The Final Exam is on Mar 18th , Time and Location TBA

NOT on Monday Mar 15th as previously announced in the Handout etc!!

Pl. make a note of this change !!

This date change is also posted in the ANNOUCEMENT section of class web page

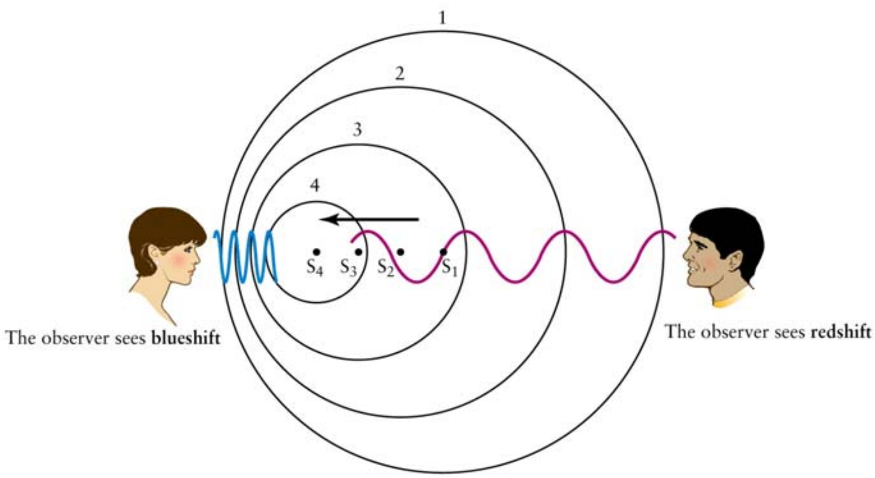
Pl. make it a point to regularly check the announcements section of class web page

Physics 2D Lecture Slides
Lecture 5: Jan 12th 2004

Vivek Sharma
UCSD Physics

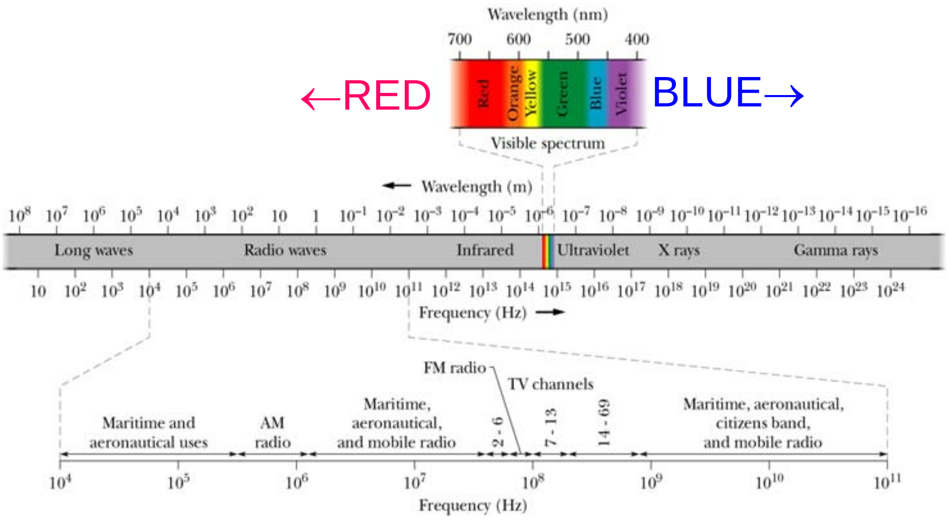
Relativistic Doppler Shift

$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}}\, f_{source}$$



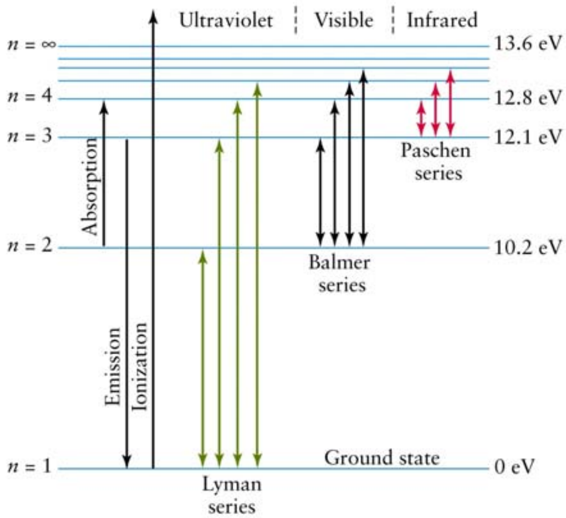
Doppler Shift & Electromagnetic Spectrum

←RED BLUE→



Fingerprint of Elements: Emission & Absorption Spectra

Example : The Atomic Energy levels of Hydrogen



Doppler Shift in Spectral Lines and Motion of Stellar Objects

Laboratory Spectrum, lines at rest wavelengths

Lines Redshifted, Object moving away from me

Larger Redshift, object moving away even faster

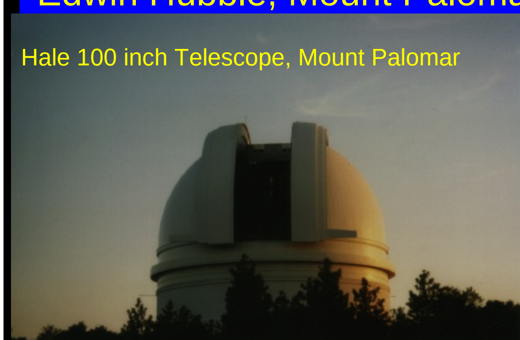
Lines blueshifted, Object moving towards me

Larger blueshift, object approaching me faster

Seeing Distant Galaxies Through Hubble Telescope

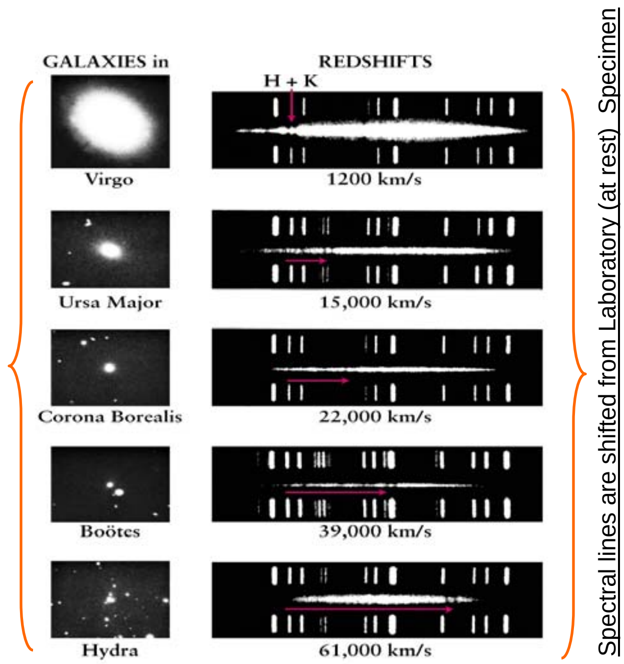
Through center of a massive galaxy clusters Abell 1689

Edwin Hubble, Mount Palomar & Expanding Universe



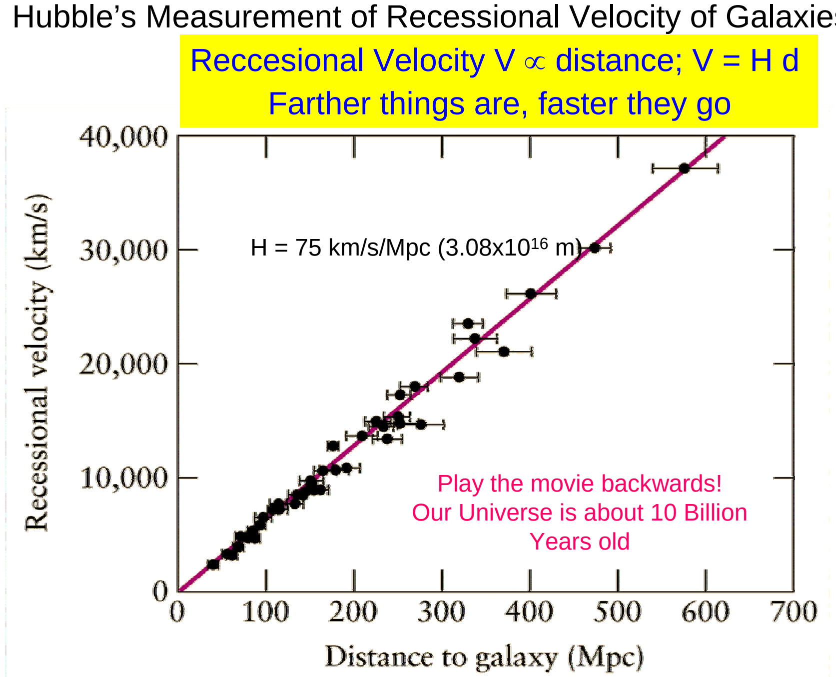
Hale 100 inch Telescope, Mount Palomar

Edwin Hubble 1920

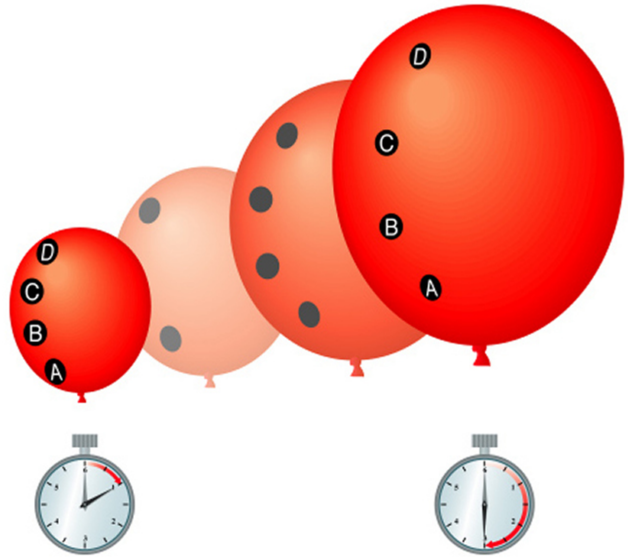


Galaxies at different locations in Universe moving away at different velocities

Hubble's Measurement of Recessional Velocity of Galaxies



Cosmological Redshift & Discovery of the Expanding Universe:
[Space itself is Expanding]



New Rules of Coordinate Transformation Needed

- The Galilean/Newtonian rules of transformation could not handles frames of refs or objects traveling fast
  - V ≈ C (like v = 0.1 c or 0.8c or 1.0c)
- Einstein's postulates led to
  - Destruction of concept of simultaneity ($\Delta t \neq \Delta t'$)
  - Moving clocks run slower
  - Moving rods shrink
- Lets formalize this in terms of <u>general rules</u> of coordinate transformation : Lorentz Transformation
  - Recall the Galilean transformation rules
    - $x' = (x-vt)$
    - $t' = t$
      - These rules that work ok for ferraris now must be modified for rocket ships with v ≈ c

Discovering The Correct Transformation Rule

$$x' = x - vt \qquad \text{guess} \rightarrow \quad x' = G(x - vt)$$

$$x = x' + vt' \qquad \text{guess} \rightarrow \quad x = G(x' + vt')$$

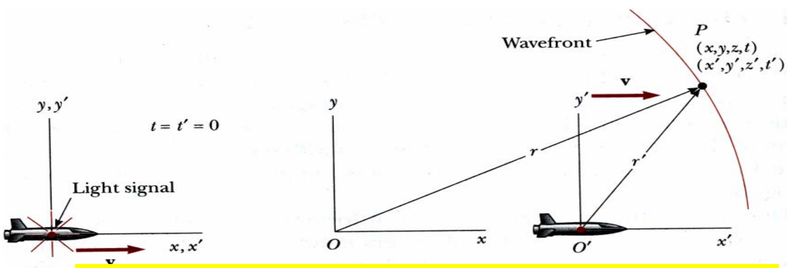
Need to figure out the functional form of G !0

- G must be dimensionless
- G does not depend on x,y,z,t
- But G depends on v/c
- G must be symmetric in velocity v
- As $v/c \rightarrow 0$, $G \rightarrow 1$

Guessing The Lorentz Transformation

Do a Thought Experiment : Watch Rocket Moving along x axis

Rocket in S' (x',y',z',t') frame moving with velocity v w.r.t observer on frame S (x,y,z,t)
Flashbulb mounted on rocket emits pulse of light at the instant origins of S,S' coincide
That instant corresponds to t = t' = 0 . Light travels as a spherical wave, origin is at O,O'



Speed of light is c for both observers: Postulate of SR

Examine a point P (at distance r from O and r' from O') on the Spherical Wavefront

The distance to point P from O : r = ct
The distance to point P from O : r' = ct'

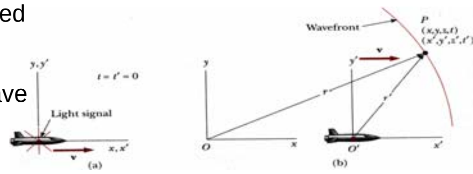
Clearly t and t' must be different
$t \neq t'$

Discovering Lorentz Transfromation for (x,y,z,t)

Motion is along x-x' axis, so y, z unchanged

y'=y,   z' = z

Examine points x or x' where spherical wave crosses the horizontal axes: x = r , x' =r'



$$x = ct = G(x'+vt')$$
$$x' = ct' = G(x - vt),$$
$$\Rightarrow t' = \frac{G}{c}(x - vt)$$
$$\therefore x = ct = G(ct'+vt')$$
$$\therefore ct = G^2\left[(ct - vt) + vt - \frac{v^2}{c}t\right]$$
$$\Rightarrow c^2 = G^2[c^2 - v^2]$$
$$\text{or } G = \frac{1}{\sqrt{1-(v/c)^2}} = \gamma$$
$$\therefore \quad x' = \gamma(x - vt)$$

$$x' = \gamma(x - vt), \quad x = \gamma(x'+vt')$$
$$\Rightarrow \quad x = \gamma(\gamma(x - vt) + vt')$$
$$\therefore \quad x - \gamma^2 x + \gamma^2 vt = \gamma vt'$$
$$\therefore \quad t' = \left[\frac{x}{\gamma v} - \frac{\gamma^2 x}{\gamma v} + \frac{\gamma^2 vt}{\gamma v}\right] = \gamma\left[\frac{x}{\gamma^2 v} - \frac{x}{v} + t\right]$$
$$\therefore \quad t' = \gamma\left[t + \frac{x}{v}\left(\frac{1}{\gamma^2} - 1\right)\right], \text{ since } \left(\frac{1}{\gamma^2} - 1\right) = -\left(\frac{v}{c}\right)^2$$
$$\Rightarrow t' = \gamma\left[t + \frac{x}{v}\left[1 - \left(\frac{v}{c}\right)^2 - 1\right]\right] = \gamma\left[t - \left(\frac{vx}{c^2}\right)\right]$$

Lorentz Transformation Between Ref Frames

Lorentz Transformation	Inverse Lorentz Transformation
$x' = \gamma(x - vt)$	$x = \gamma(x'+vt')$
$y' = y$	$y = y'$
$z' = z$	$z = z'$
$t' = \gamma\left(t - \frac{vx}{c^2}\right)$	$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

As v→0 , Galilean Transformation is recovered, as per requirement

Notice : SPACE and TIME Coordinates mixed up !!!

Not just Space, Not just Time

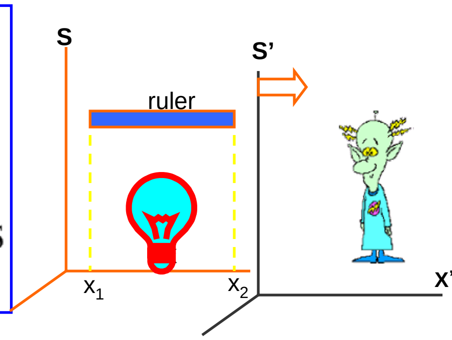
New Word, new concept !

SPACETIME

Lorentz Transform for Pair of Events

$$\left.\begin{aligned}\Delta x' &= \gamma(\Delta x - v\,\Delta t)\\ \Delta t' &= \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)\end{aligned}\right\} S \rightarrow S'$$

$$\left.\begin{aligned}\Delta x &= \gamma(\Delta x' + v\,\Delta t')\\ \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)\end{aligned}\right\} S' \rightarrow S$$



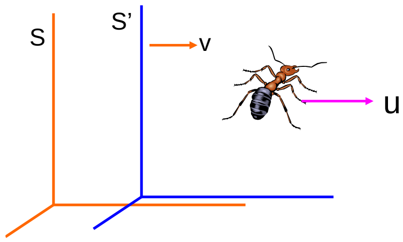
Can understand Simultaneity, Length contraction & Time dilation formulae from this

Time dilation: Bulb in S frame turned on at t_1 & off at t_2 : What **Δt'** did S' measure ?
two events occur at same place in S frame => $\Delta x = 0$
Δt' = γ Δt (Δt = proper time)

Length Contraction: Ruler measured in S between x_1 & x_2 : What **Δx'** did S' measure ?
two ends measured at same time in S' frame => $\Delta t' = 0$
Δx = γ (Δx' + 0) => Δx' = Δx / γ (Δx = proper length)

Lorentz Velocity Transformation Rule

S and S' are measuring ant's speed u along x, y, z axes



In S' frame, $u_{x'} = \frac{x_2' - x_1'}{t_2' - t_1'} = \frac{dx'}{dt'}$

$dx' = \gamma(dx - vdt),\ dt' = \gamma(dt - \frac{v}{c^2}dx)$

$u_{x'} = \frac{dx - vdt}{dt - \frac{v}{c^2}dx}$, divide by dt'

$u_{x'} = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$

For v << c, $u_{x'} = u_x - v$

(Galilean Trans. Restored)

Velocity Transformation Perpendicular to S-S' motion

$dy' = dy, \quad dt' = \gamma(dt - \frac{v}{c^2}dx)$

$u_y' = \frac{dy'}{dy'} = \frac{dy}{\gamma(dt - \frac{v}{c^2}dx)}$

divide by dt on RHS

$u_y' = \frac{u_y}{\gamma(1 - \frac{v}{c^2}u_x)}$

There is a change in velocity in the direction $\perp$ to S-S' motion !

Similarly Z component of Ant's velocity transforms as

$u_z' = \frac{u_z}{\gamma(1 - \frac{v}{c^2}u_x)}$

Inverse Lorentz Velocity Transformation

Inverse Velocity Transform:

$$u_x = \frac{u_{x'} + v}{1 + \frac{v u_{x'}}{c^2}}$$

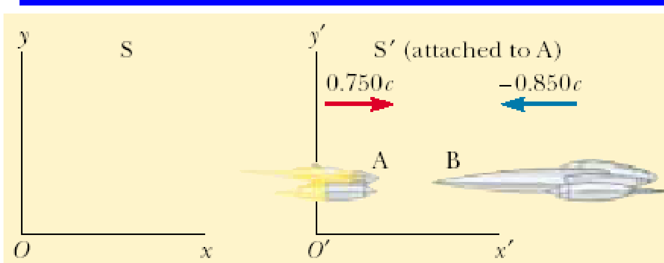
$$u_y = \frac{u_y'}{\gamma(1 + \frac{v}{c^2} u_x')}$$

$$u_z = \frac{u_z'}{\gamma(1 + \frac{v}{c^2} u_x')}$$

As usual, replace

v ⇒ - v

Does Lorentz Transform "work" ?



Two rockets travel in opposite directions

An observer on earth (S) measures speeds = 0.75c And 0.85c for A & B respectively

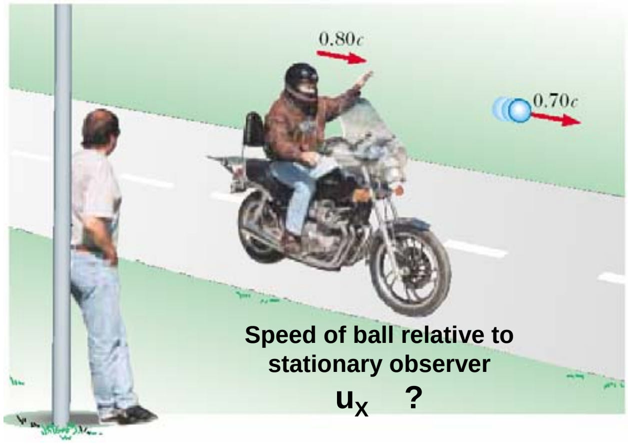
What does A measure as B's speed?

Place an imaginary S' frame on Rocket A ⇒ v = 0.75c relative to Earth Observer S

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.977c$$

Consistent with Special Theory of Relativity

Example of Inverse velocity Transform



Biker moves with speed = 0.8c past stationary observer

Throws a ball forward with speed = 0.7c

What does stationary observer see as velocity of ball ?

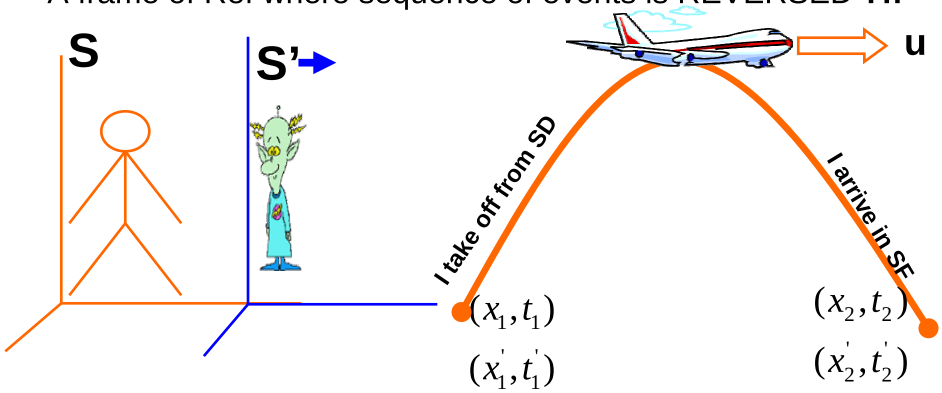
Place S' frame on biker
Biker sees ball speed
$u_{X'}$ =0.7c

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} = \frac{0.70c + 0.80c}{1 + \frac{(0.70c)(0.80c)}{c^2}} = 0.96c$$

Hollywood Yarns !

Terminator : Can you be seen to be born before your mother?

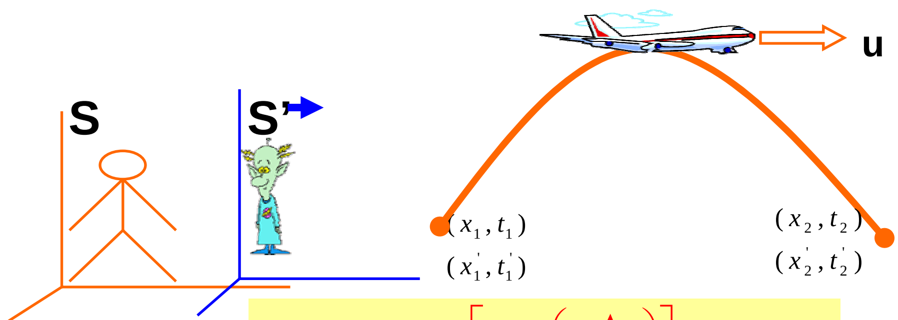
A frame of Ref where sequence of events is REVERSED **?!!**



$$\Delta t' = t_2' - t_1' = \gamma\left[\Delta t - \left(\frac{v\,\Delta x}{c^2}\right)\right]$$

Reversing sequence of events $\Rightarrow \Delta t' < 0$

I Cant 'be seen to arrive in SF before I take off from SD

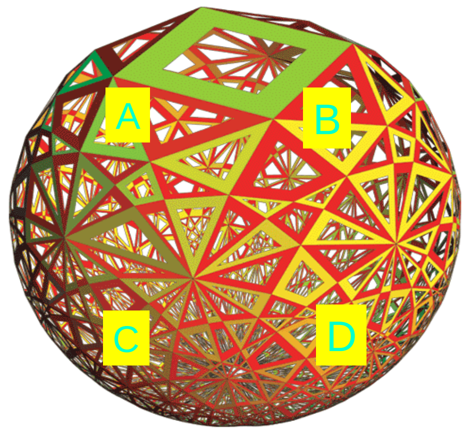


$$\Delta t' = t_2' - t_1' = \gamma\left[\Delta t - \left(\frac{v\,\Delta x}{c^2}\right)\right]$$

For what value of v can $\Delta t' < 0$

$$\Delta t' < 0 \Rightarrow \Delta t < \frac{v\,\Delta x}{c^2} \quad \Rightarrow 1 < \frac{v\,\Delta x}{c^2 \Delta t} = \frac{v\,\mathrm{u}}{c^2}$$

$$\Rightarrow \frac{\mathrm{v}}{\mathrm{c}} > \frac{c}{u} \qquad \Rightarrow v > c : \text{Not allowed}$$


A
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