

Quiz 1 Blue Books will be returned to Student Affairs office
(Ground Floor, Urey Hall Extension)
this Wednesday by 10am
(usually you can pick it up Tuesday 2pm)

Regrade Requests must get to TA within 7 days
Frivolous regrade requests will lead to loss of points

Quiz 2 is This Friday
Quiz 2 will emphasize Sections 1.6-1.10 (inclusive)
[But you will need to know concepts from earlier sections]



Physics 2D Lecture Slides

Lecture 8: Jan 20th 2004

Vivek Sharma
UCSD Physics

Relativistic Force & Acceleration

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

Relativistic Force And Acceleration

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt} \left(\frac{m\vec{u}}{\sqrt{1-(u/c)^2}} \right)$$

$$F = \left[\frac{m}{(1-(u/c)^2)^{3/2}} \right] \frac{du}{dt} : \text{Relativistic Force}$$

Since Acceleration $\vec{a} = \frac{d\vec{u}}{dt}$, [rate of change of velocity]

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m} [1-(u/c)^2]^{3/2}$$

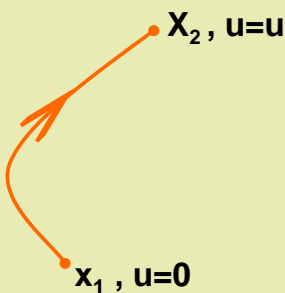
Reason why you cant quite get up to the speed of light no matter how hard you try!



Note: As $u/c \rightarrow 1$, $\vec{a} \rightarrow 0$!!!!
Its harder to accelerate when you get closer to speed of light

Relativistic Work Done & Change in Energy

$$W = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{x} = \int_{x_1}^{x_2} \frac{d\vec{p}}{dt} \cdot d\vec{x}$$



$$p = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}} \quad \therefore \quad \frac{d\vec{p}}{dt} = \frac{m \frac{du}{dt}}{\left[1-\frac{u^2}{c^2}\right]^{3/2}}, \text{ substitute in } W,$$

$$\therefore W = \int_0^u \frac{m \frac{du}{dt} u dt}{\left[1-\frac{u^2}{c^2}\right]^{3/2}} \quad (\text{change in var } x \rightarrow u)$$

$$W = \int_0^u \frac{mudu}{\left[1-\frac{u^2}{c^2}\right]^{3/2}} = \frac{mc^2}{\left[1-\frac{u^2}{c^2}\right]^{1/2}} - mc^2 = \gamma mc^2 - mc^2$$

Work done is change in Kinetic energy K

$$K = \gamma mc^2 - mc^2 \text{ or}$$

$$\text{Total Energy } E = \gamma mc^2 = K + mc^2$$

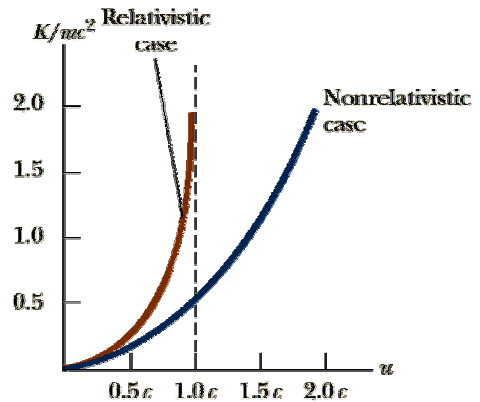
But Professor... Why Can's ANYTHING go faster than light ?

$$K = \frac{mc^2}{\left[1 - \frac{u^2}{c^2}\right]^{1/2}} - mc^2 \Rightarrow (K + mc^2)^2 = \left(\frac{mc^2}{\left[1 - \frac{u^2}{c^2}\right]^{1/2}} \right)^2$$

$$\Rightarrow \left[1 - \frac{u^2}{c^2}\right] = m^2 c^4 [K + mc^2]^{-2}$$

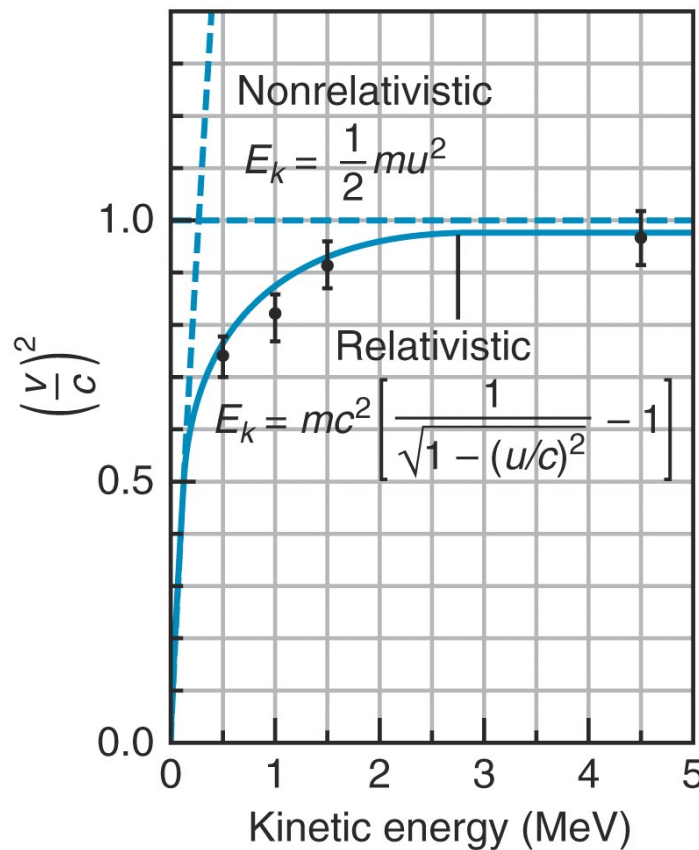
$$\Rightarrow u = c \sqrt{1 - \left(\frac{K}{mc^2} + 1\right)^{-2}} \quad \text{(Parabolic in } u \text{ Vs } \frac{K}{mc^2}\text{)}$$

As $K \rightarrow \infty$, $u \rightarrow c$



Non-relativistic case: $K = \frac{1}{2}mu^2 \Rightarrow u = \sqrt{\frac{2K}{m}}$

Relativistic Kinetic Energy Vs Velocity



Relativistic Kinetic Energy & Newtonian Physics

$$\text{Relativistic KE } K = \gamma mc^2 - mc^2$$

Remember Binomial Theorem

$$\text{for } x \ll 1; (1+x)^n = \left(1 + \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 + \text{smaller terms}\right)$$

$$\therefore \text{ When } u \ll c, \left[1 - \frac{u^2}{c^2}\right]^{-\frac{1}{2}} \cong 1 + \frac{1}{2} \frac{u^2}{c^2} + \dots \text{smaller terms}$$

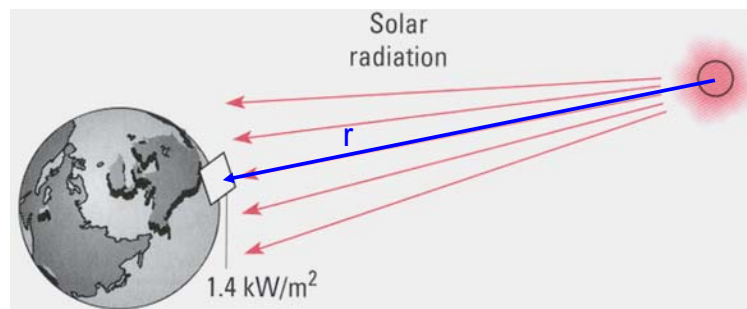
$$\text{so } K \cong mc^2 \left[1 + \frac{1}{2} \frac{u^2}{c^2}\right] - mc^2 = \frac{1}{2} mu^2 \quad (\text{classical form recovered})$$

$$\text{Total Energy of a Particle } E = \gamma mc^2 = KE + mc^2$$

$$\text{For a particle at rest, } u = 0 \Rightarrow \text{Total Energy } E = mc^2$$

$E=mc^2 \Rightarrow$ Sunshine Won't Be Forever !

Q: Solar Energy reaches earth at rate of 1.4kW per square meter of surface perpendicular to the direction of the sun. by how much does the mass of sun decrease per second owing to energy loss? The mean radius of the Earth's orbit is $1.5 \times 10^{11}\text{m}$.



- Surface area of a sphere of radius r is $A = 4\pi r^2$
- Total Power radiated by Sun = power received by a sphere whose radius is equal to earth's orbit radius

$$P = \frac{P}{A} A = \frac{P}{A} 4\pi r^2 = (1.4 \times 10^3 \text{ W/m}^2)(4\pi)(1.5 \times 10^{11})^2$$

$$P = 4.0 \times 10^{26} \text{ W}$$

So Sun loses $E = 4.0 \times 10^{26} \text{ J}$ of rest energy per second

$$\text{Its mass decreases by } m = \frac{E}{c^2} = \frac{4.0 \times 10^{26} \text{ J}}{(3.0 \times 10^8)^2} = 4.4 \times 10^9 \text{ kg !!}$$

If the Sun's Mass = $2.0 \times 10^{30} \text{ kg}$ So how long with the Sun last ?

$$E = \gamma mc^2 \Rightarrow E^2 = \gamma^2 m^2 c^4$$

Relationship between P and E

$$p = \gamma mu \Rightarrow p^2 c^2 = \gamma^2 m^2 u^2 c^2$$

$$\begin{aligned} \Rightarrow E^2 - p^2 c^2 &= \gamma^2 m^2 c^4 - \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^2 (c^2 - u^2) \\ &= \frac{m^2 c^2}{1 - \frac{u^2}{c^2}} (c^2 - u^2) = \frac{m^2 c^4}{c^2 - u^2} (c^2 - u^2) = m^2 c^4 \end{aligned}$$

$$E^2 = p^2 c^2 + (mc^2)^2 \text{important relation}$$

For particles with zero rest mass like photon (EM waves)

$$E = pc \text{ or } p = \frac{E}{c} \text{ (light has momentum!)}$$

Relativistic Invariance : $E^2 - p^2 c^2 = m^2 c^4$: In all Ref Frames

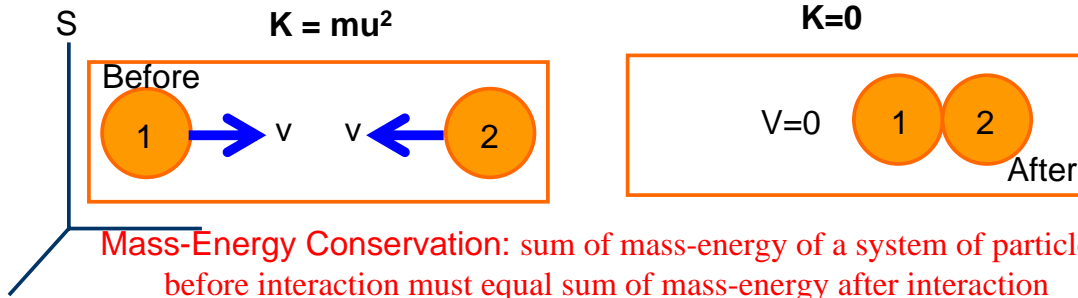
Rest Mass is a "finger print" of the particle

Mass Can "Morph" into Energy & Vice Verca

- Unlike in Newtonian mechanics
- In relativistic physics : Mass and Energy are the same thing
- New word/concept : MassEnergy , just like SpaceTime
- It is the mass-energy that is always conserved in every reaction : Before & After a reaction has happened
- Like squeezing a balloon : Squeeze here, it grows elsewhere
 - If you "squeeze" mass, it becomes (kinetic) energy & vice verca !
 - CONVERSION FACTOR = C²
 - This exchange rate never changes !

Mass is Energy, Energy is Mass : Mass-Energy Conservation

Examine Kinetic energy Before and After Inelastic Collision: Conserved?



$$E_{\text{before}} = E_{\text{after}}$$

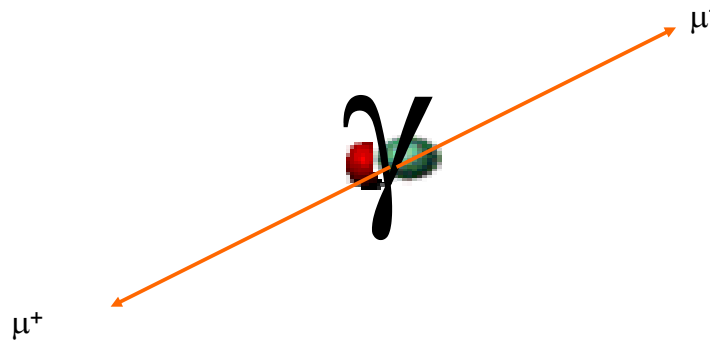
$$\frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} + \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} = Mc^2 \Rightarrow M = \frac{2m}{\sqrt{1-\frac{u^2}{c^2}}} > 2m$$

Kinetic energy has been transformed into mass increase

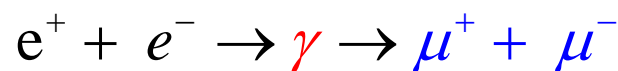
Kinetic energy is not lost, its transformed into more mass in final state

$$\Delta M = M - 2m = \frac{2K}{c^2} = \frac{2}{c^2} \left(\frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} - mc^2 \right)$$

Creation and Annihilation of Particles

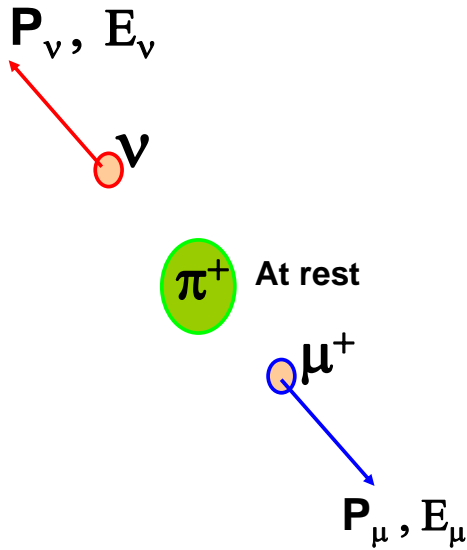


Sequence of events in a matter-antimatter collision:



Relativistic Kinematics of Subatomic Particles

Reconstructing Decay of a π Meson



The decay of a stationary $\pi^+ \rightarrow \mu^+ \nu$ happens quickly, ν is invisible, has $m \cong 0$; μ^+ leaves a trace in a B field
 μ^+ mass = $106 \text{ MeV}/c^2$, KE = 4.6 MeV

What was mass of the fleeting π^+ ?

Energy Conservation:

$$E_\pi = E_\mu + E_\nu \Rightarrow m_\pi c^2 = \sqrt{(m_\mu c^2)^2 + p_\mu^2 c^2} + p_\nu c$$

Momentum Conservation : $p_\mu = p_\nu$

$$\Rightarrow m_\pi c^2 = \sqrt{(m_\mu c^2)^2 + p_\mu^2 c^2} + p_\mu c$$

$$\text{also } p_\mu^2 c^2 = E_\mu^2 - (m_\mu c^2)^2 = (K_\mu + m_\mu c^2)^2 - (m_\mu c^2)^2 \\ = K_\mu^2 + 2K_\mu m_\mu c^2$$

Substituting for $p_\mu^2 c^2 \Rightarrow$

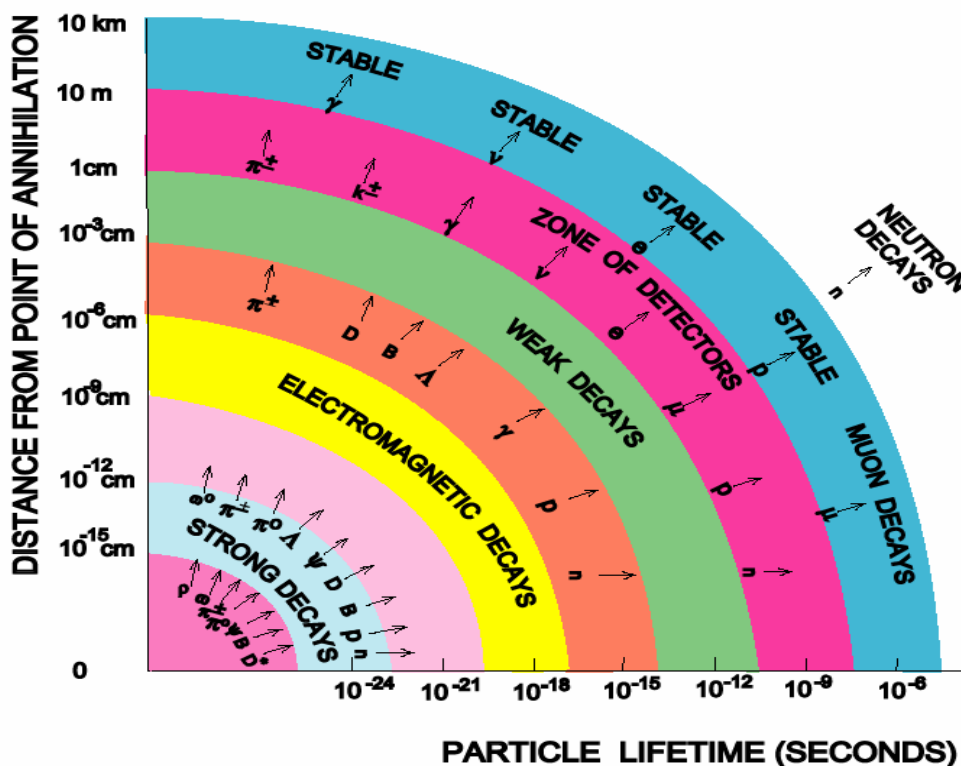
$$m_\pi c^2 = \sqrt{m_\mu^2 c^4 + K_\mu^2 + 2K_\mu m_\mu c^2} + \sqrt{K_\mu^2 + 2K_\mu m_\mu c^2}$$

Put in all the known #s

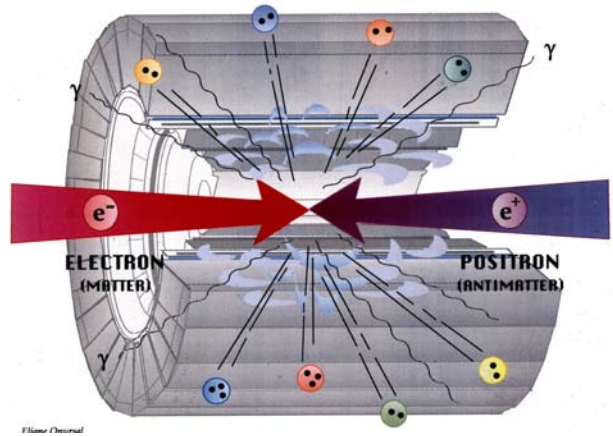
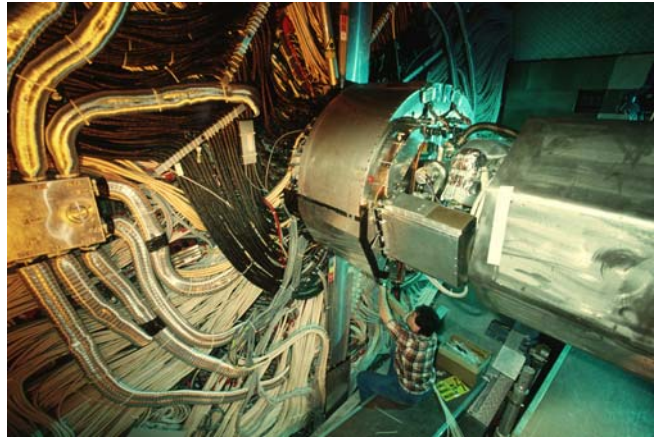
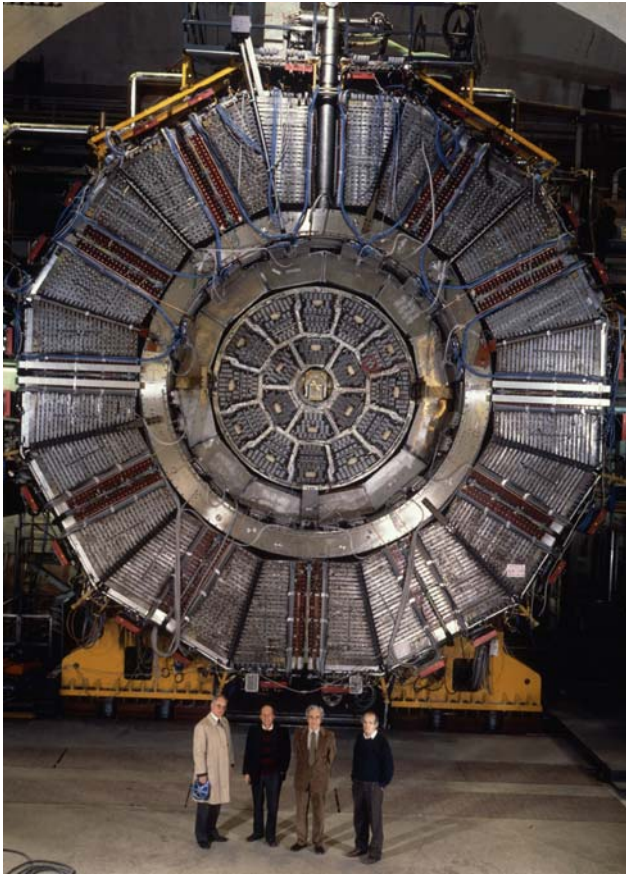
$$\Rightarrow m_\pi c^2 = 111 \text{ MeV} + 31 \text{ MeV} = 141 \text{ MeV}$$

$$\Rightarrow m_\pi = 141 \text{ MeV} / c^2$$

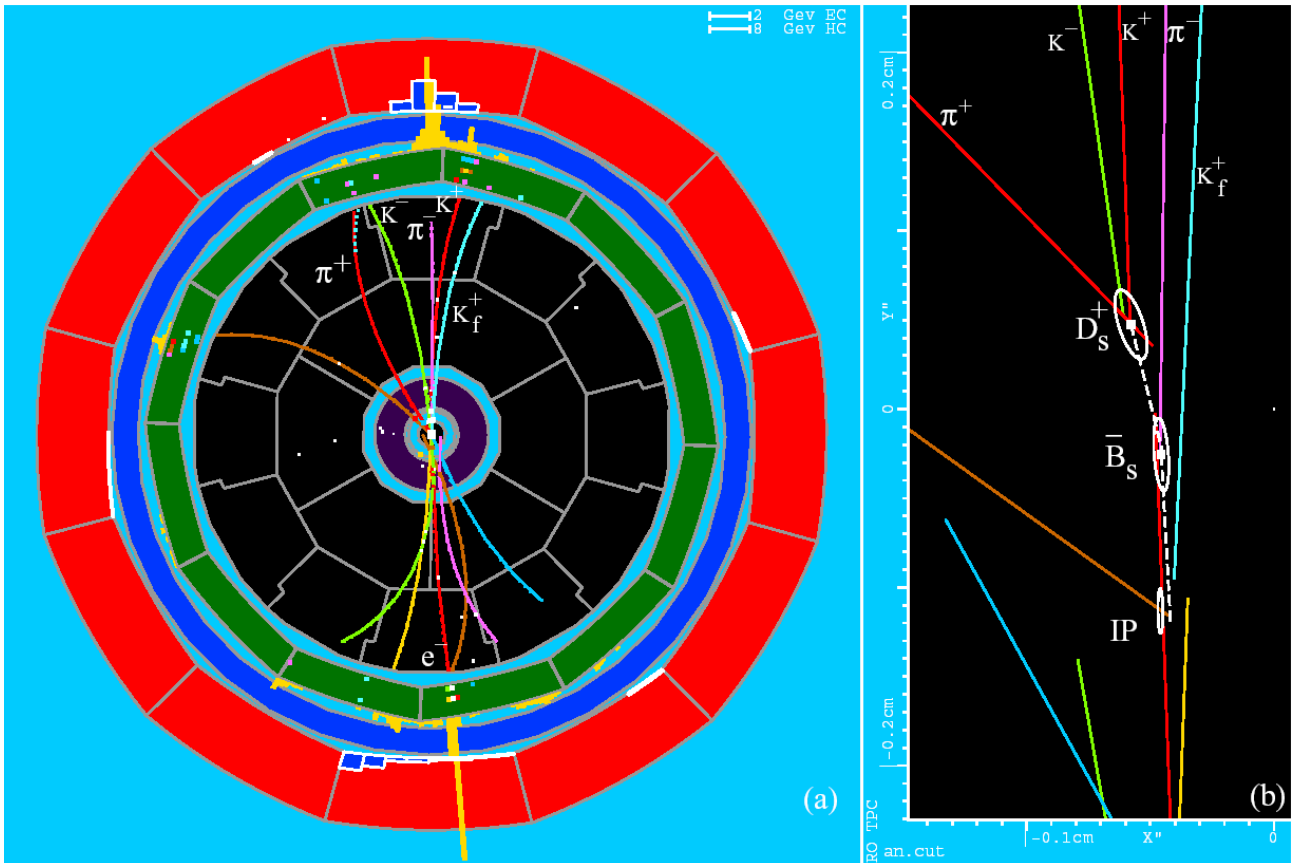
THE SEQUENCE OF PARTICLE DECAY FOLLOWING e^+e^- ANNIHILATION



Detecting Baby Universes : Need a "Camera"



My Discovery (1993): Beauty With Strangeness

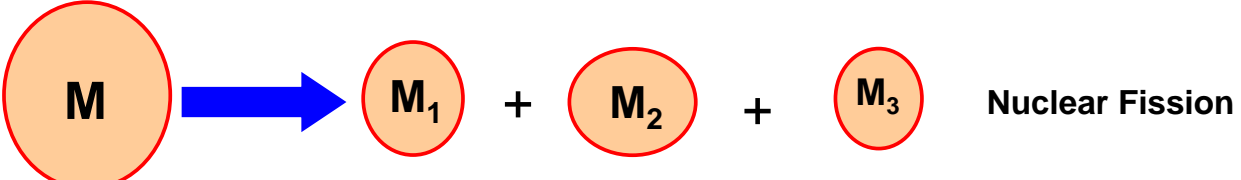


Two Faced Particle : Beauty With Strangeness (Bs)



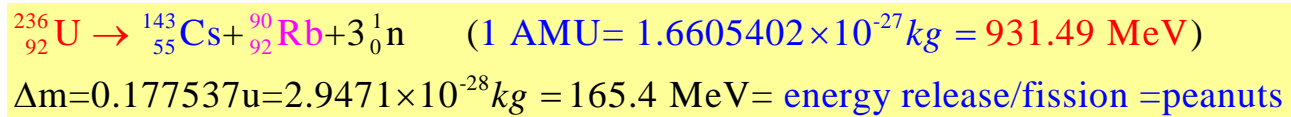
Sometimes Matter
Sometimes Antimatter

Conservation of Mass-Energy: Nuclear Fission



$$Mc^2 = \frac{M_1c^2}{\underbrace{\sqrt{1-\frac{u_1^2}{c^2}}}_{<1}} + \frac{M_2c^2}{\underbrace{\sqrt{1-\frac{u_2^2}{c^2}}}_{<1}} + \frac{M_3c^2}{\underbrace{\sqrt{1-\frac{u_3^2}{c^2}}}_{<1}} \Rightarrow M > M_1 + M_2 + M_3$$

Loss of mass shows up as kinetic energy of final state particles
Disintegration energy per fission $Q = (M - (M_1 + M_2 + M_3))c^2 = \Delta Mc^2$



What makes it explosive is 1 mole of Uranium = 6.023×10^{23} Nuclei !!

Energy Released by 1 Kg of Fissionable Uranium

1 Mole of Uranium = 236 gm, Avagadro's # = 6.023×10^{23} Nuclei

So in 1 kg N = $\frac{6.023 \times 10^{23}}{236 \text{ g / mole}} \times 1000 \text{ g} = 2.55 \times 10^{24}$ nuclei

1 Nuclear fission = 165.4 MeV $\therefore 10^3 \text{ g} = 2.55 \times 10^{24} \times 165.4 \text{ MeV}$

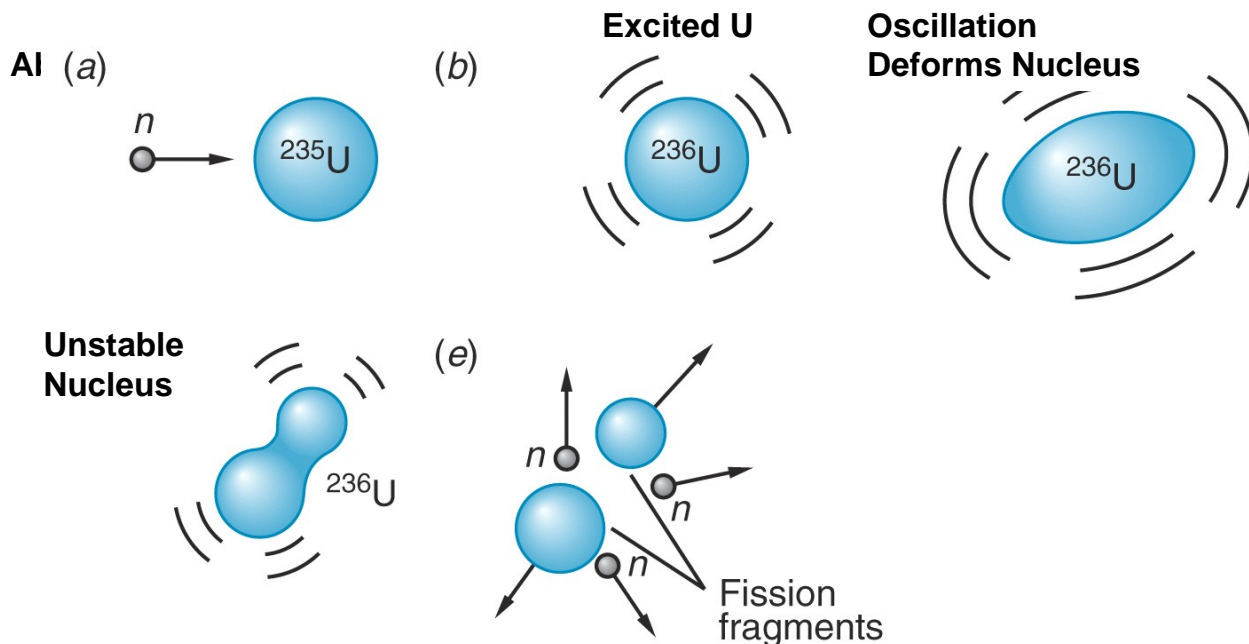
Note 1 MeV = $4.45 \times 10^{-20} \text{ kWh}$

If the power plant has conversion efficiency = 40%

Energy Transformed = $748 \times 10^6 \text{ kWh}$

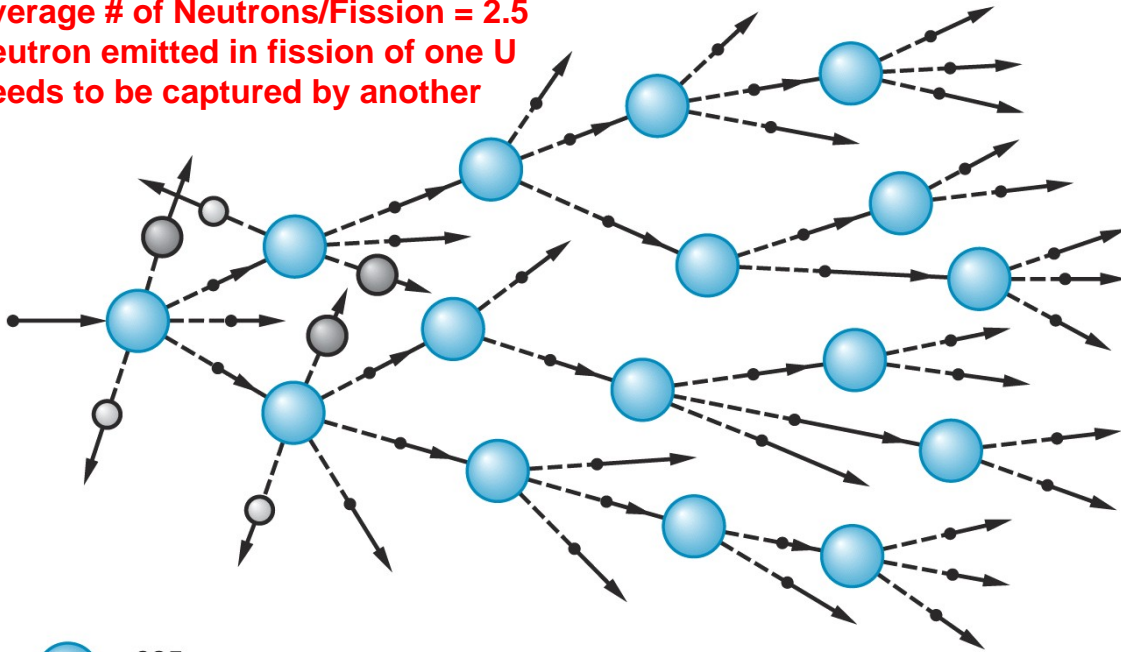
\Rightarrow 1 100W lamp can be lit for 8500 years !

Nuclear Fission Schematic



Sustaining Chain Reaction: 1st three Fissions

Average # of Neutrons/Fission = 2.5
 Neutron emitted in fission of one U
 Needs to be captured by another



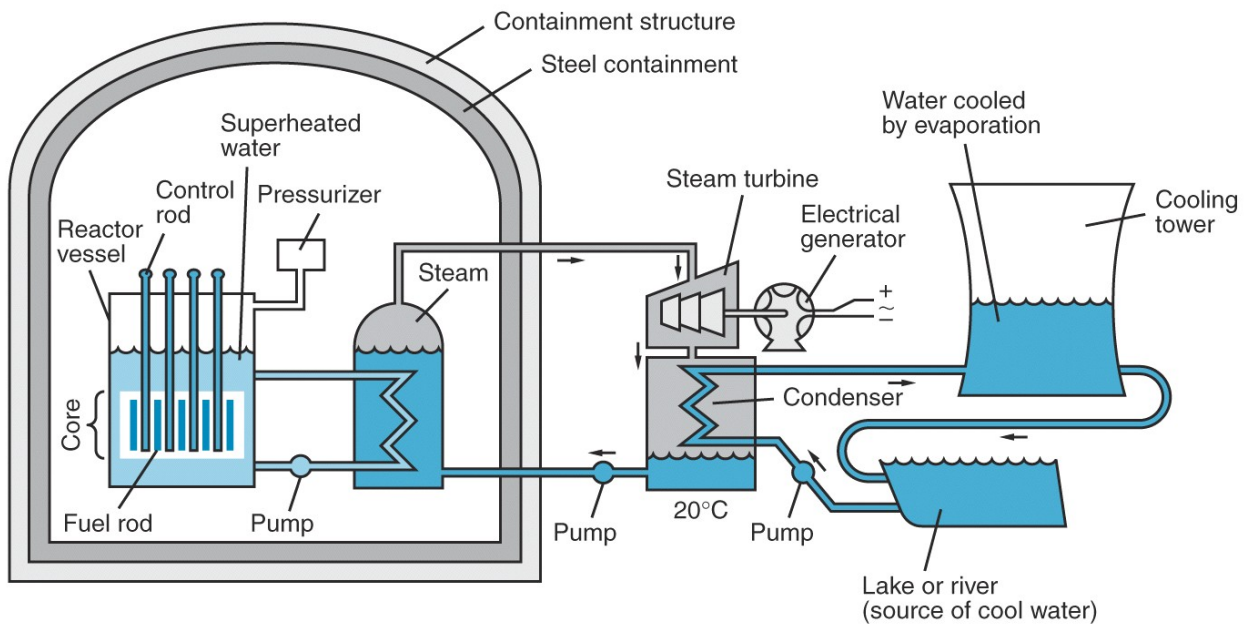
- ^{235}U nucleus
- Fission fragments
- Neutron

To control reaction => define factor K

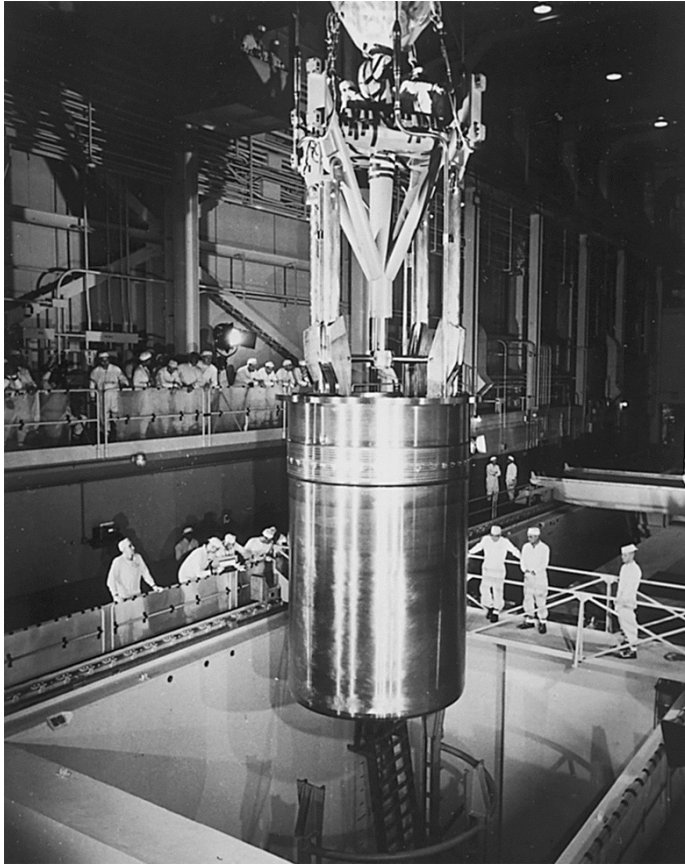
Supercritical $K \gg 1$ in a Nuclear Bomb
 Critical $K = 1$ in a Nuclear Reactor

Schematic of a Pressurized-Water Reactor

Water in contact with reactor core serves as a moderator and heat transfer Medium. Heat produced in fission drives turbine



Lowering Fuel Core in a Nuclear Reactor



First Nuke Reactor :Pennsylvania
1957

Pressure Vessel contains :
14 Tons of Natural Uranium
+ 165 lb of enriched Uranium

Power plant rated at 90MW,
Retired (82)

Pressure vessel packed with
Concrete now sits in **Nuclear Waste**
Facility in Hanford, Washington

Nuclear Fusion : What Powers the Sun

Opposite of Fission

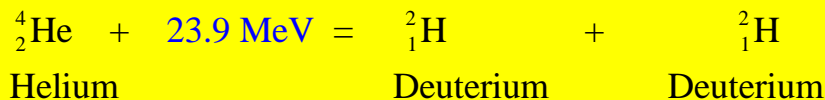
Mass of a Nucleus < mass of its component protons+Neutrons

Nuclei are stable, bound by an attractive "Strong Force"

Think of Nuclei as molecules and proton/neutron as atoms making it

Binding Energy: Work/Energy required to pull a bound system (M) apart leaving its components (m) free of the attractive force and at rest:

$$Mc^2 + BE = \sum_{i=1}^n m_i c^2$$



Think of energy released in Fusion as **Dissociation energy** in Chem

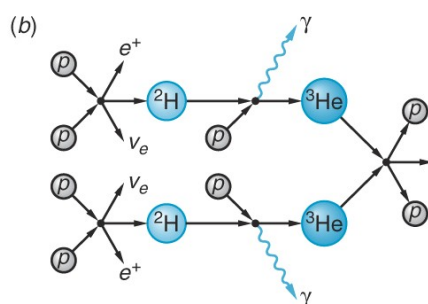
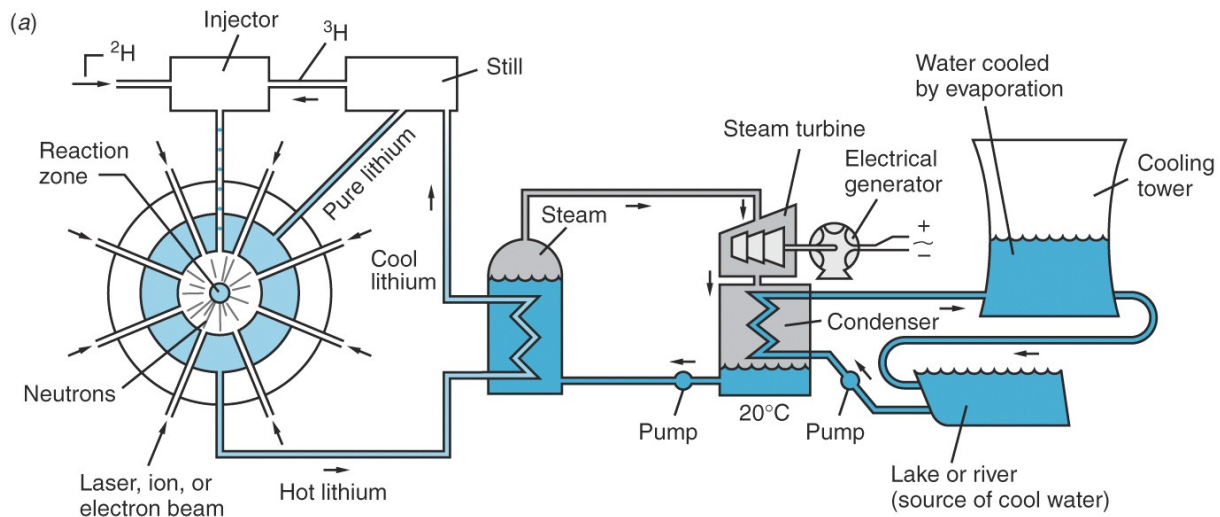
Sun's Power Output = 4×10^{26} Watts $\Rightarrow 10^{38}$ Fusion/Second

No wonder Sun is considered a God in many cultures !

Nuclear Fusion: Wishing For The Star

- Fusion is eminently desirable because
 - More Energy/Nucleon
 - (3.52 MeV in fusion Vs 1 MeV in fission)
 - ${}^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + \text{n} + 17.6 \text{ MeV}$
 - Relatively abundant fuel supply
 - No danger like nuclear reactor going supercritical
- Unfortunately technology not commercially available
 - What's inside nuclei => protons and Neutrons
 - Need Large KE to overcome Coulomb repulsion between nuclei
 - About 1 MeV needed to bring nuclei close enough together for Strong Nuclear Attraction → fusion
 - Need to
 - heat particle to high temp such that $kT \approx 10\text{keV} \rightarrow$ tunneling
 - High density plasma at high temp $T \approx 10^8 \text{ K}$ like in stars
 - Confine Plasma (\pm ions) long enough for fusion
 - » In stars, enormous gravitational field confines plasma

Inertial Fusion Reactor : Schematic



Pellet of frozen-solid Deuterium & tritium bombarded from all sides with intense pulsed laser beam with energy $\approx 10^6$ Joules lasting 10^{-8} S

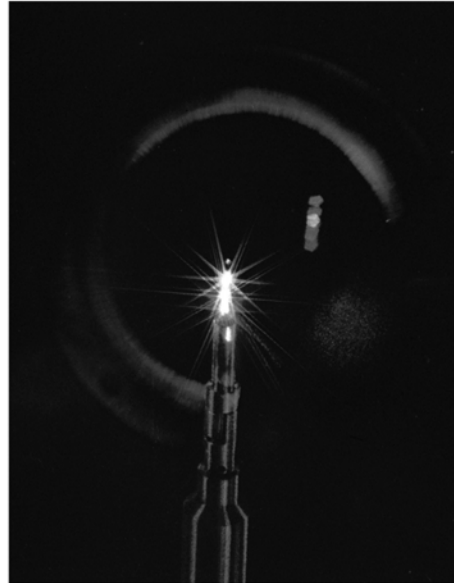
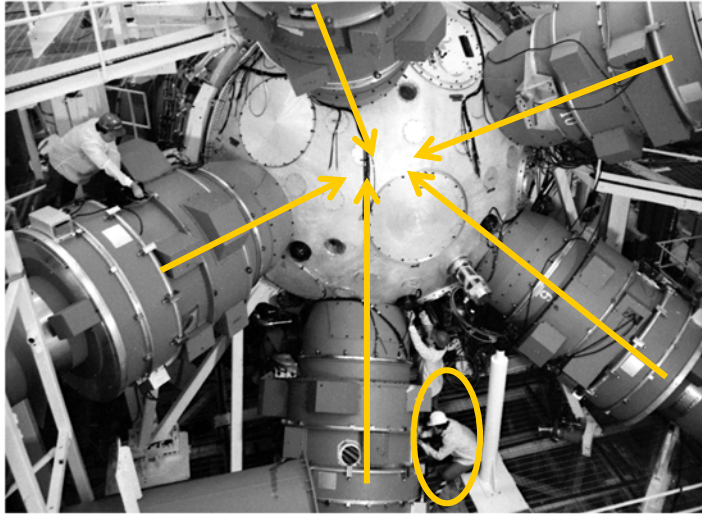
Momentum imparted by laser beam compresses pellet by 1/10000 of normal density and heats it to temp $T \approx 10^8 \text{ K}$ for 10^{-10} S

Burst of fusion energy transported away by liquid Li

World's Most Powerful Laser : NOVA @ LLNL

Size of football field, 3 stories tall

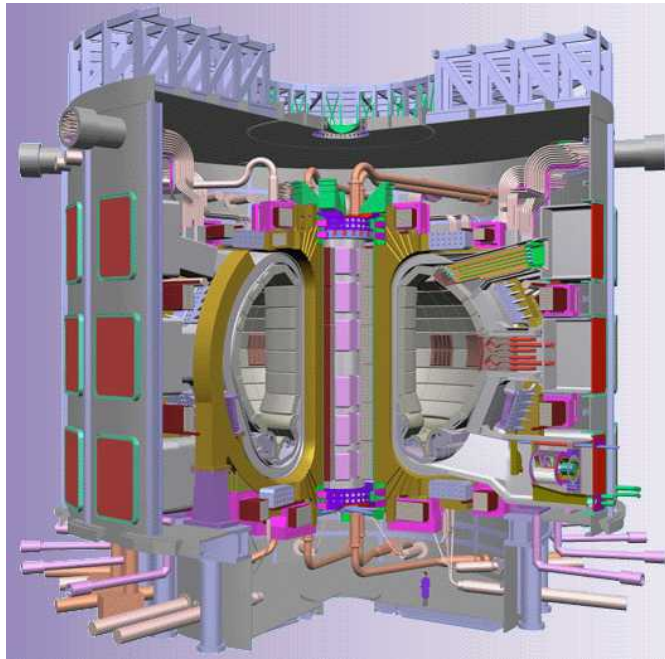
Generates 1.0×10^{14} watts (100 terawatts)



10 laser beams converge onto H pellet (0.5mm diam)

Fusion reaction is visible as a starlight lasting 10^{-10} S
Releasing 10^{13} neutrons

ITER: The Next Big Step in Nuclear Fusion



Visit www.iter.org for Details of this mega Science & Engineering Project
This may be future of cheap, clean Nuclear Energy for Earthlings