$, $S' = \text{student frame}$

Event A : arrival of right end of pole at left end of barn: $(t=0, t'=0)$ is reference

$L'_0 = \text{proper length of pole in } S'$

$l_0 = \text{length of barn in } S \text{ frame} < L'_0$

In S : length of pole $L = L'_0 \sqrt{1 - (v/c)^2}$

The times in two frames are related:

$$t'_B = \frac{l'}{v} = \frac{l_0}{v} \sqrt{1 - (v/c)^2} = t_{BC} \sqrt{1 - (v/c)^2}$$

$$t'_C = \frac{L'_0}{v} = \frac{l'}{v} \frac{1}{1 - (v/c)^2} = \frac{t_{BC}}{\sqrt{1 - (v/c)^2}}$$

\Rightarrow Time gap in S' by which events B and C fail to be simultaneous

Simultaneity Required !

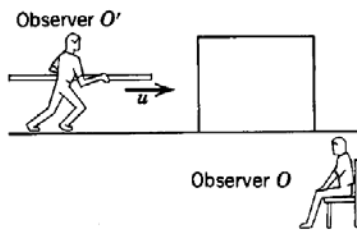
Events

B: Arrival of left end of pole at left end of barn

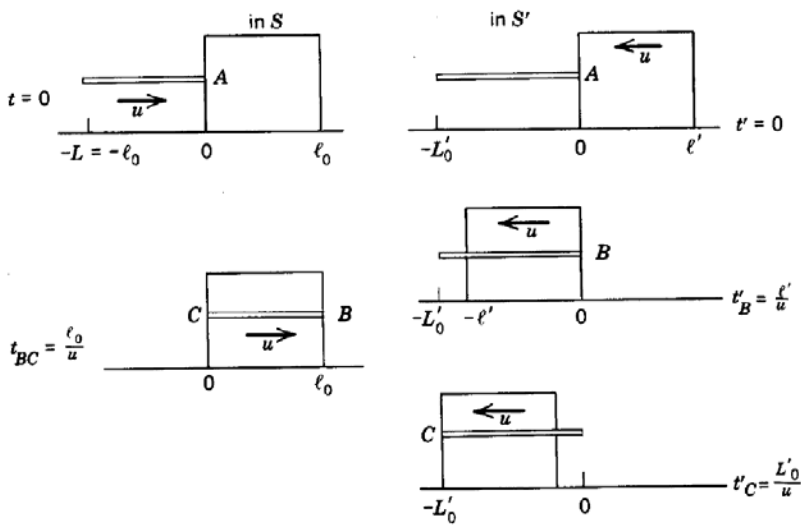
C: Arrival of right end of pole at right end of barn

Farmboy sees two events as simultaneous
2D student can not agree
Fitting of the pole in barn is relative !

Farmboy Vs 2D Student

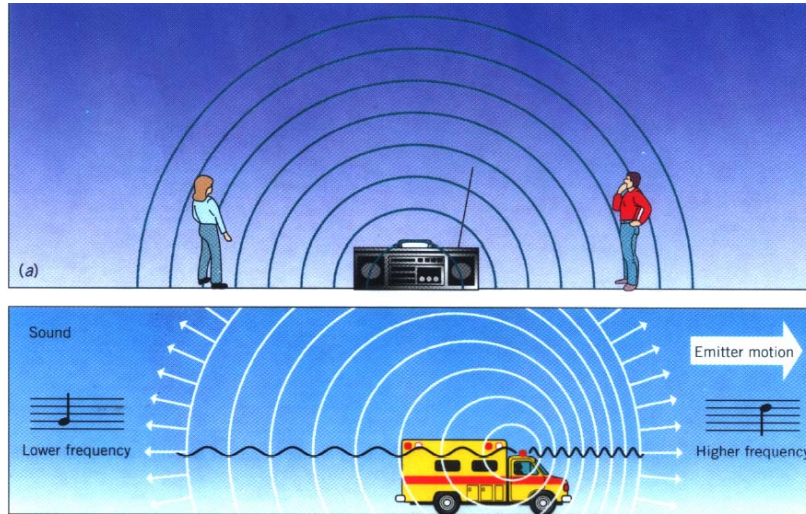


Pole and barn are in relative motion u such that Lorentz contracted length of pole = Proper length of barn



In rest frame of pole, Event B precedes C

Doppler Effect In Sound : Reminder from 2A

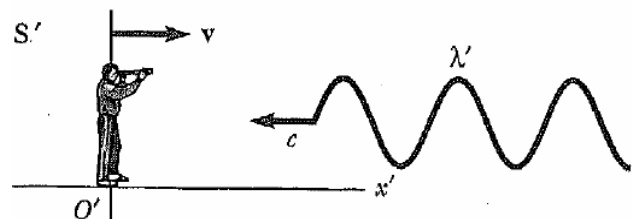
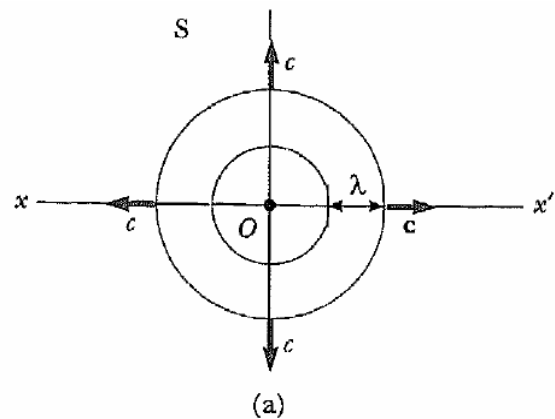


Observed **Frequency** of sound **INCREASES** if emitter moves towards the Observer
 Observed **Wavelength** of sound **DECREASES** if emitter moves towards the Observer

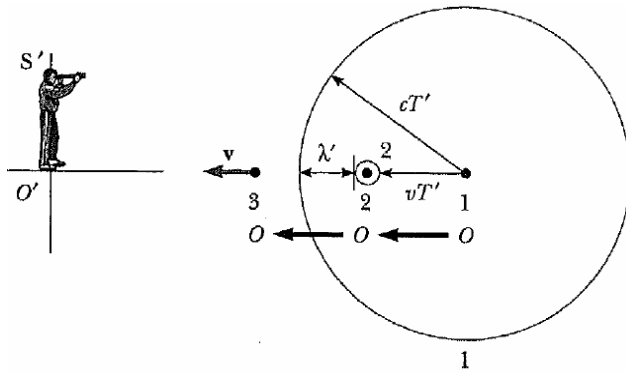
$$v = f \lambda$$

Time Dilation Example: Relativistic Doppler Shift

- Light : velocity $c = f \lambda$, $f=1/T$
- A source of light S at rest
- Observer S' approaches S with velocity v
- S' measures f' or λ' , $c = f' \lambda'$
- Expect $f' > f$ since more wave crests are being crossed by Observer S' due to its approach direction than if it were at rest w.r.t source S



Relativistic Doppler Shift



Examine two successive wavefronts emitted by S at location 1 and 2

In S' frame, T' = time between two wavefronts

In time T' , the Source moves by cT' w.r.t 1

Meanwhile Light Source moves a distance vT'

Distance between successive wavefront

$$\lambda' = cT' - vT'$$

$$\lambda' = cT' - vT', \text{ use } f = c / \lambda$$

$$f' = \frac{c}{(c-v)T'}, \quad T' = \frac{T}{\sqrt{1 - (v/c)^2}}$$

Substituting for T' , use $f = 1/T$

$$\Rightarrow f' = \frac{\sqrt{1 - (v/c)^2}}{1 - (v/c)} f$$

$$\Rightarrow f' = \frac{\sqrt{1 + (v/c)}}{\sqrt{1 - (v/c)}} f$$

better remembered as:

$$f_{\text{obs}} = \frac{\sqrt{1 + (v/c)}}{\sqrt{1 - (v/c)}} f_{\text{source}}$$

f_{obs} = Freq measured by
observer approaching
light source

Relativistic Doppler Shift

$$f_{\text{obs}} = \frac{\sqrt{1 + (v/c)}}{\sqrt{1 - (v/c)}} f_{\text{source}}$$

