

Quiz 1 is This Friday

Bring **Blue Book**, check calculator battery



Physics 2D Lecture Slides
Lecture 6: Jan 13th 2004

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Lorentz Transformation Between Ref Frames

Lorentz Transformation

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma \left(t - \frac{vx}{c^2} \right)$$

Inverse Lorentz Transformation

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

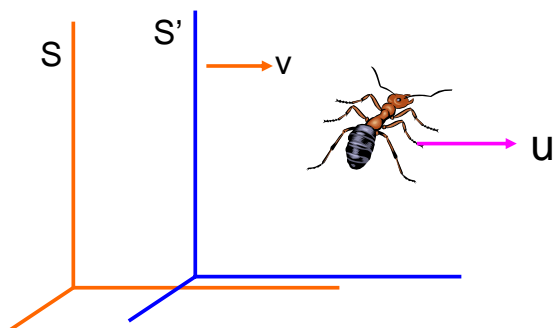
$$t = \gamma \left(t' + \frac{vx'}{c^2} \right)$$

As $v \rightarrow 0$, Galilean Transformation is recovered, as per requirement

Notice : SPACE and TIME Coordinates mixed up !!!

Lorentz Velocity Transformation Rule

S and S' are measuring ant's speed u along x, y, z axes



$$\text{In } S' \text{ frame, } u_{x'} = \frac{x'_2 - x'_1}{t'_2 - t'_1} = \frac{dx'}{dt'}$$

$$dx' = \gamma(dx - vdt), \quad dt' = \gamma \left(dt - \frac{v}{c^2} dx \right)$$

$$u_{x'} = \frac{dx - vdt}{dt - \frac{v}{c^2} dx}, \quad \text{divide by } dt$$

$$u_{x'} = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

For $v \ll c$, $u_{x'} = u_x - v$

(Galilean Trans. Restored)

Velocity Transformation Perpendicular to S-S' motion

$$dy' = dy, \quad dt' = \gamma \left(dt - \frac{v}{c^2} dx \right)$$

$$u'_y = \frac{dy'}{dt'} = \frac{dy}{\gamma \left(dt - \frac{v}{c^2} dx \right)}$$

divide by dt on RHS

$$u'_y = \frac{u_y}{\gamma \left(1 - \frac{v}{c^2} u_x \right)}$$

There is a change in velocity in the direction \perp to S-S' motion !

Similarly

Z component of Ant' s velocity transforms as

$$u'_z = \frac{u_z}{\gamma \left(1 - \frac{v}{c^2} u_x \right)}$$

Inverse Lorentz Velocity Transformation

Inverse Velocity Transform:

$$u_x = \frac{u_{x'} + v}{1 + \frac{v u_{x'}}{c^2}}$$

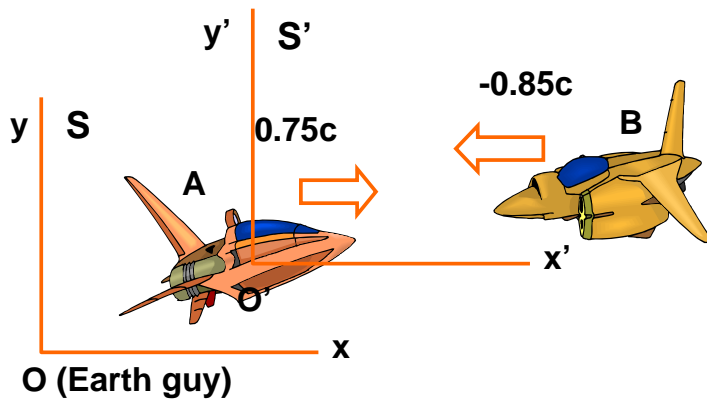
$$u_y = \frac{u'_y}{\gamma \left(1 + \frac{v}{c^2} u'_x \right)}$$

$$u_z = \frac{u'_z}{\gamma \left(1 + \frac{v}{c^2} u'_x \right)}$$

As usual,
replace

$$v \Rightarrow -v$$

Does Lorentz Transform “work” For Topgun ?



Two rockets A & B travel in opposite directions

An observer on earth (S) measures speeds = $0.75c$ And $0.85c$ for A & B respectively

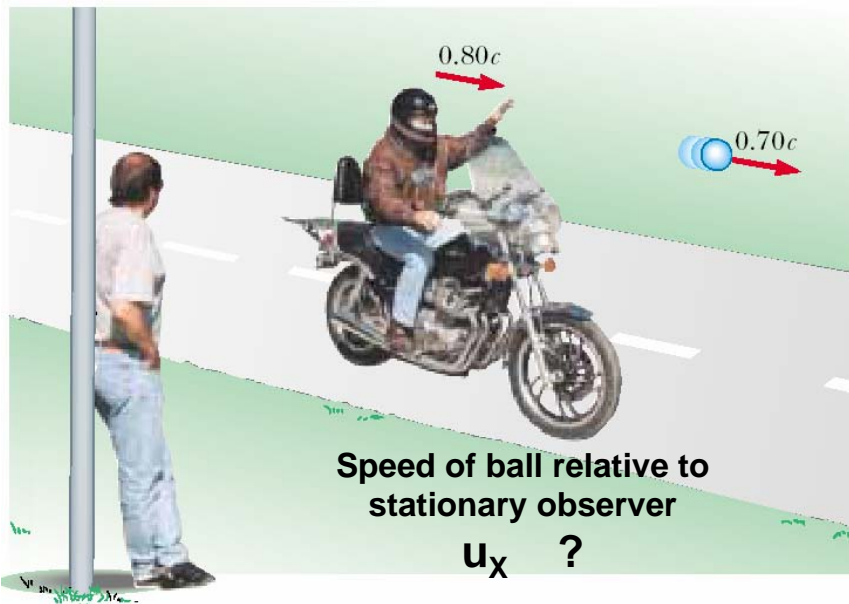
What does A measure as B's speed?

Place an imaginary S' frame on Rocket A $\Rightarrow v = 0.75c$ relative to Earth Observer S

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.977c$$

Consistent with Special Theory of Relativity

Example of Inverse velocity Transform



Biker moves with speed = $0.8c$ past stationary observer

Throws a ball forward with speed = $0.7c$

What does stationary observer see as velocity of ball ?

Place S' frame on biker
Biker sees ball speed

$$u_{x'} = 0.7c$$

$$u_x = \frac{u'_{x'} + v}{1 + \frac{u'_{x'} v}{c^2}} = \frac{0.70c + 0.80c}{1 + \frac{(0.70c)(0.80c)}{c^2}} = 0.96c$$

Hollywood Yarns Of Time Travel !

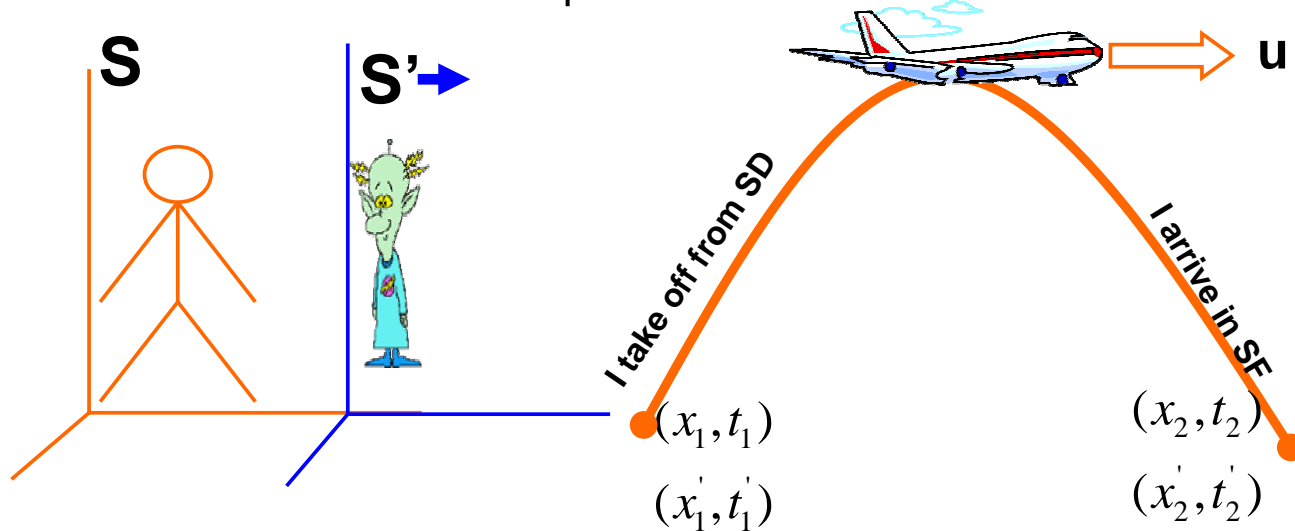


IT'S NOTHING PERSONAL.
TERMINATOR 2
JUDGMENT DAY



Terminator : Can you be **seen** to be born before your mother?

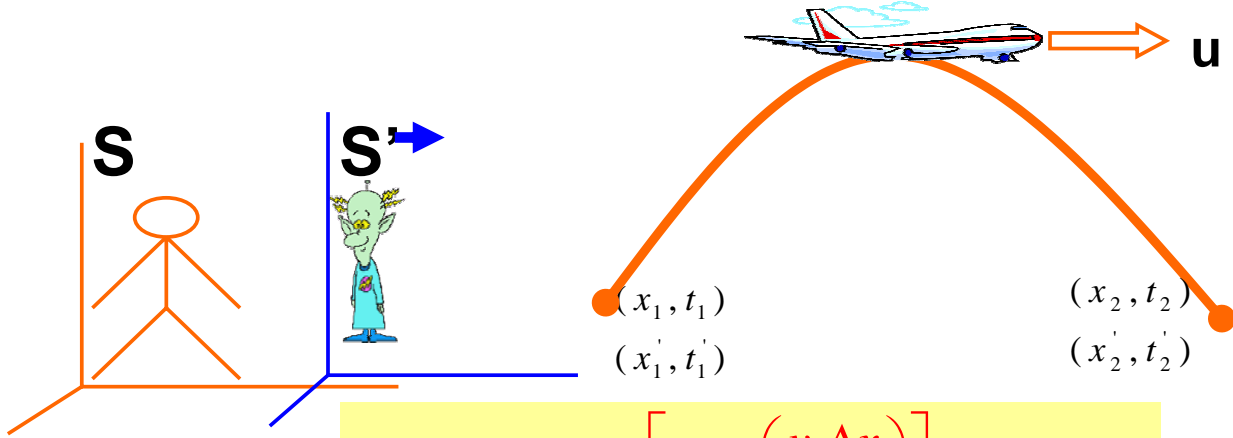
A frame of Ref where sequence of events is REVERSED ?!!



$$\Delta t' = t'_2 - t'_1 = \gamma \left[\Delta t - \left(\frac{v \Delta x}{c^2} \right) \right]$$

Reversing sequence of events $\Rightarrow \Delta t' < 0$

I Cant 'be seen to arrive in SF before I take off from SD



$$\Delta t' = t'_2 - t'_1 = \gamma \left[\Delta t - \left(\frac{v \Delta x}{c^2} \right) \right]$$

For what value of v can $\Delta t' < 0$

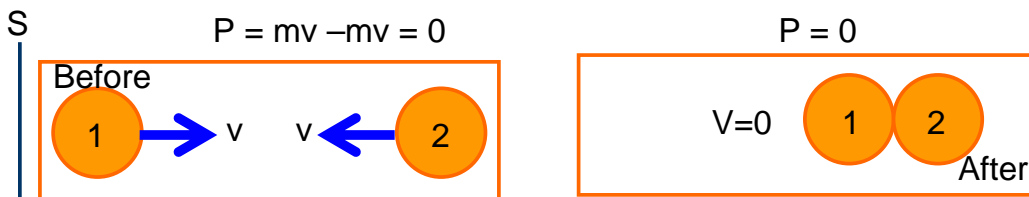
$$\Delta t' < 0 \Rightarrow \Delta t < \frac{v \Delta x}{c^2} \Rightarrow 1 < \frac{v \Delta x}{c^2 \Delta t} = \frac{v u}{c^2}$$

$$\Rightarrow \frac{v}{c} > \frac{c}{u} \Rightarrow v > c : \text{Not allowed}$$

Relativistic Momentum and Revised Newton's Laws

Need to generalize the laws of Mechanics & Newton to confirm to Lorentz Transform and the Special theory of relativity: Example : $\vec{p} = m\vec{u}$

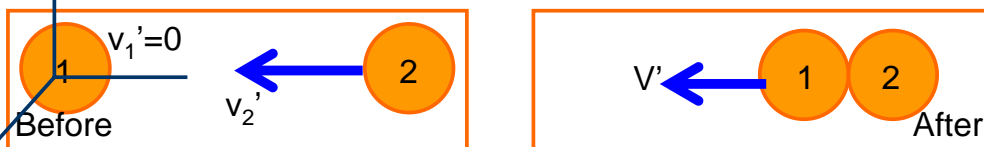
Watching an Inelastic Collision between two putty balls



$$v'_1 = \frac{v_1 - v}{1 - \frac{v_1 v}{c^2}} = 0, \quad v'_2 = \frac{v_2 - v}{1 - \frac{v_1 v}{c^2}} = \frac{-2v}{1 + \frac{v^2}{c^2}}, \quad V' = \frac{V - v}{1 - \frac{Vv}{c^2}} = -v$$

$$p'_{\text{before}} = mv'_1 + mv'_2 = \frac{-2mv}{1 + \frac{v^2}{c^2}}, \quad p'_{\text{after}} = 2mV' = -2mv$$

$$p'_{\text{before}} \neq p'_{\text{after}}$$



Definition (without proof) of Relativistic Momentum

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

With the new definition relativistic momentum is conserved in all frames of references : Do the exercise

New Concepts

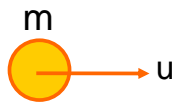
Rest mass = mass of object measured
In a frame of ref. where object is at rest

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}}$$

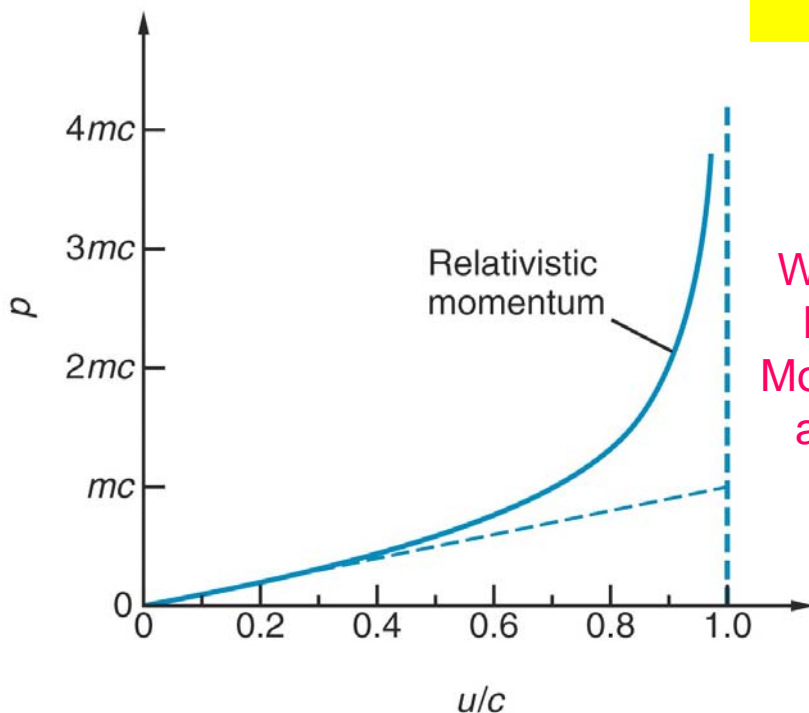
u is velocity of the object

NOT of a reference frame !

Nature of Relativistic Momentum



$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$



With the new definition of
Relativistic momentum
Momentum is conserved in
all frames of references

Relativistic Force & Acceleration

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

Relativistic Force And Acceleration

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt} \left(\frac{m\vec{u}}{\sqrt{1-(u/c)^2}} \right) \quad \text{use} \quad \frac{d}{dt} = \frac{du}{dt} \frac{d}{du}$$

$$F = \left[\frac{m}{\sqrt{1-(u/c)^2}} + \frac{mu}{(1-(u/c)^2)^{3/2}} \times \left(\frac{-1}{2} \right) \left(\frac{-2u}{c^2} \right) \right] \frac{du}{dt}$$

$$F = \left[\frac{mc^2 - mu^2 + mu^2}{c^2 (1-(u/c)^2)^{3/2}} \right] \frac{du}{dt}$$

$$F = \left[\frac{m}{(1-(u/c)^2)^{3/2}} \right] \frac{du}{dt} \quad \text{: Relativistic Force}$$

Since Acceleration $\vec{a} = \frac{d\vec{u}}{dt}$, [rate of change of velocity]

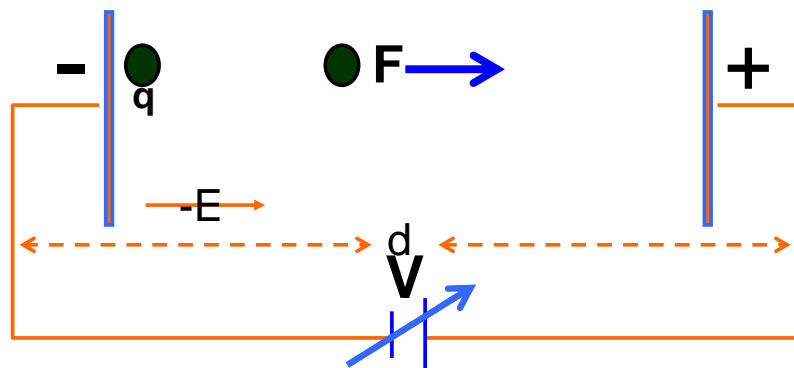
$$\Rightarrow \vec{a} = \frac{\vec{F}}{m} [1-(u/c)^2]^{3/2}$$

Reason why you cant quite get up to the speed of light no matter how hard you try!



Note: As $u/c \rightarrow 1$, $\vec{a} \rightarrow 0$!!!!
Its harder to accelerate when you get closer to speed of light

A Linear Particle Accelerator



Parallel Plates

$$E = V/d$$

$$F = eE$$

Charged particle q moves in straight line in a uniform electric field \vec{E} with speed \vec{u} accelerates under force $\vec{F} = q\vec{E}$

$$|\vec{a}| = \left| \frac{d\vec{u}}{dt} \right| = \left| \frac{\vec{F}}{m} \right| \left(1 - \frac{u^2}{c^2} \right)^{3/2} = \left| \frac{q\vec{E}}{m} \right| \left(1 - \frac{u^2}{c^2} \right)^{3/2}$$

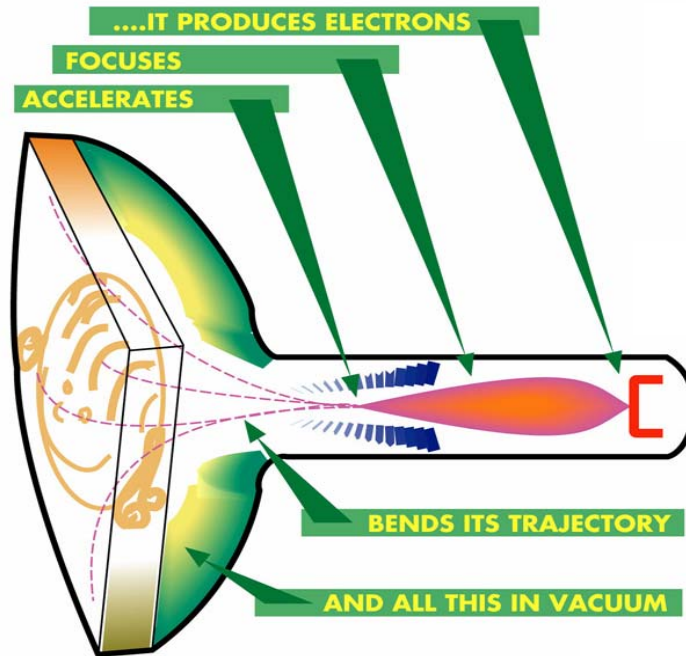
larger the potential difference V across plates, larger the force on particle

Under force, work is done on the particle, it gains Kinetic energy

New Unit of Energy

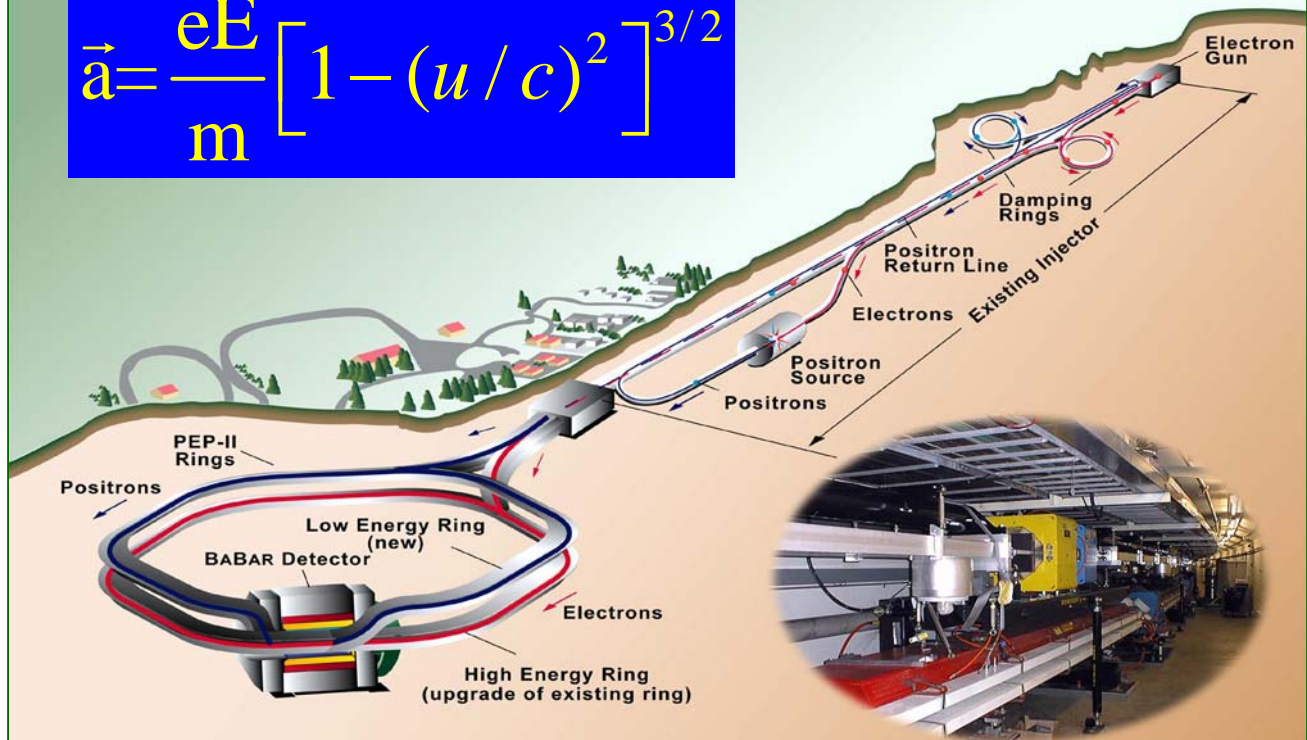
- 1 eV = 1.6×10^{-19} Joules
- 1 MeV = 1.6×10^{-13} Joules
- 1 GeV = 1.6×10^{-10} Joules

Your Television (the CRT type) is a Small Particle Accelerator !



Linear Particle Accelerator : 50 GigaVolts Accelerating Potential

$$\vec{a} = \frac{e\vec{E}}{m} \left[1 - (u/c)^2 \right]^{3/2}$$



Both Rings Housed in Current PEP Tunnel