

Discussion Session Wednesday 3 pm, WLH 2005

Problem Solving Session Tomorrow only:
5pm @ WLH 2001 (Not Peterson)

& Starting Thursday 15th: 5pm , Peterson 108

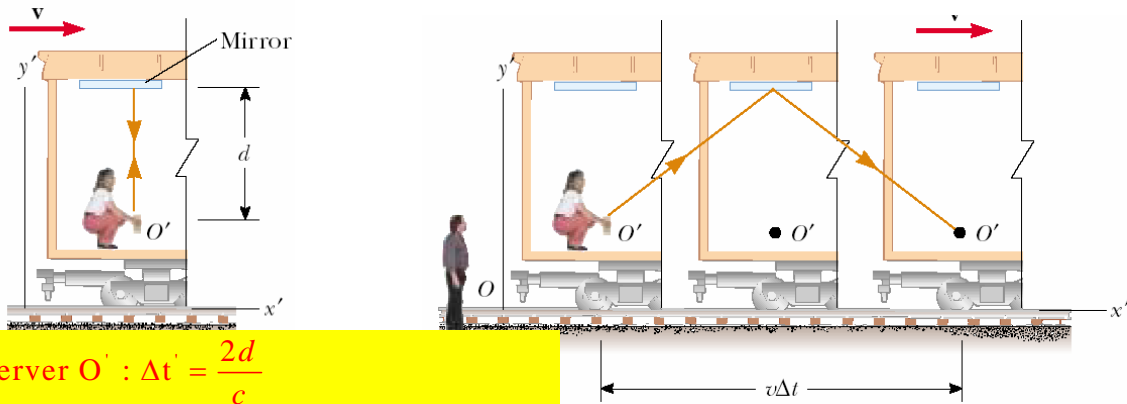


Physics 2D Lecture Slides
Lecture 3: Jan 7 2004

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UCSD Physics

Time Dilation and Proper Time

Watching a time interval (between 2 events) with a simple clock



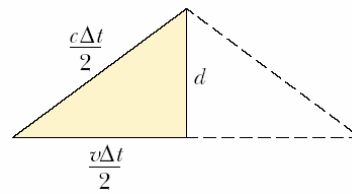
Observer O' : $\Delta t' = \frac{2d}{c}$

Observer O : Apply Pythagoras Theorem

$$\left(\frac{c\Delta t}{2}\right)^2 = (d)^2 + \left(\frac{v\Delta t}{2}\right)^2, \text{ but } d = \left(\frac{c\Delta t'}{2}\right)$$

$$\therefore c^2(\Delta t)^2 = c^2(\Delta t')^2 + v^2(\Delta t)^2$$

$$\therefore \Delta t = \frac{\Delta t'}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma \Delta t', \Delta t > \Delta t'$$

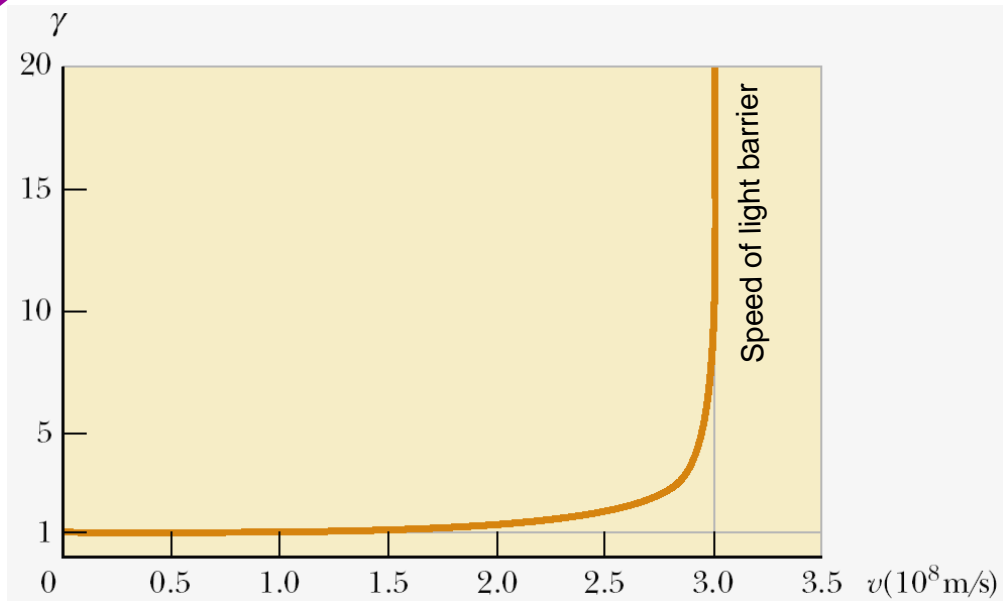


The γ factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

as $v \rightarrow 0$, $\gamma \rightarrow 1$

as $v \rightarrow c$, $\gamma \rightarrow \infty$



Pop Quiz !



- What happens when I reverse the clocks being watched ?
 - Sally now watches Sam's clock
 - Sally is moving w.r.t. Sam's clock. Sam is at rest w.r.t the clock.
 - What does she make of time intervals as measured by his clock ?

Measuring Time: Period of a Pendulum

- Period of a pendulum is 3.0 s in the **rest frame** of the pendulum
- What is period of the pendulum as seen by an **observer moving at $v=0.95c$**

Answer:



- Proper time $T' = 3.0s$
- Since motion is relative and time dilation does not distinguish between
 - relative motion $\rightarrow\rightarrow (V)$ from relative motion $\leftarrow\leftarrow (-V)$
- lets reformulate the problem like this (??)
 - A pendulum in a rocket is flying with velocity $V = 0.95c$ past a stationary observer
 - Moving clocks runs slower [w.r.t clock in observer's hand (rest)] by factor γ
 - \rightarrow Period T measured by observer = $\gamma T'$

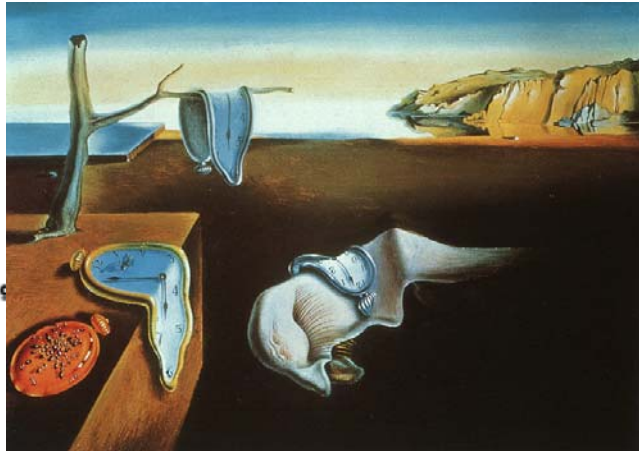
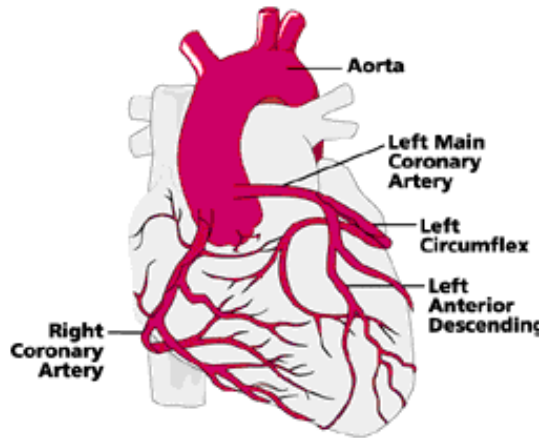
$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.95)^2}} = 3.2$$

$$\Rightarrow T = \gamma T' = 3.2 \times 3.0s = 9.6s$$

Moving pendulum slows down \rightarrow takes longer to complete a period

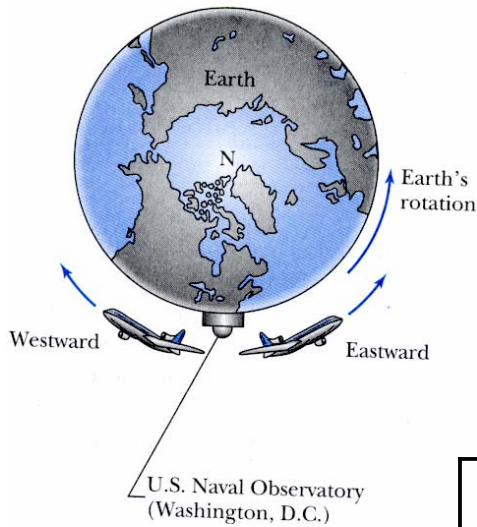
All Measures of Time Slow down from a Moving Observer's Perspective !

- Your heartbeat or your pulse



- Mitosis and Biological growth
- Growth of an inorganic crystal
- ‘...Watching the river flow’
- ...all measures of time interval

Round The World With An Atomic Clock !

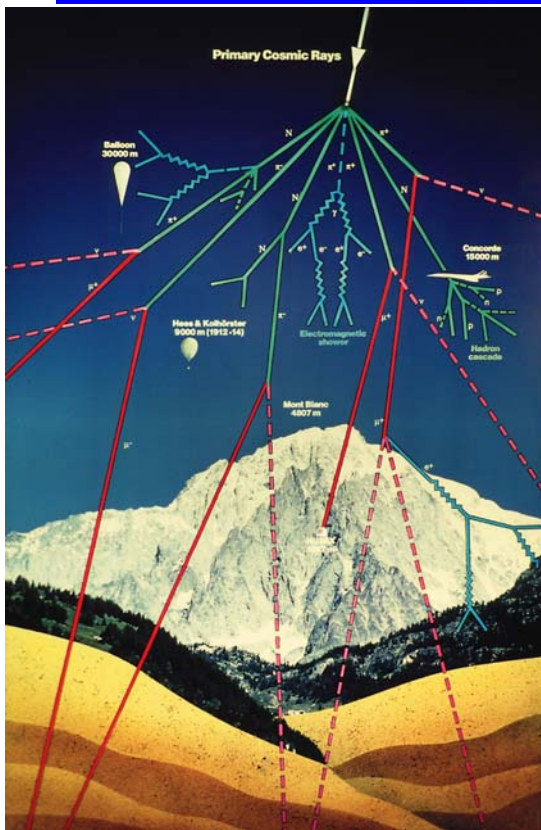


- Atomic Clock : measure time interval for certain atomic level transitions in Cesium atom
- Two planes take off from DC, travel east and west with the atomic clock
 - Eastward trip took 41.2 hrs
 - Westward trip took 48.6
- Atomic clocks compared to similar ones kept in DC
- Need to account for Earth's rotation + GR etc

Travel	Predicted	Measured
Eastward	-40 ± 23 ns	-59 ± 10 ns
Westward	275 ± 21 ns	273 ± 7 ns

Flying clock ticked faster or slower than reference clock. Slow or fast is due to Earth's rotation

Cosmic Rain !



- Cosmic “rays” are messengers from space
- Produced in violent collisions in the cosmos
- Typical Kinetic energy ~ 100 GeV
- Smash into Earth’s outer atmosphere
 - 4700 m from sea level
- Sometimes produce short lived **Muons (μ)**

Muon is electron like charged particle

- ~ 200 times heavier , same charge
- Lifetime $\tau = 2.2\mu\text{s} = 2.2 \times 10^{-6} \text{ s}$
- Produced with speed $v \approx c$
- Distance traveled in its lifetime

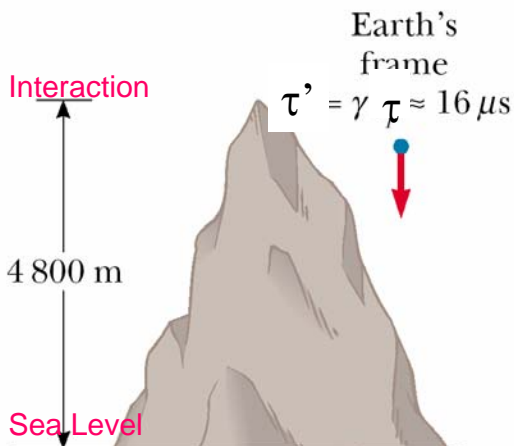
$$d = c\tau = 650\text{m}$$

- Yet they seem to reach the surface!!
 - Why => **Time Dilation**
 - Must pay attention to **frames of references** involved

Cosmic Rays Are Falling On Earth : Example of Time Dilation



(a)



- Consider Two frames of references
 1. You Riding on the Muon Particle
 2. Your twin watching On surface of earth

– Muon Rider has “Proper Time”

- Time measured by observer moving along with clock
 - $\Delta t' = \tau = 2.2 \mu\text{s}$
 - $D' = v \Delta t' = 650\text{m}$

– Earthling watches a moving clock (muon’s) run slower

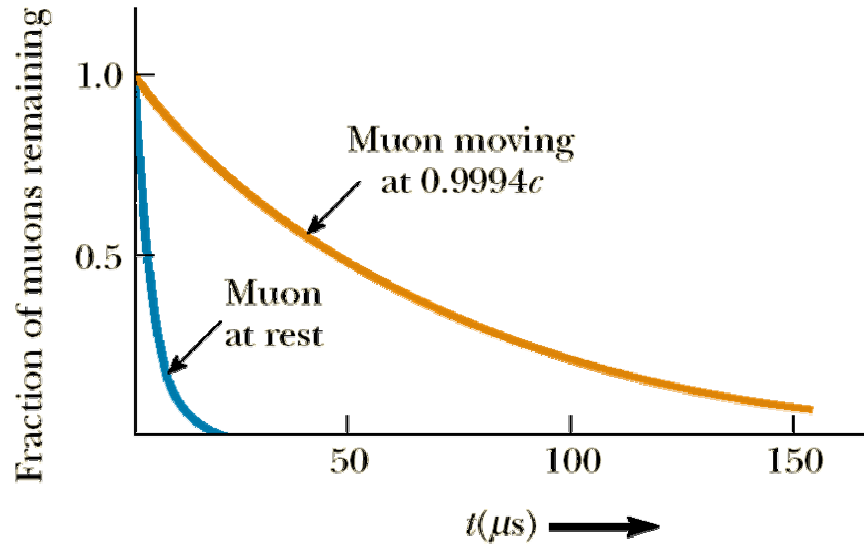
- $\Delta t = \gamma \tau$
 - $v = 0.99c, \Rightarrow \gamma = 7.1$
 - $D = v \Delta t = 4700\text{m}$

Muon Decay Distance Distribution

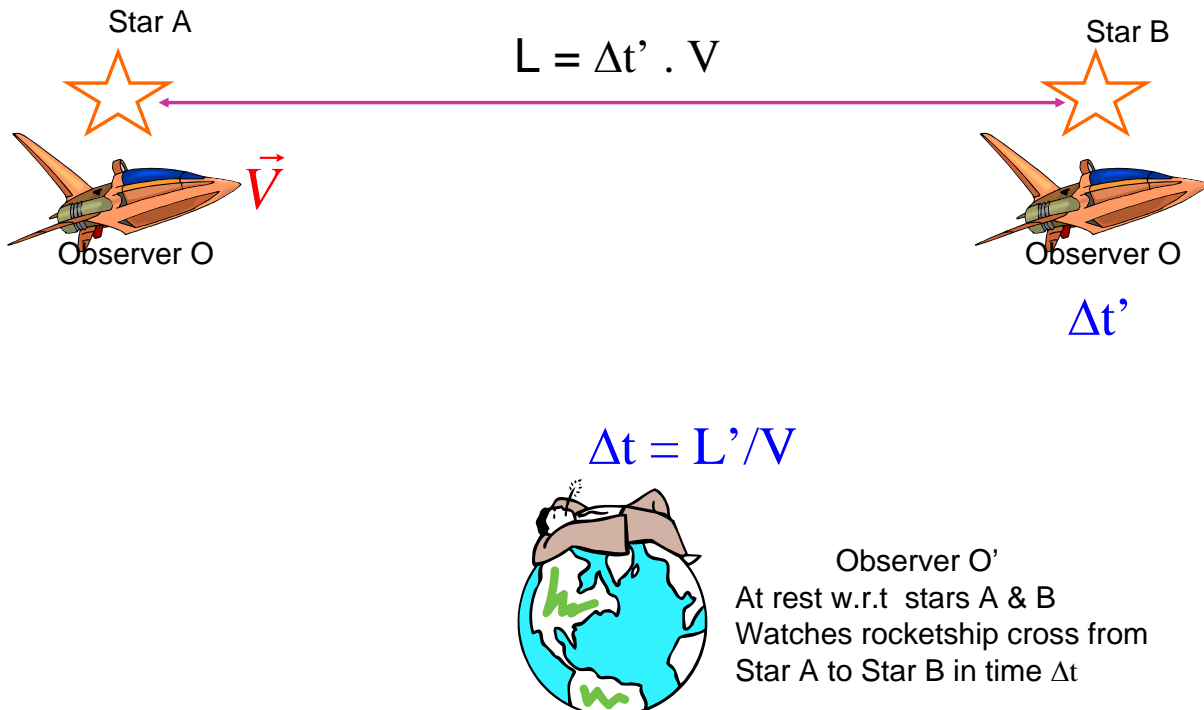
Relative to Observer on Earth Muons have a lifetime

$$t = \gamma\tau = 7.1 \tau$$

Exponential Decay time Distribution : As in Radioactivity



Offsetting Penalty : Length Contraction



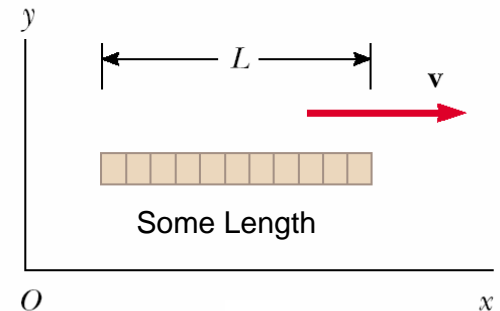
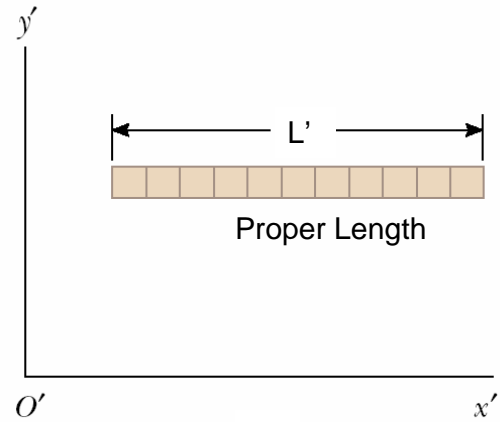
Rocketman Vs The Earthling

- Earth Observer saw rocketman take time $\Delta t = (L' / V)$
- Rocketman says he is at rest, Star B moving towards him with speed V from right passed him by in time $\Delta t'$, so
 - $L = \Delta t' \cdot V$
 - But $\Delta t' = \Delta t / \gamma$ (time dilation)
 - $\Rightarrow L = V \cdot (\Delta t / \gamma)$
 $= L' / \gamma$

$$L = L' \cdot \sqrt{1 - \frac{V^2}{c^2}}$$

$$L \leq L'$$

Moving Rods Contract in direction
Of relative motion



Immediate Consequences of Einstein's Postulates: Recap

- Events that are simultaneous for one Observer are **not simultaneous** for another Observer in relative motion
- **Time Dilation** : Clocks in motion relative to an Observer appear to slow down by factor γ
- **Length Contraction** : Lengths of Objects in motion appear to be contracted in the direction of motion by factor γ^{-1}
- **New Definitions** :
 - Proper Time (who measures this ?)
 - Proper Length (who measures this ?)
 - Different clocks for different folks !