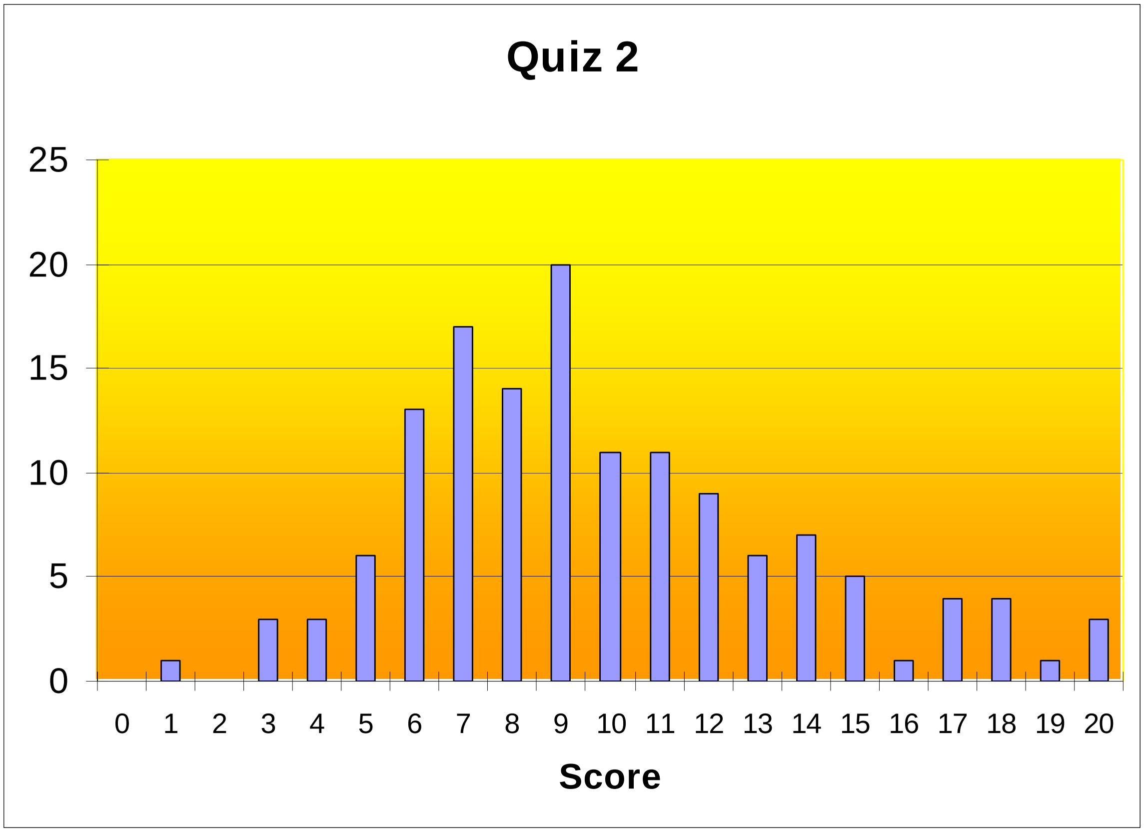
Brian Wecht, the TA, is away this week. I will substitute for his office hours (in my office 3314 Mayer Hall, discussion and PS session.

Pl. give all regrade requests to me this week

Quiz 3 is This Friday

Physics 2D Lecture Slides
Lecture 10: Jan 26^{th} 2004

Vivek Sharma
UCSD Physics



Ch 2 : Quantum Theory Of Light

- What is the nature of light ?
  - When it propagates ?
  - When it interacts with Matter?
- What is Nature of Matter ?
  - When it interacts with light ?
  - As it propagates ?
- Revolution in Scientific Thought
  - Like a firestorm of new ideas (every body goes nuts!..not like Evolution)
    - Old concepts violently demolished , new ideas born
      - Interplay of experimental findings & scientific reason
- One such revolution happened at the turn of 20th Century
  - Led to the birth of Quantum Theory & Modern Physics

Classical Picture of Light : Maxwell's Equations

- Maxwell's Equations:

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$\oint_S \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

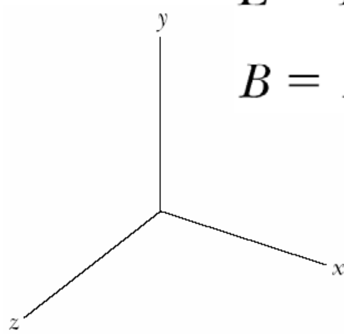
$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I + \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2}$$

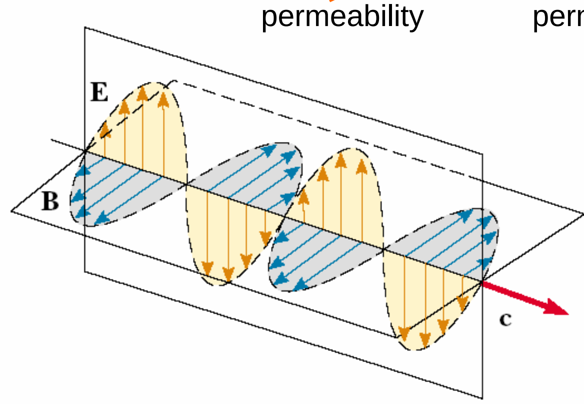
$$E = E_{\max}\cos(kx - \omega t)$$

$$B = B_{\max}\cos(kx - \omega t)$$

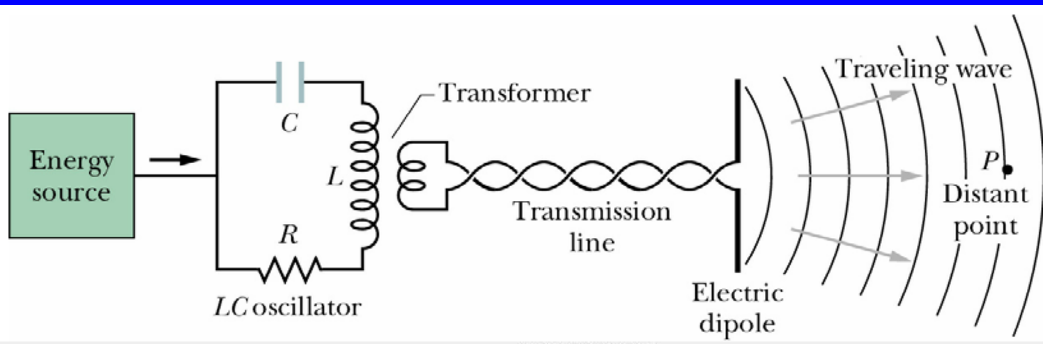


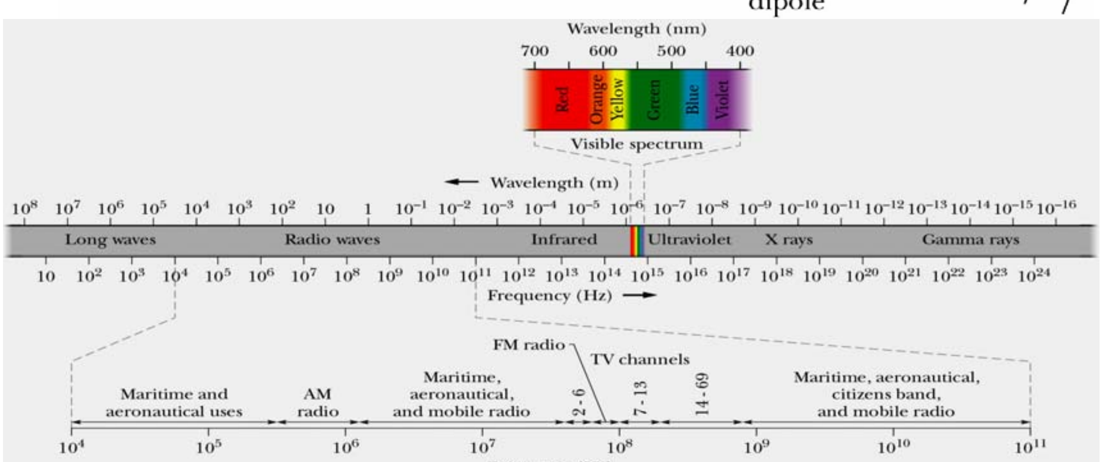
$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

permeability permittivity

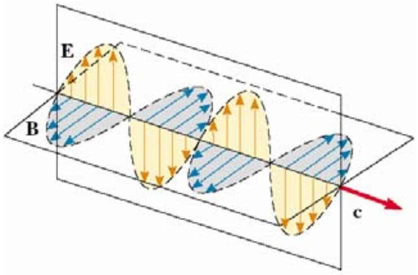


Hertz & Experimental Demo of Light as EM Wave





Properties of EM Waves: Maxwell's Equations



Energy Flow in EM Waves :

Poynting Vector $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$

Power incident on an area A $= \vec{S}.\vec{A} = \frac{1}{\mu_0}\left(AE_0 B_0 Sin^2(kx - \omega t)\right)$

Intensity of Radiation $I = \frac{1}{2\mu_0 c} E_0^2$

Larger the amplitude of Oscillation
More intense is the radiation

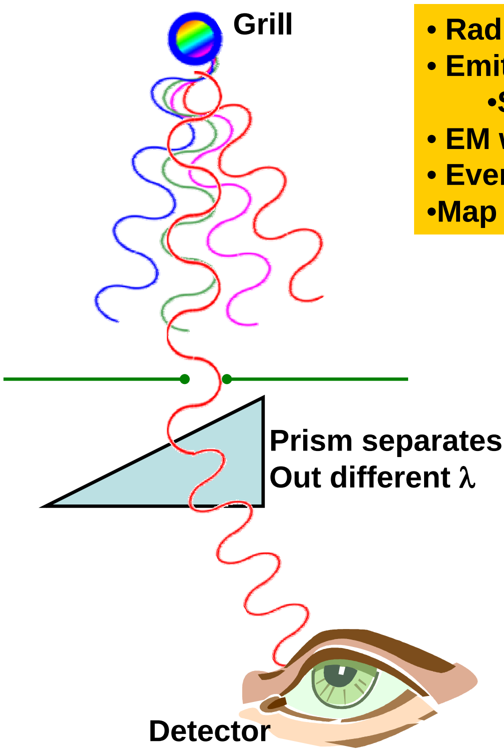
Disasters in Classical Physics (1899-1922)

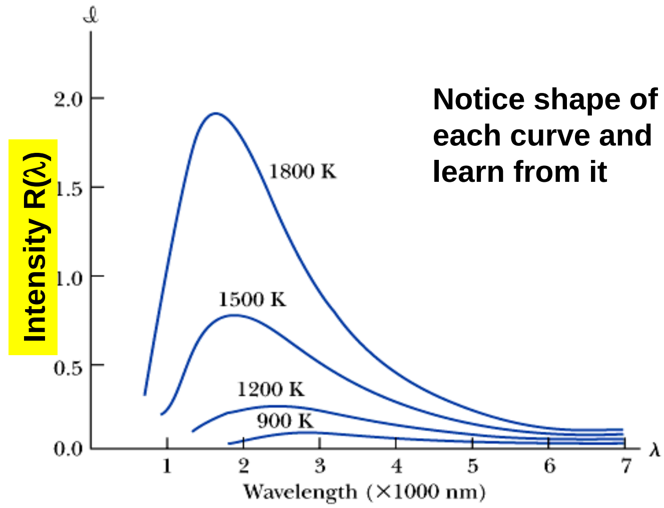
- Disaster → Experimental observation that could not be explained by Classical theory (Phys 2A, 2B, 2C)
  - Disaster # 1 : Nature of Blackbody Radiation from your BBQ grill
  - Disaster # 2: Photo Electric Effect
  - Disaster # 3: Scattering light off electrons (Compton Effect)
- Resolution of Experimental Observation will require radical changes in how we think about nature
  - → QUANTUM MECHANICS
    - The Art of Conversation with Subatomic Particles

Nature of Radiation: An Expt with BBQ Grill

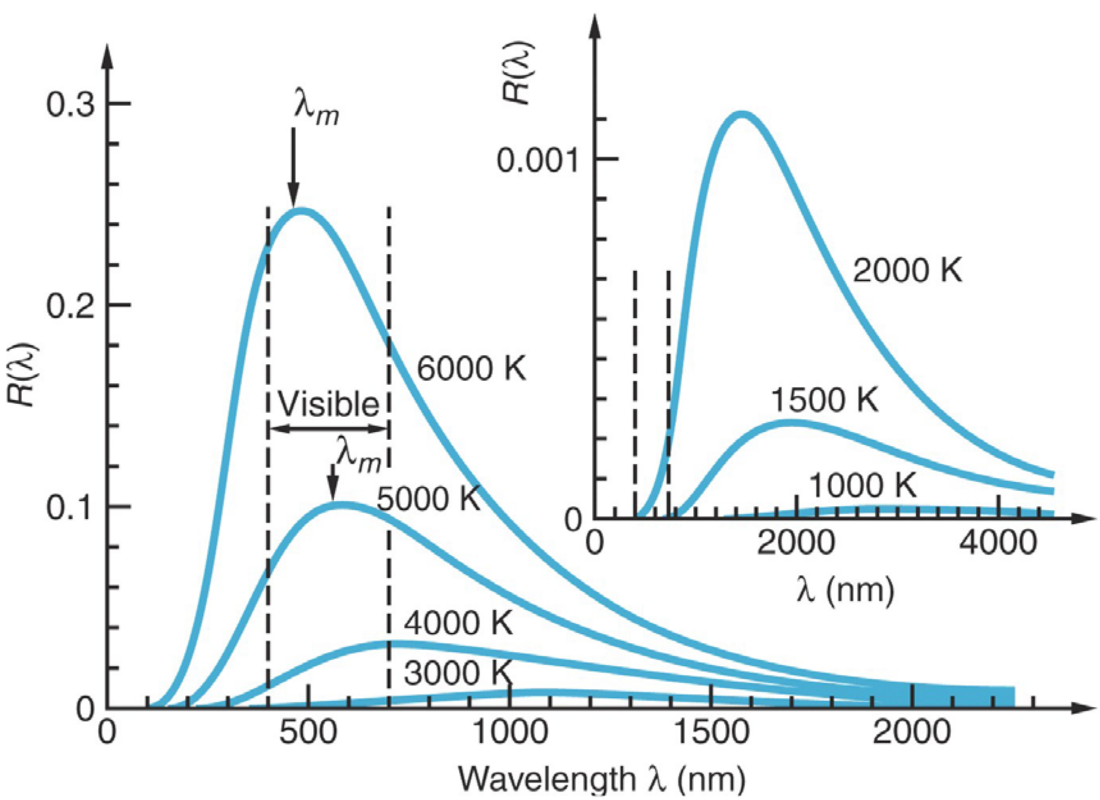
Question : Distribution of Intensity of EM radiation Vs T & λ

- Radiator (grill) at some temp T
- Emits variety of wavelengths
  - Some with more intensity than others
- EM waves of diff. λ bend differently within prism
- Eventually recorded by a detector (eye)
- Map out emitted Power / area Vs λ





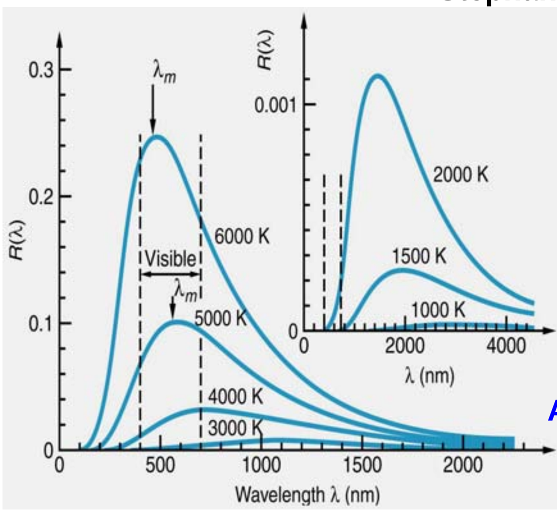
Radiation from A Blackbody



(a) Intensity of Radiation $\mathbf{I} = \int R(\lambda)d\lambda \propto T^4$

$$I = \sigma T^4 (\text{Area under curve})$$

Stephan-Boltzmann Constant $\sigma = 5.67\ 10^{-8}\ \mathrm{W / m^2 K^4}$



(b) Higher the temperature of BBQ
Lower is the λ of PEAK intensity

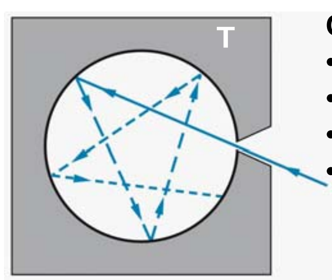
$$\lambda_{MAX} \propto 1 / T$$

Wein's Law $\lambda_{MAX}\ T = \text{const} = 2.898\ 10^{-3}\ \text{mK}$

As a body gets hotter it gets more RED then White

Reason for different shape of R(λ) Vs λ for different temperature?
Can one explain in on basis of Classical Physics (2A,2B,2C) ??

Blackbody Radiator: An Idealization



Classical Analysis:

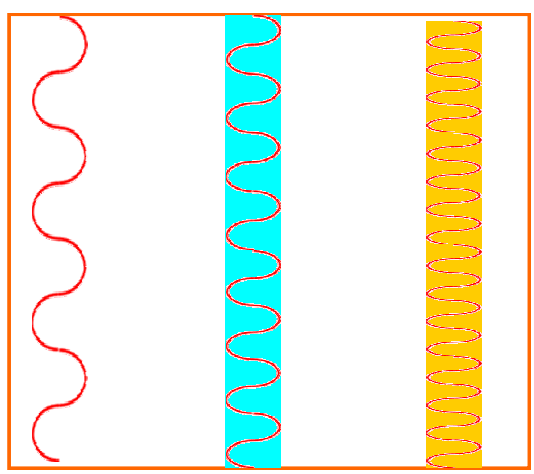
- **Box is filled with EM standing waves**
- **Radiation reflected back-and-forth between walls**
- **Radiation in thermal equilibrium with walls of Box**
- **How may waves of wavelength λ can fit inside the box ?**

Blackbody Absorbs everything
Reflects nothing
All light entering opening gets absorbed (ultimately) by the cavity wall

Cavity in equilibrium T w.r.t. surrounding. So it radiates everything It absorbs

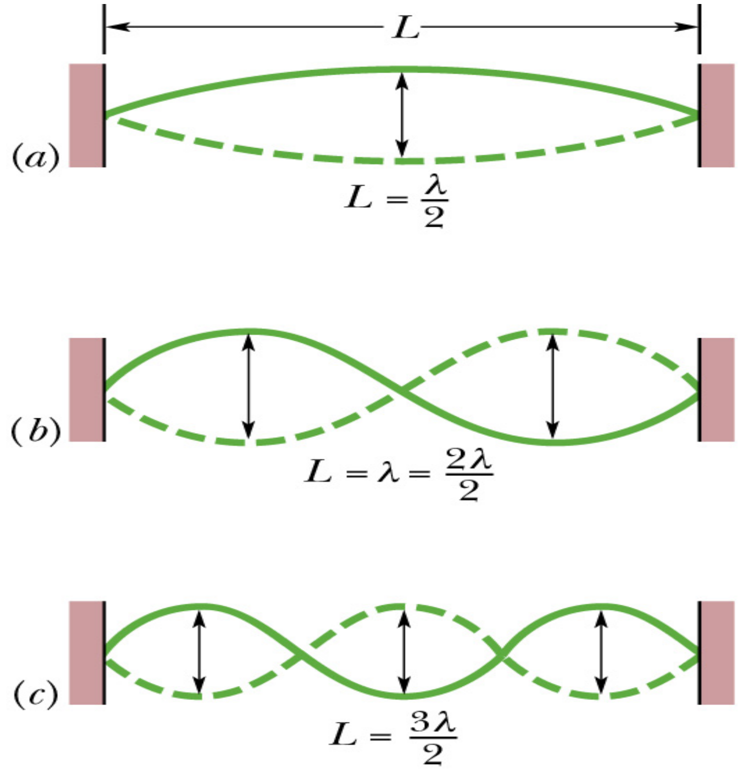
Emerging radiation is a sample of radiation inside box at temp T

Predict nature of radiation inside Box ?



less **more** **Even more**

Standing Waves



The Beginning of The End ! How BBQ Broke Physics

Classical Calculation

# of standing waves between Wavelengths λ and λ+dλ are

$$N(\lambda)d\lambda = \frac{8\pi V}{\lambda^4} \bullet d\lambda \; ; \; V = \text{Volume of box} = L^3$$

Each standing wave contributes energy E= kT to radiation in Box

Energy density $u(\lambda)$= [# of standing waves/volume]× Energy/Standing Wave

$$= \frac{8\pi V}{\lambda^4} \times \frac{1}{V} \times kT = \frac{8\pi}{\lambda^4} kT$$

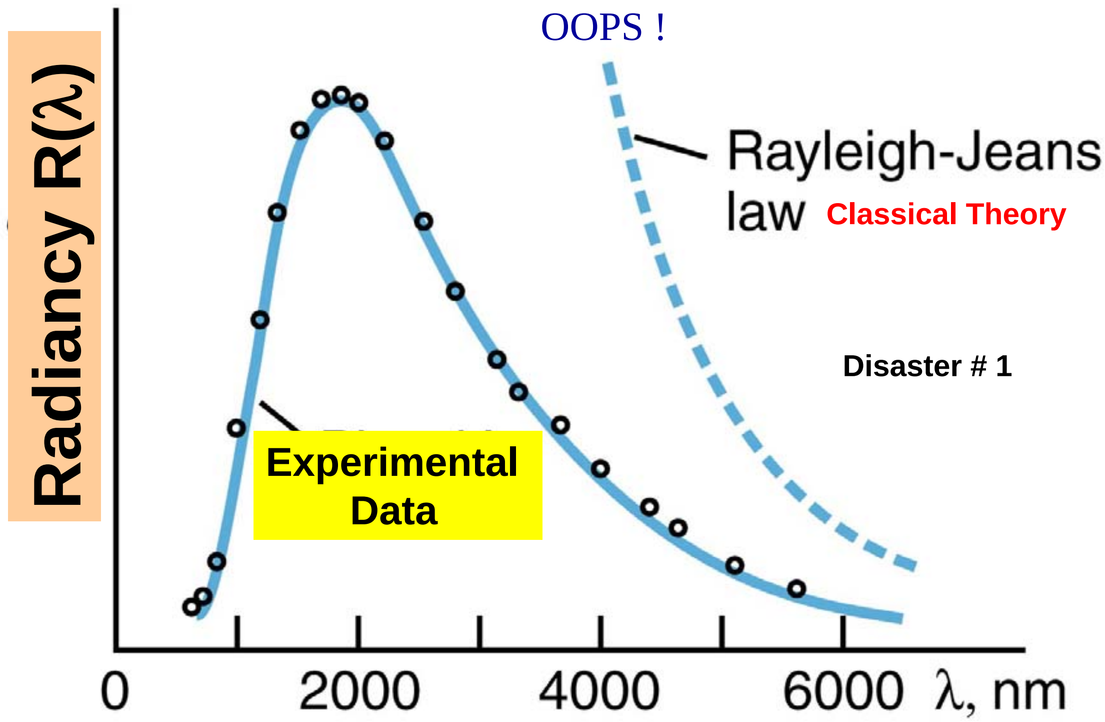
$$\text{Radiancy } R(\lambda) = \frac{c}{4} u(\lambda) = \frac{c}{4}\frac{8\pi}{\lambda^4} kT = \frac{2\pi c}{\lambda^4} kT$$

Radiancy is Radiation intensity per unit λ interval: Lets plot it

Prediction : as $\lambda \rightarrow 0$ (high frequency) $\Rightarrow$ R(λ) → Infinity !

Oops !

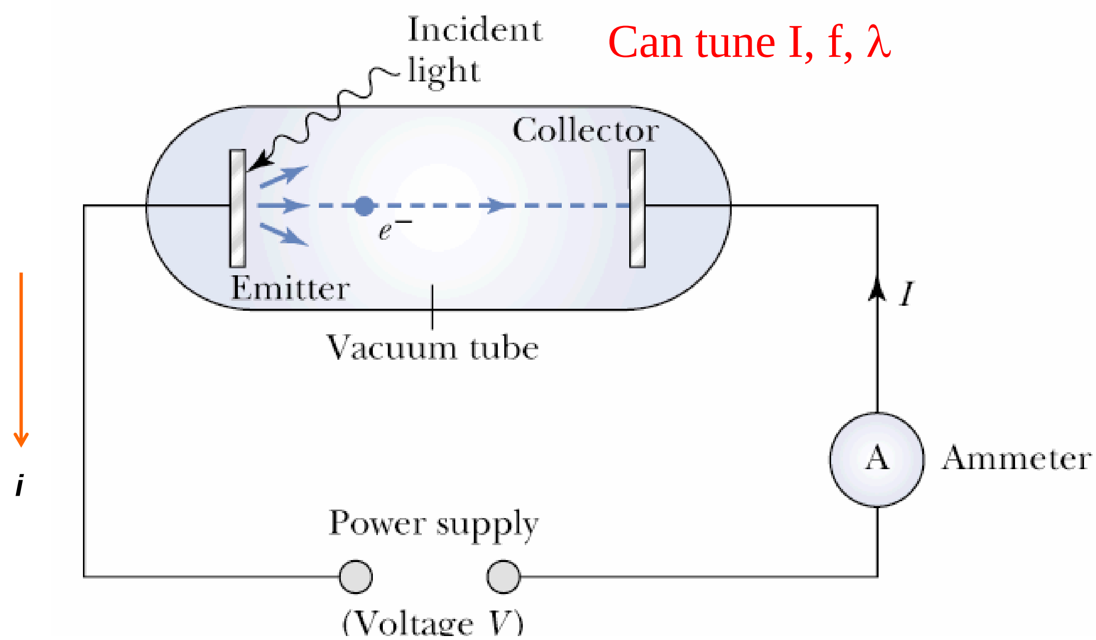
Ultra Violet (Frequency) Catastrophe



That was a Disaster ! (#1)

Disaster # 2 : Photo-Electric Effect

Light of intensity I, wavelength λ and frequency ν incident on a photo-cathode

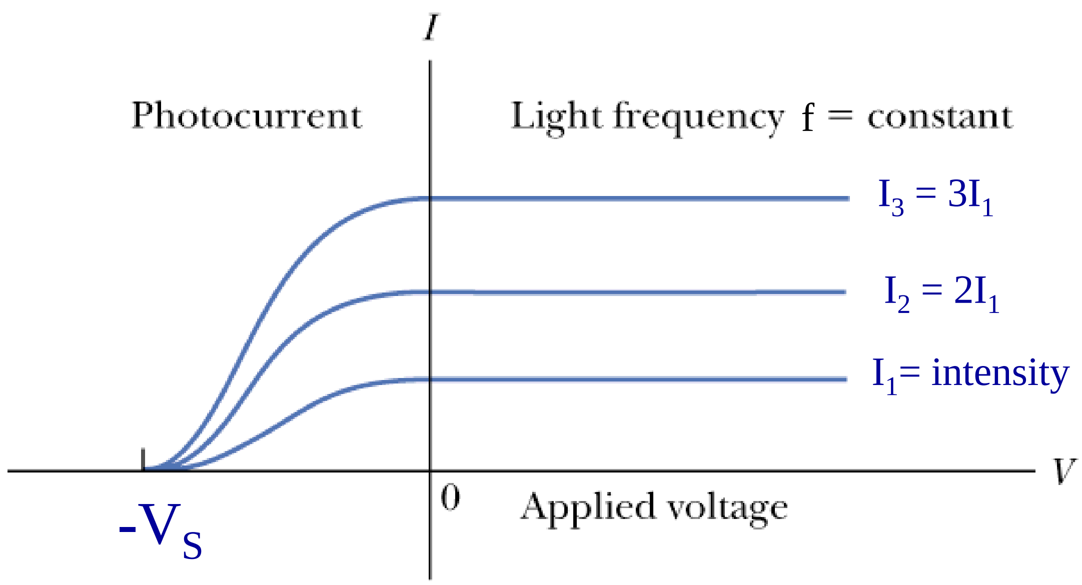


Measure characteristics of current in the circuit as a fn of I, f, λ

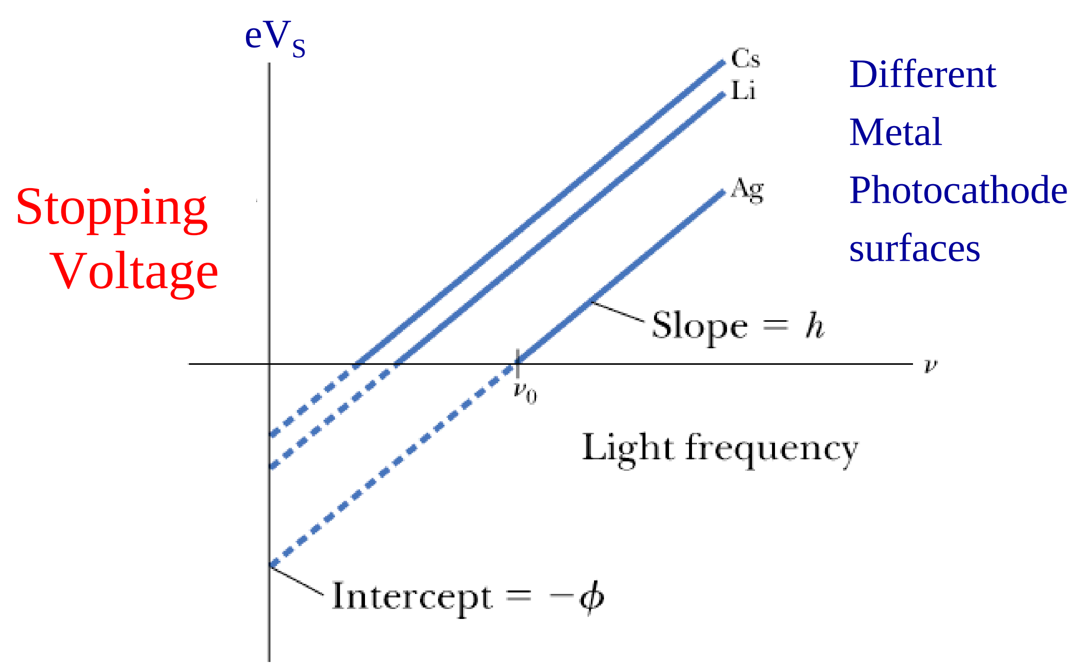
Photo Electric Effect: Measurable Properties

- Rate of electron emission from cathode
  - From current *i* seen in ammeter
- Maximum kinetic energy of emitted electron
  - By applying retarding potential on electron moving towards Collector plate
    - » **$K_{MAX} = eV_S$ (V_S = Stopping voltage)**
    - » **Stopping voltage → no current flows**
- Effect of different types of photo-cathode metal
- Time between shining light and first sign of photo-current in the circuit

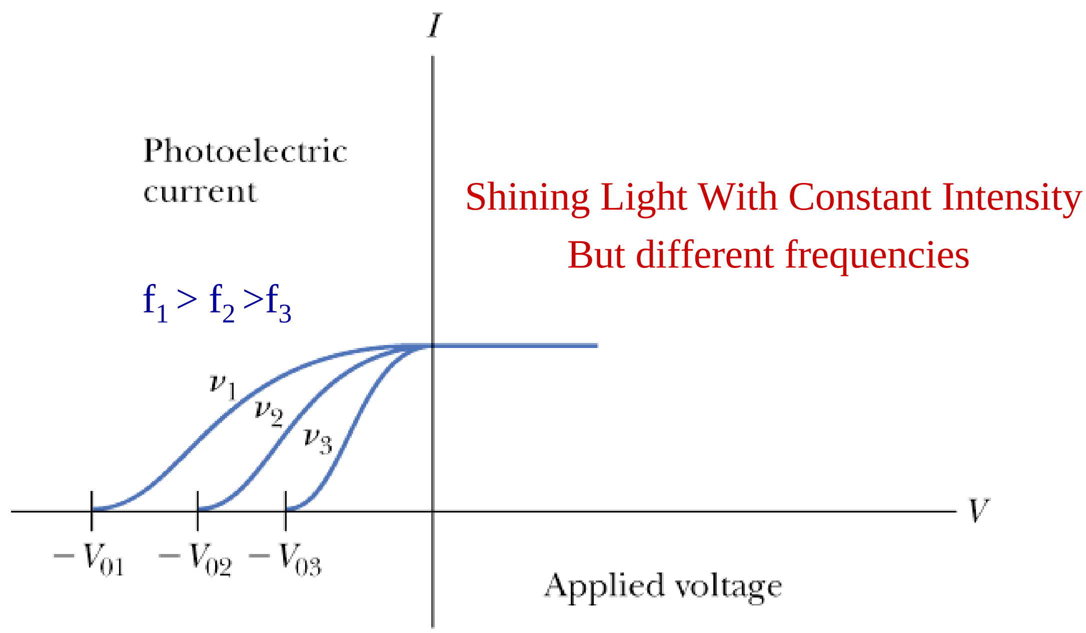
Observations : Current Vs Frequency of Incident Light



Stopping Voltage V_s Vs Incident Light Frequency



Retarding Potential Vs Light Frequency



Conclusions from the Experimental Observation

- Max Kinetic energy K_{MAX} independent of Intensity I for light of same frequency
- No photoelectric effect occurs if light frequency f is below a threshold no matter how high the intensity of light
- For a particular metal, light with $f > f_0$ causes photoelectric effect IRRESPECTIVE of light intensity.
  - f_0 is characteristic of that metal
- Photoelectric effect is instantaneous !...not time delay

Can one Explain all this Classically !

Classical Explanation of Photo Electric Effect

- As light Intensity increased $\Rightarrow \vec{E}$ field amplitude larger
  - E field and electrical force seen by the "charged subatomic oscillators" Larger
    - $\vec{F} = e\vec{E}$
    - More force acting on the subatomic charged oscillator
    - $\Rightarrow$ More energy transferred to it
    - $\Rightarrow$ Charged particle "hooked to the atom" should leave the surface with more Kinetic Energy KE !! The intensity of light shining rules !
- As long as light is intense enough , light of ANY frequency f should cause photoelectric effect
- Because the Energy in a Wave is uniformly distributed over the Spherical wavefront incident on cathode, thould be a noticeable time lag ΔT between time is incident & the time a photo-electron is ejected : Energy absorption time
  - How much time ? Lets calculate it classically.

Classical Physics: Time Lag in Photo-Electric Effect

- Electron absorbs energy incident on a surface area where the electron is confined ≅ size of atom in cathode metal
- Electron is "bound" by attractive Coulomb force in the atom, so it must absorb a minimum amount of radiation before its stripped off
- Example : Laser light Intensity I = 120W/m^2 on Na metal
  - Binding energy = 2.3 eV= "Work Function"
  - Electron confined in Na atom, size ≅ 0.1nm ..how long before ejection ?
  - Average Power Delivered $P_{AV} = \mathbf{I} \cdot \mathbf{A}$, $A = \pi r^2 \cong 3.1 \times 10^{-20}$ m^2
  - If all energy absorbed then $\Delta E = P_{AV} \cdot \Delta T \Rightarrow \Delta T = \Delta E / P_{AV}$

$$\Delta T = \frac{(2.3eV)(1.6\times10^{-19} J/eV)}{(120W/m^2)(3.1\times10^{-20} m^2)} = 0.10\ S$$

  - Classical Physics predicts Measurable delay even by the primitive clocks of 1900
  - But in experiment, the effect was observed to be instantaneous !!
  - Classical Physics fails in explaining all results

Disaster # 2 !

Now we need a Hero with New Ideas → Modern Physics !

Max Planck & Birth of Quantum Physics

Back to Blackbody Radiation Discrepancy

Planck noted the UltraViolet Catastrophe at high frequency

"Cooked" calculation with new "ideas" so as bring:

$R(\lambda) \rightarrow 0$ as $\lambda \rightarrow 0$

$f \rightarrow \infty$

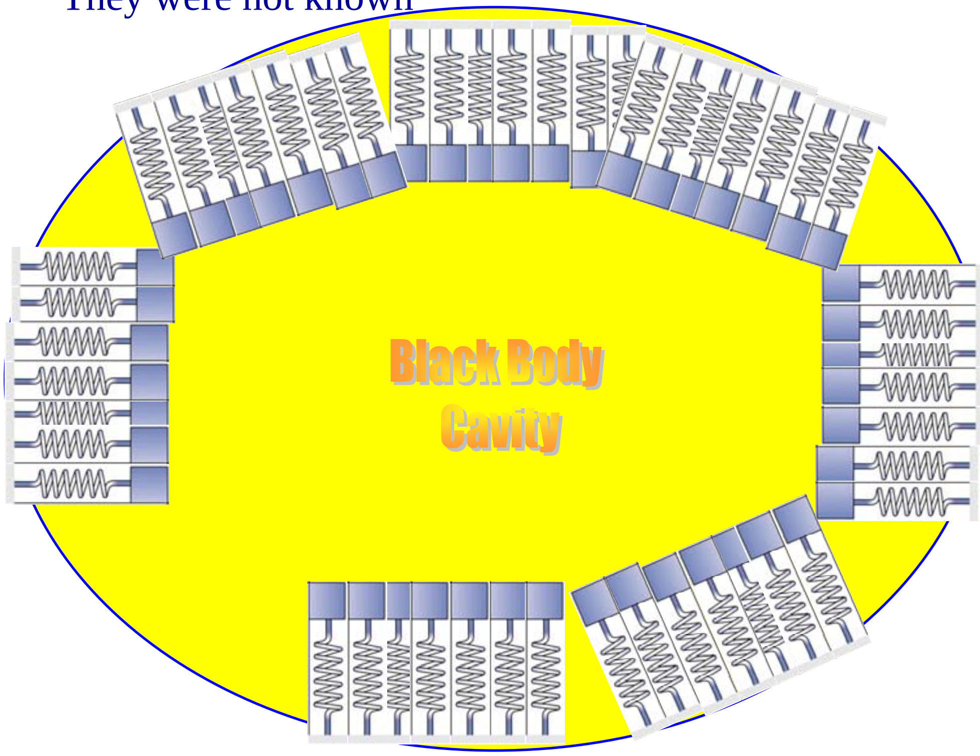
- Cavity radiation as equilibrium exchange of energy between EM radiation & "atomic" oscillators present on walls of cavity
- Oscillators can have any frequency f
- But the Energy exchange between radiation and oscillator NOT continuous and arbitarary…it is discrete …in packets of same amount
- $E = n\, hf$, with $n = 1,2\ 3\ldots\ \infty$

  h = constant he invented, a very small number he made up

Planck's "Charged Oscillators" in a Black Body Cavity

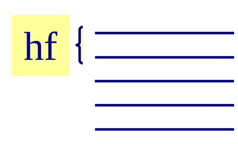
Planck did not know about electrons, Nucleus etc:

They were not known

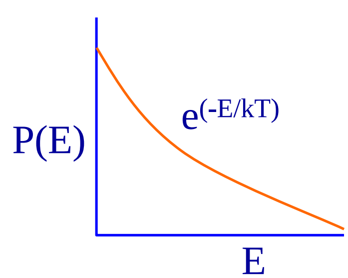


Planck, Quantization of Energy & BB Radiation

- Keep the rule of counting how many waves fit in a BB Volume
- Radiation Energy in cavity is quantized
- EM standing waves of frequency f have energy
  - E = n hf (n = 1,2 ,3 …10 ….1000…)



- Probability Distribution: At an equilibrium temp T, possible Energy of wave is distributed over a spectrum of states: $P(E) = e^{(-E/kT)}$
- Modes of Oscillation with :
  - Less energy E=hf = favored
  - More energy E=hf = disfavored



By this statistics, large energy, high f modes of EM disfavored

Planck's Calculation

$$R(\lambda)=\left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\left[\frac{hc}{\lambda}\left(\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\right)\right]$$

Odd looking form

When $\lambda \to$ large $\Rightarrow \frac{\text{hc}}{\lambda\text{kT}} \to$ small

Recall $\boxed{e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots}$

$$\Rightarrow e^{\frac{hc}{\lambda kT}} - 1 = (1 + \frac{hc}{\lambda kT} + \frac{1}{2}\left(\frac{hc}{\lambda kT}\right)^2 + \ldots] - 1$$

$$= \frac{hc}{\lambda kT}$$ plugging this in R(λ) eq:

$$\boxed{R(\lambda)=\left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\frac{hc}{\lambda kT}}$$

Graph & Compare
With BBQ data

Planck's Formula and Small λ

When λ is small (large f)

$$\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \cong \frac{1}{e^{\frac{hc}{\lambda kT}}} = e^{-\frac{hc}{\lambda kT}}$$

Substituting in R(λ) eqn:

$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right) e^{-\frac{hc}{\lambda kT}}$$

As $\lambda \to 0$, $e^{-\frac{hc}{\lambda kT}} \to 0$

$$\Rightarrow R(\lambda) \to 0$$

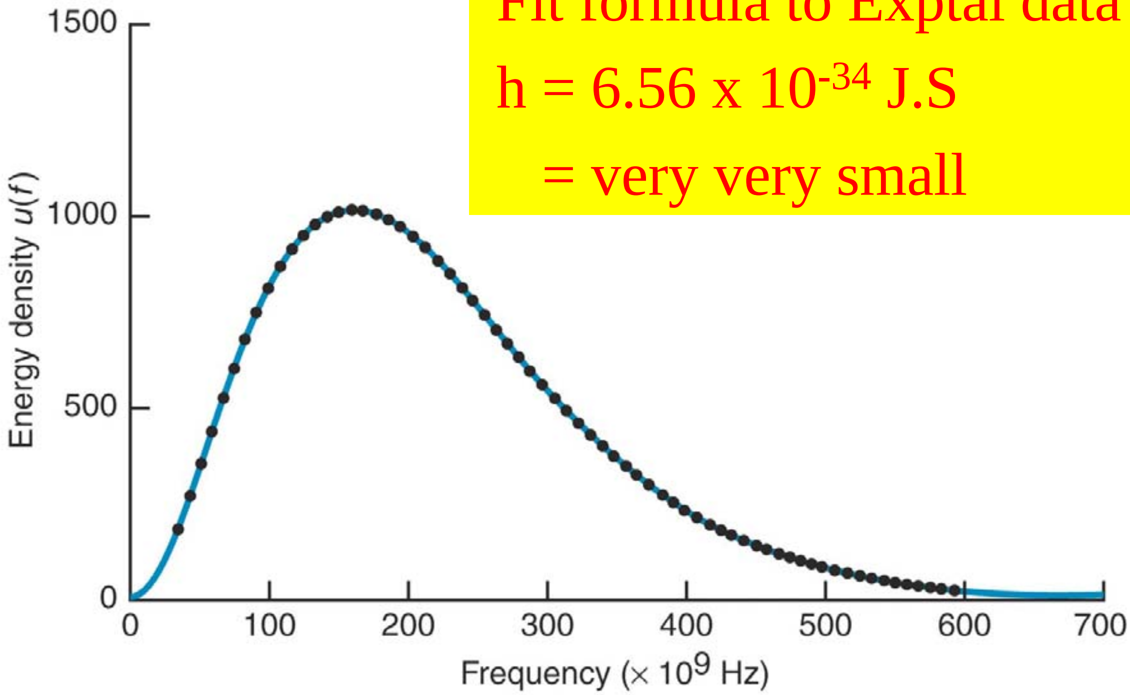
Just as seen in the experimental data

Planck's Explanation of BB Radiation

Fit formula to Exptal data

$h = 6.56 \times 10^{-34}$ J.S

= very very small



Consequence of Planck's Formula

Quantization of Energy !