Quiz 1 is This Friday

Quiz 1 will cover Sections 1.1-1.6 (inclusive)
Remaining material will be carried over to Quiz 2

Bring Blue Book, check calculator battery

Write all answers in indelible ink else no grade!

Write answers on consecutive pages: Don't leave blank pages

All nighters don't work for these quizzes
Get plenty of Sleep….come with a fresh mind!

Physics 2D Lecture Slides
Lecture 7: Jan 14th 2004

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Relativistic Force & Acceleration

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

Relativistic
Force
And
Acceleration

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-(u/c)^2}}\right)$$

$$F = \left[\frac{m}{\left(1-(u/c)^2\right)^{3/2}}\right]\frac{du}{dt}$$: Relativistic Force

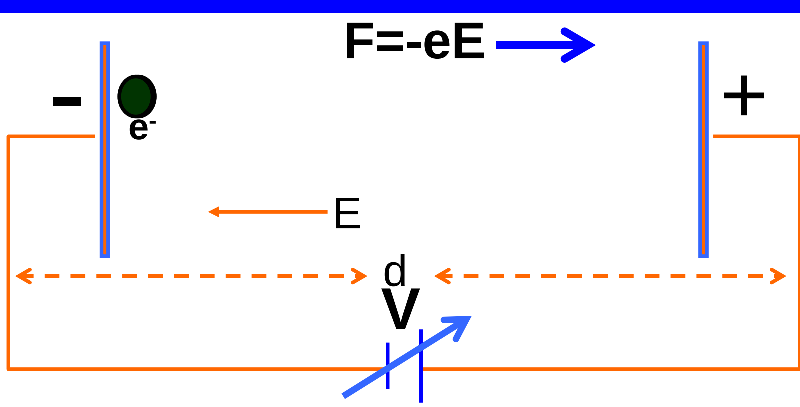
Since Acceleration $\vec{a} = \frac{d\vec{u}}{dt}$, [rate of change of velocity]

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m}\left[1-(u/c)^2\right]^{3/2}$$

Reason why you cant quite get up to the speed of light no matter how hard you try!

Note: As $u/c \to 1$, $\vec{a} \to 0$!!!!
Its harder to accelerate when you get closer to speed of light

Linear Particle Accelerator : Parallel Plates With Potential Difference



Parallel Plates

E= V/d
F= -eE

Charged particle q moves in straight line in a uniform electric field $\vec{E}$ with speed $\vec{u}$ accelarates under force $\vec{F}=q\vec{E}$

$$|\vec{a}| = \left|\frac{d\vec{u}}{dt}\right| = \left|\frac{\vec{F}}{m}\right|\left(1-\frac{u^2}{c^2}\right)^{3/2} = \left|\frac{q\vec{E}}{m}\right|\left(1-\frac{u^2}{c^2}\right)^{3/2}$$

larger the potential difference V across plates, larger the force on particle

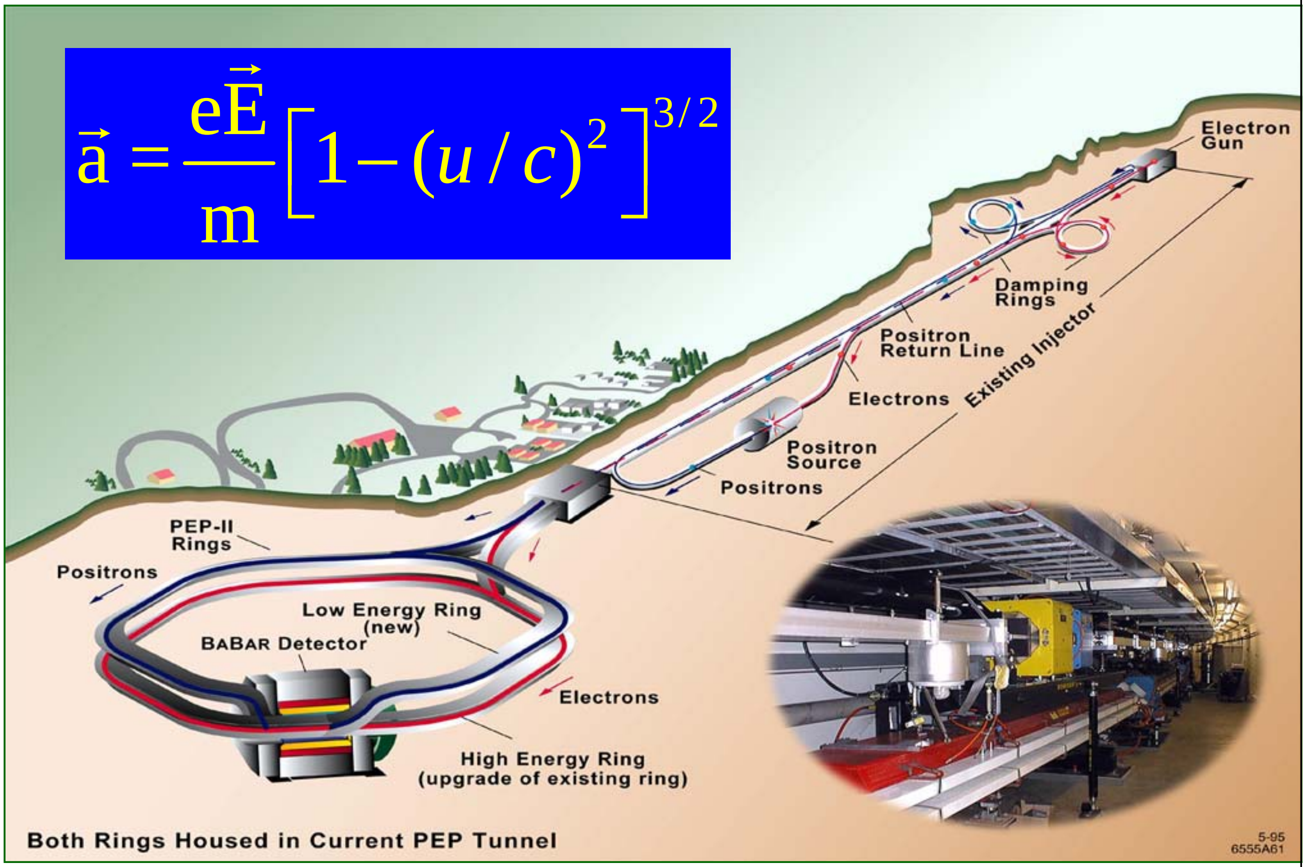
Under force, work is done on the particle, it gains Kinetic energy

New Unit of Energy

1 eV = 1.6x10^{-19} Joules
1 MeV = 1.6x10^{-13} Joules
1 GeV = 1.6x10-10 Joules

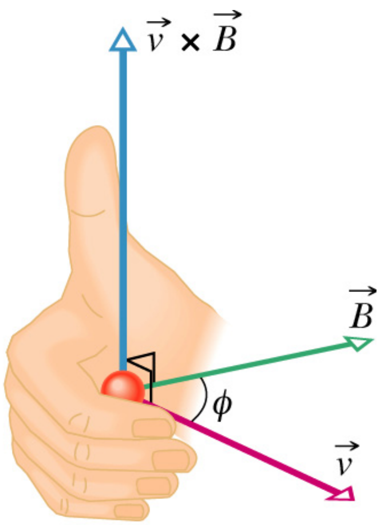
Linear Particle Accelerator : 50 GigaVolts Accelating Potential

$$\vec{a} = \frac{e\vec{E}}{m}\left[1-(u/c)^2\right]^{3/2}$$



PEP-II accelerator schematic and tunnel view

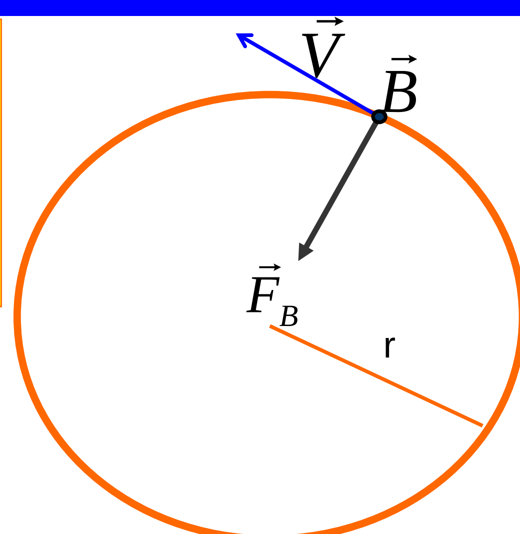
Magnetic Confinement & Circular Particle Accelerator



Classically

$$F = m\frac{v^2}{r}$$

$$qvB = m\frac{v^2}{r}$$

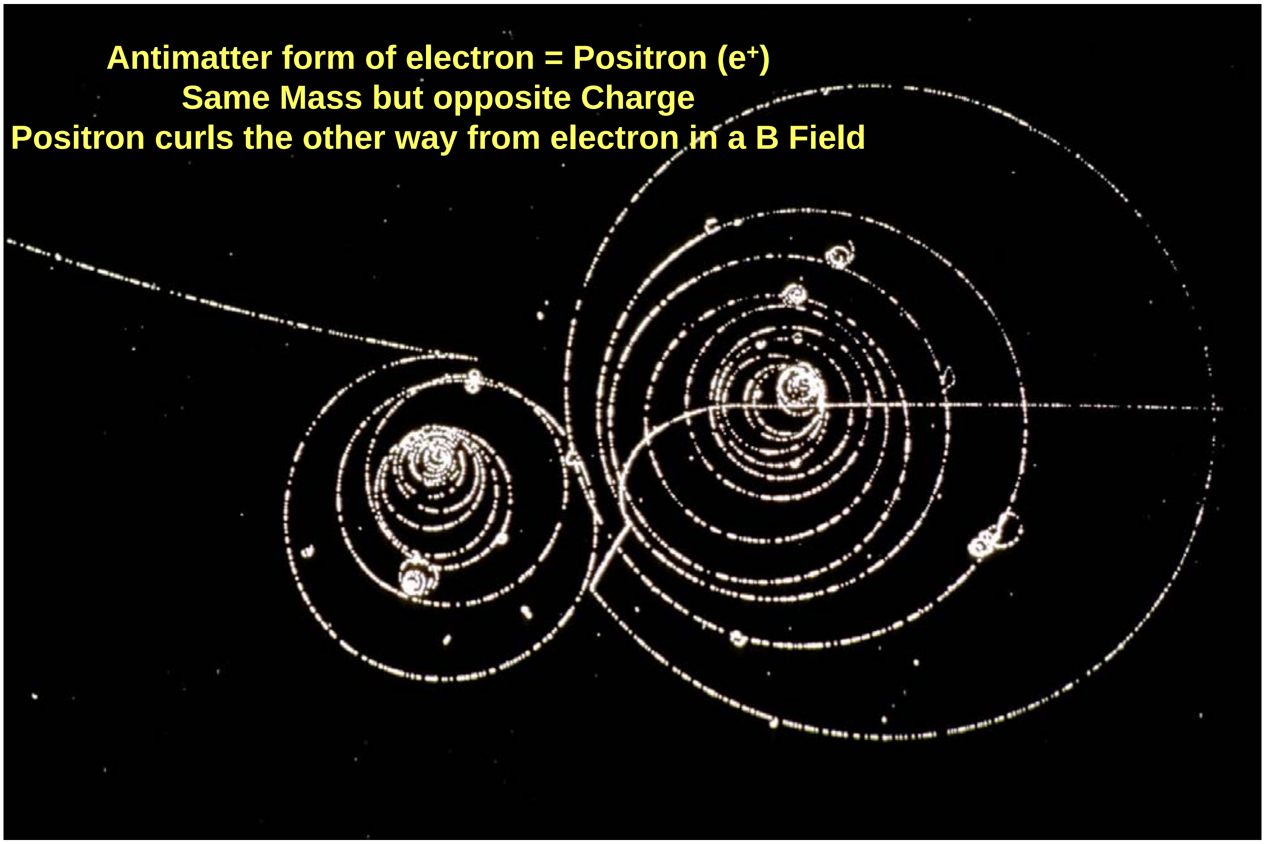


$$F = \frac{dp}{dt} = \frac{d(\gamma mu)}{dt} = \gamma m\frac{du}{dt} = quB$$

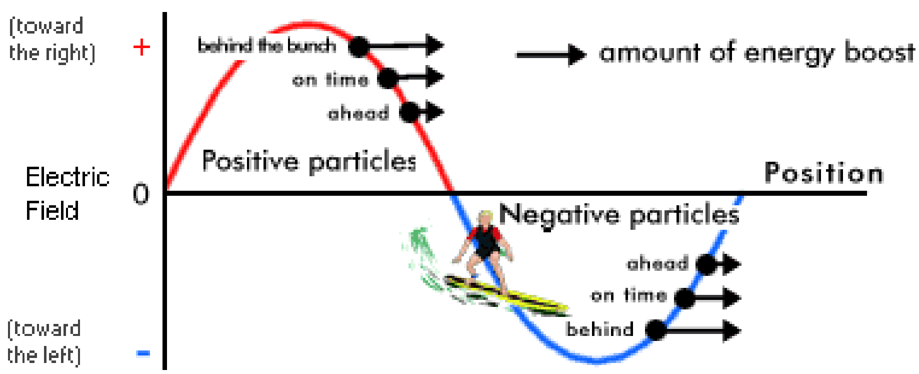
$$\frac{du}{dt} = \frac{u^2}{r} \quad \text{(Centripetal accelaration)}$$

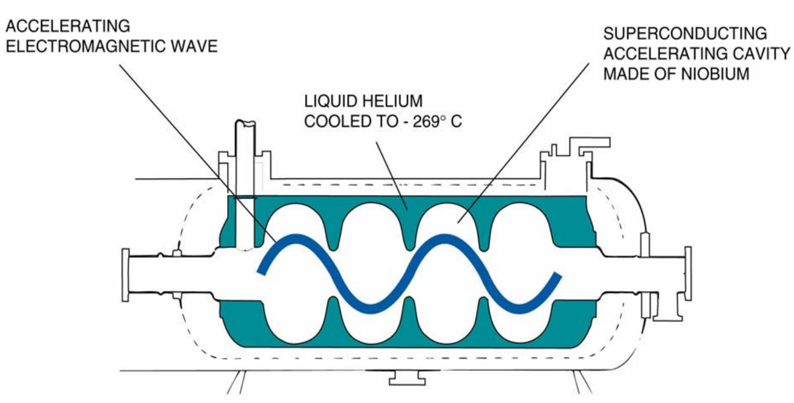
$$\gamma m\frac{u^2}{r} = quB \quad \Rightarrow \gamma mu = qBr \Rightarrow p = qBr$$

Charged Form of Matter & Anti-Matter in a B Field

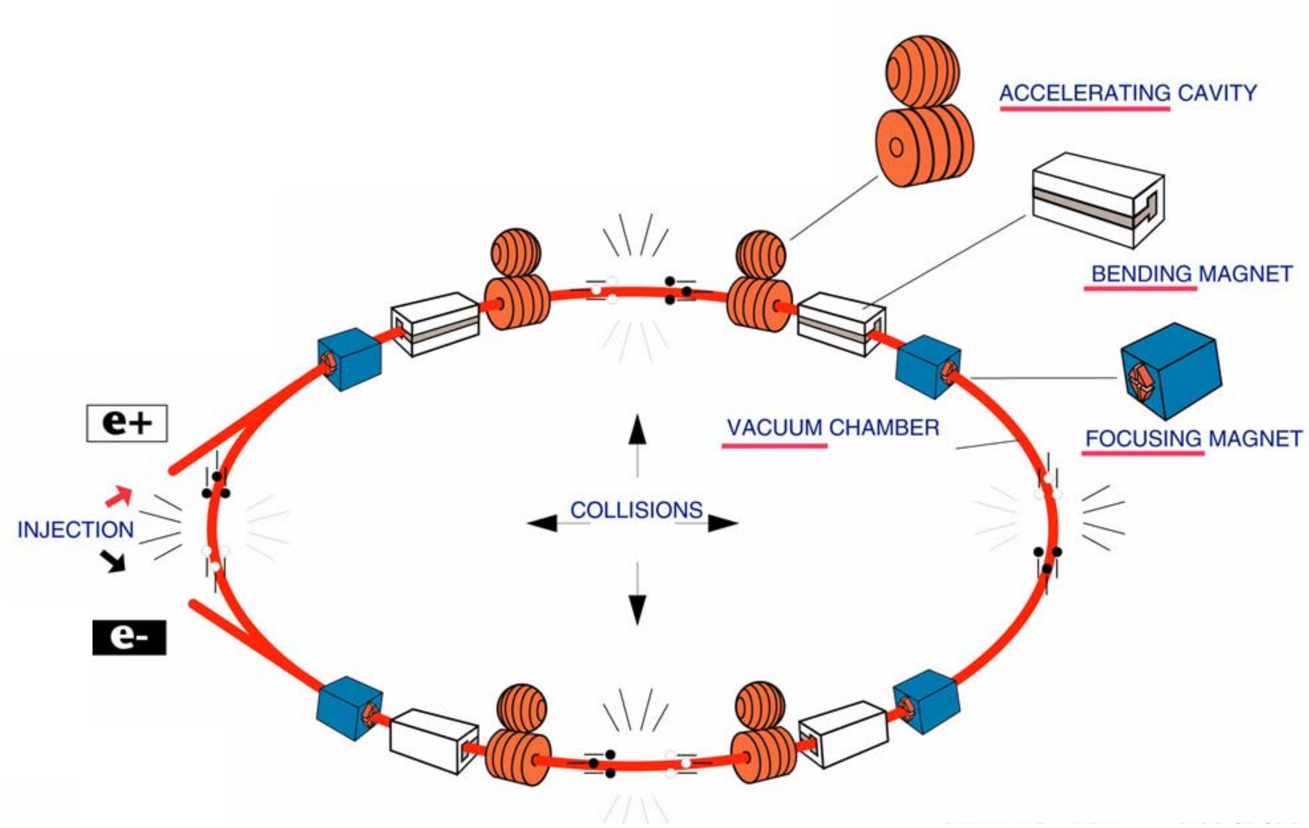


Accelerating Electrons Thru RF Cavities

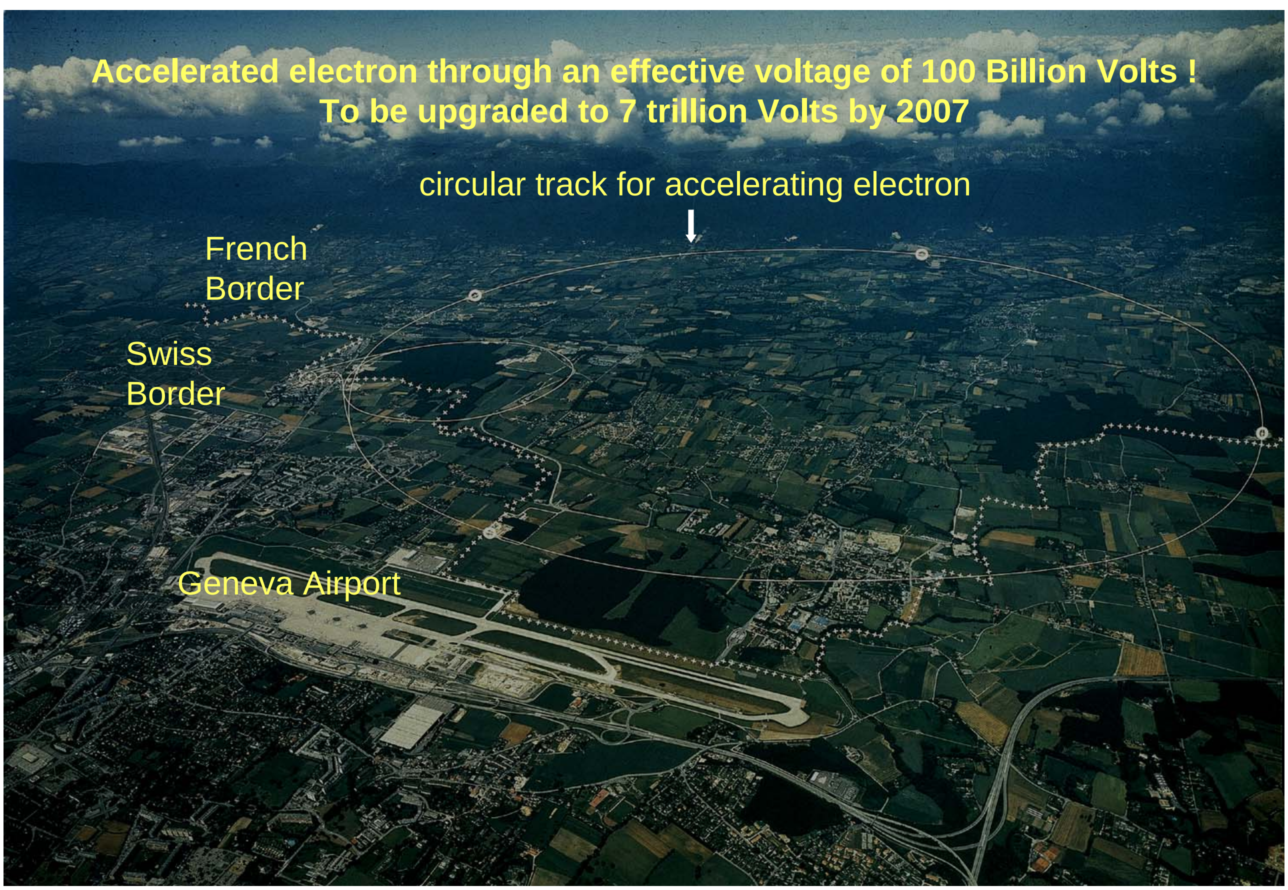




A Circular Accelerator : Using B Field to Confine the electron and RF cavity to power it



Circular Particle Accelerator: LEP @ CERN, Geneve



In Tunnel 150m underground, 27km ring of Magnets Keep electron in Circular Orbit

Inside A Circular Particle Accelerator Tunnel : Monorail !

Test of Relativistic Momentum In Circular Accelerator

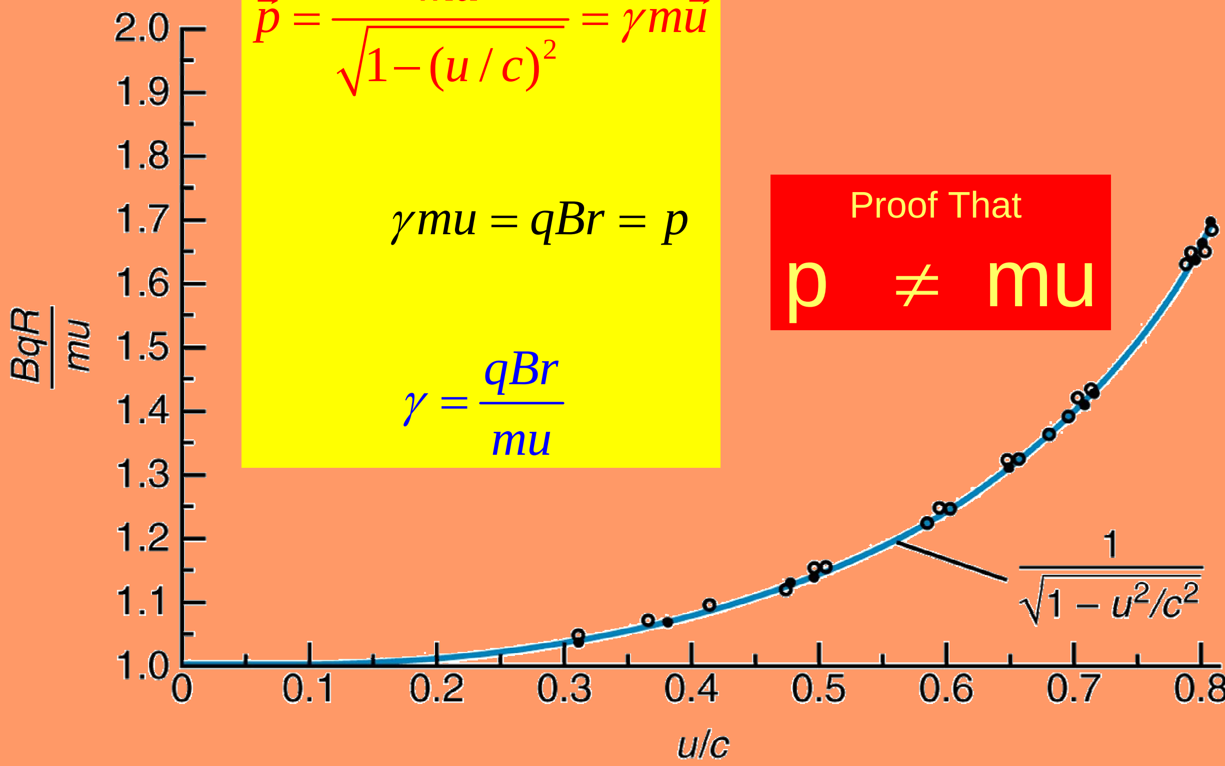
$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

$$\gamma mu = qBr = p$$

$$\gamma = \frac{qBr}{mu}$$

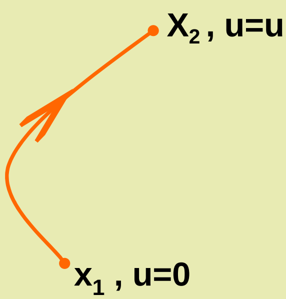
Proof That

p ≠ mu



Relativistic Work Done & Change in Energy

$$W = \int_{x_1}^{x_2} \vec{F}.d\vec{x} = \int_{x_1}^{x_2} \frac{d\vec{p}}{dt}.d\vec{x}$$



$$p = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}} \quad \therefore \quad \frac{d\vec{p}}{dt} = \frac{m\frac{du}{dt}}{\left[1-\frac{u^2}{c^2}\right]^{3/2}},$$ substitute in W,

$$\therefore W = \int_0^u \frac{m\frac{du}{dt}udt}{\left[1-\frac{u^2}{c^2}\right]^{3/2}}$$ (change in var x → u)

$$W = \int_0^u \frac{mudu}{\left[1-\frac{u^2}{c^2}\right]^{3/2}} = \frac{mc^2}{\left[1-\frac{u^2}{c^2}\right]^{1/2}} - mc^2 = \gamma mc^2 - mc^2$$

Work done is change in Kinetic energy K

$$\boxed{K = \gamma mc^2 - mc^2}$$ or

Total Energy $\boxed{E = \gamma mc^2 = K + mc^2}$

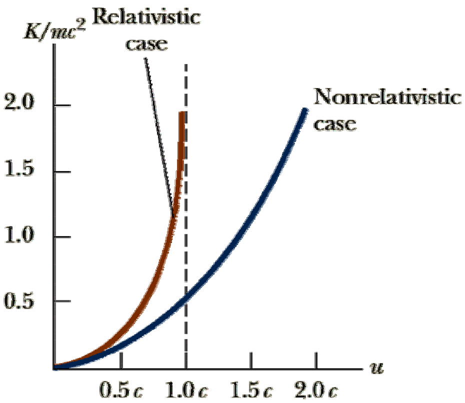
But Professor… Why Can's ANYTHING go faster than light ?

$$K = \frac{mc^2}{\left[1-\frac{u^2}{c^2}\right]^{1/2}} - mc^2 \Rightarrow \left(K+mc^2\right)^2 = \left(\frac{mc^2}{\left[1-\frac{u^2}{c^2}\right]^{1/2}}\right)^2$$

$$\Rightarrow \left[1-\frac{u^2}{c^2}\right] = m^2c^4\left[K+mc^2\right]^{-2}$$

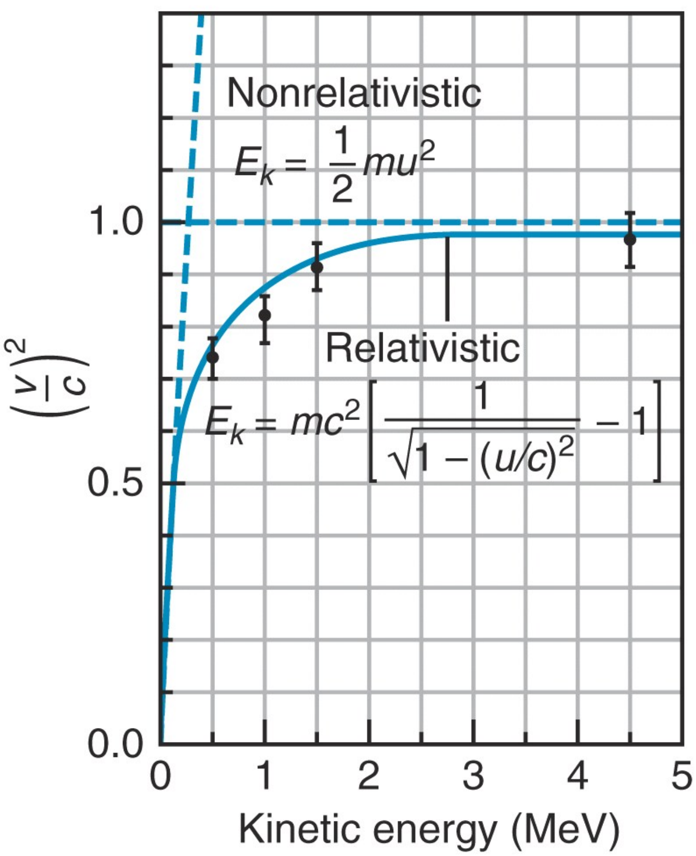
$$\Rightarrow u = c\sqrt{1-(\frac{K}{mc^2}+1)^{-2}}$$ (Parabolic in u Vs $\frac{K}{mc^2}$)

As K $\rightarrow \infty$, u $\rightarrow$ c



Non-relativistic case: $K = \frac{1}{2}mu^2 \Rightarrow u = \sqrt{\frac{2K}{m}}$

Relativistic Kinetic Energy Vs Velocity



A Digression: How to Handle Large/Small Numbers

- Example: consider very energetic particle with very large Energy E

$$\gamma = \frac{E}{mc^2} = \frac{mc^2 + K}{mc^2} = 1 + \frac{K}{mc^2}$$

- Lets Say $\gamma = 3\times10^{11}$, Now calculate u from → $\frac{u}{c} = \left[1 - \frac{1}{\gamma^2}\right]^{1/2}$
- Try this on your el-cheapo calculator, you will get u/c =1, u=c due to limited precision.
- In fact u ≅c but not exactly!, try to get this analytically

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{(1-\beta)(1+\beta)}}$$

Since $\beta = \frac{u}{c} \cong 1, \quad 1+\beta = 2$

$$\gamma \approx \frac{1}{\sqrt{2}\sqrt{1-\beta}}$$

$$\Rightarrow 1-\beta = \frac{1}{2\gamma^2} = 5\times10^{-24}, \quad u = \beta c$$

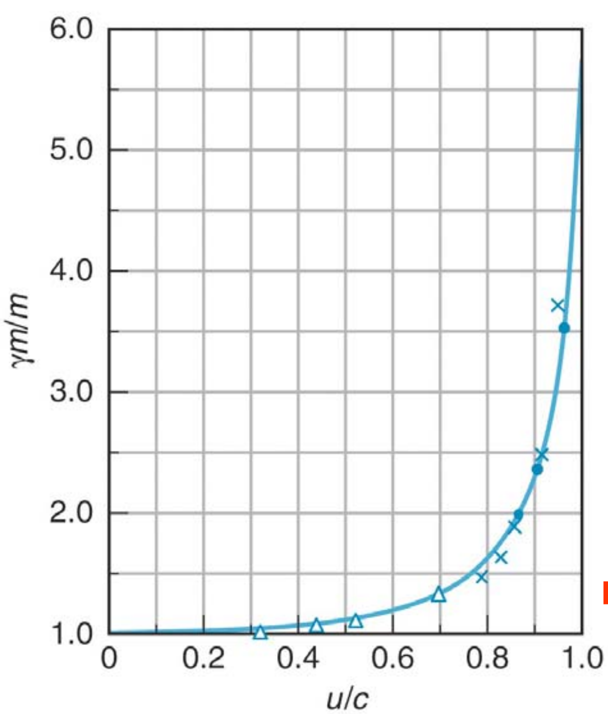
$\Rightarrow u = 0.999\ 999\ 999\ 999\ 999\ 999\ 999\ 995c$!!

Such particles are routinely produced in violent cosmic collisions

In Quizzes, you are Expected to perform Such simple approximations

When Electron Goes Fast it Gets "Fat"

Total Energy $E = \gamma mc^2 = K + mc^2$



$$E = \gamma mc^2$$

As $\frac{v}{c} \to 1, \quad \gamma \to \infty$

Apparent Mass approaches ∞

New Concept
Rest Mass = particle mass when its at rest

Relativistic Kinetic Energy & Newtonian Physics

Relativistic KE $K = \gamma mc^2 - mc^2$

$$\left[\text{Remember Binomial Theorem} \quad \text{for } x \ll 1;\ (1+x)^n = (1+\frac{nx}{1!}+\frac{n(n-1)}{2!}x^2 + \text{smaller terms})\right]$$

$$\therefore \text{ When } u \ll c,\ \left[1-\frac{u^2}{c^2}\right]^{-\frac{1}{2}} \cong 1+\frac{1}{2}\frac{u^2}{c^2}+\ldots\text{smaller terms}$$

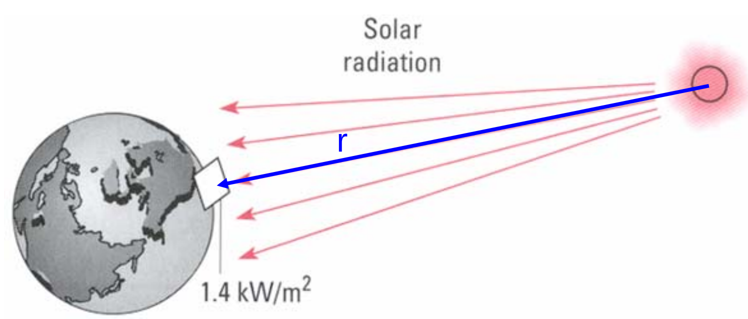
$$\text{so } K \cong mc^2[1+\frac{1}{2}\frac{u^2}{c^2}]-mc^2 = \frac{1}{2}mu^2 \quad \text{(classical form recovered)}$$

Total Energy of a Particle $E = \gamma mc^2 = KE + mc^2$

For a particle at rest, u = 0 $\Rightarrow$ Total Energy $E = mc^2$

$E=mc^2$: Sunshine Won't Be Forever

Q: Solar Energy reaches earth at rate of 1.4kW per square meter of surface perpendicular to the direction of the sun. by how much does the mass of sun decrease per second owing to energy loss? The mean radius of the Earth's orbit is 1.5×10^{11}m.



- Surface area of a sphere of radius r is $A = 4\pi r^2$
- Total Power radiated by Sun = power received by a sphere whose radius is equal to earth's orbit radius

$$P = \frac{P}{A}A = \frac{P}{A}4\pi r^2 = (1.4\times10^3 W/m^2)(4\pi)(1.5\times10^{11})^2$$

$$P = 4.0\times10^{26} W$$

So Sun loses $E=4.0\times10^{26}$ J of rest energy per second

Its mass decreases by $m=\frac{E}{c^2} = \frac{4.0\times10^{26}\text{ J}}{(3.0\times10^8)^2} = 4.4\times10^9 kg$!!

If the Sun's Mass = $2.0 \times 10^{30} kg$ So how long with the Sun last ?

Relationship between P and E

$$E = \gamma mc^2 \Rightarrow E^2 = \gamma^2 m^2 c^4$$

$$p = \gamma mu \Rightarrow p^2c^2 = \gamma^2 m^2 u^2 c^2$$

$$\Rightarrow E^2 - p^2c^2 = \gamma^2 m^2 c^4 - \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^2 (c^2 - u^2)$$

$$= \frac{m^2c^2}{1 - \frac{u^2}{c^2}}(c^2 - u^2) = \frac{m^2c^4}{c^2 - u^2}(c^2 - u^2) = m^2c^4$$

$$E^2 = p^2c^2 + (mc^2)^2$$ ....important relation

For particles with zero rest mass like photon (EM waves)

$E = pc$ or $p = \frac{E}{c}$ (light has momentum!)

Relativistic Invariance : $E^2 - p^2c^2 = m^2c^4$: In all Ref Frames

Rest Mass is a "finger print" of the particle

Mass Can "Morph" into Energy & Vice Verca

- Unlike in Newtonian mechanics
- In relativistic physics : Mass and Energy are the same thing
- New word/concept : MassEnergy , just like SpaceTime
- It is the mass-energy that is always conserved in every reaction : Before & After a reaction has happened
- Like squeezing a balloon : Squeeze here, it grows elsewhere
  - **If you "squeeze" mass, it becomes (kinetic) energy & vice verca !**
    - **CONVERSION FACTOR = C^2**
    - **This exchange rate never changes !**