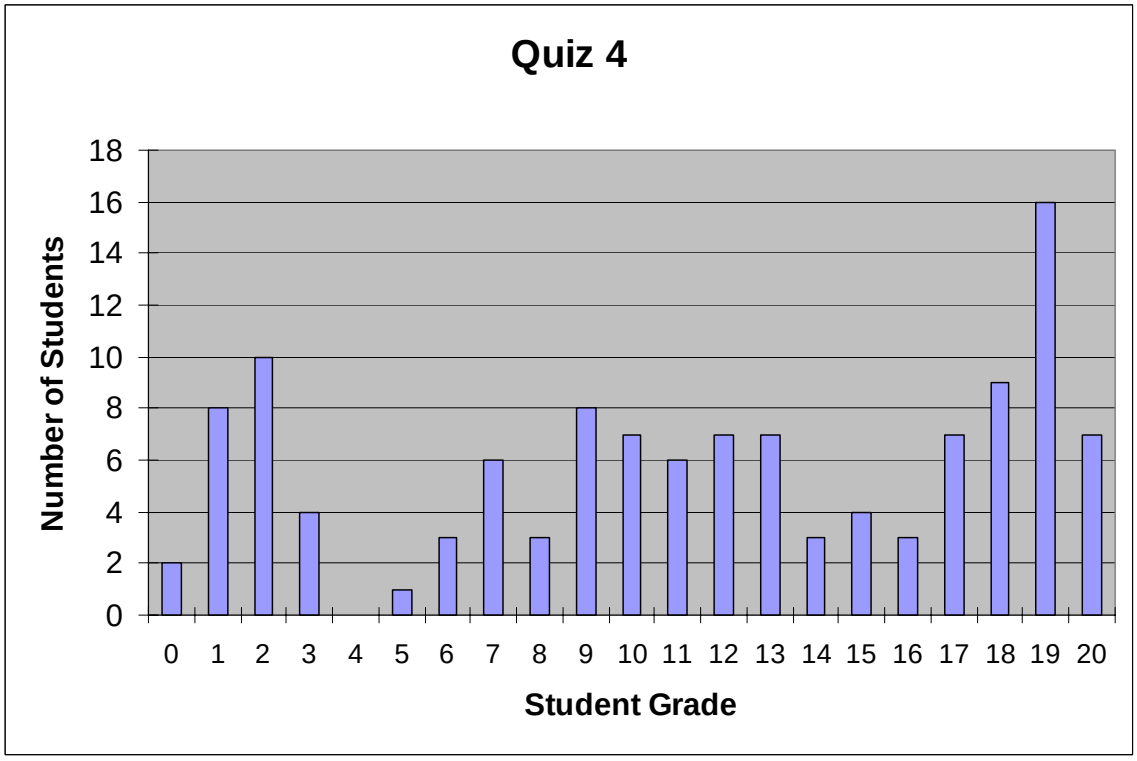
Physics 2D Lecture Slides
Lecture 16: Feb 9th

Vivek Sharma
UCSD Physics



Bohr's Explanation of Hydrogen like atoms

- Bohr's Semiclassical theory explained some spectroscopic data → Nobel Prize : 1922
- The "hotch-potch" of clasical & quantum attributes left many (Einstein) unconvinced
  - "appeared to me to be a miracle – and appears to me to be a miracle today ..... One ought to be ashamed of the successes of the theory"
- Problems with Bohr's theory:
  - Failed to predict INTENSITY of spectral lines
  - Limited success in predicting spectra of Multi-electron atoms (He)
  - Failed to provide "time evolution " of system from some initial state
  - Overemphasized Particle nature of matter-could not explain the wave-particle duality of light
  - No general scheme applicable to non-periodic motion in subatomic systems
- "Condemned" as a one trick pony ! Without fundamental insight …raised the question : Why was Bohr successful?

Prince Louise de Broglie & Matter Waves

- Key to Bohr atom was Angular momentum quantization
- Why this Quantization: $mvr = |L| = nh/2\pi$?
- Invoking symmetry in nature, Louise de Broglie (Da Prince of France !) conjectured:

Because photons have wave and particle like nature → particles may have wave like properties !!

Electrons have accompanying "pilot" wave (not EM) which guide particles thru spacetime

A PhD Thesis Fit For a Prince

- Matter Wave !
  - **"Pilot wave" of $\lambda = h/p = h / (\gamma mv)$**
  - **frequency $f = E/h$**
- **Consequence:**
  - **If matter has wave like properties then there would be interference (destructive & constructive)**
    - **Use analogy of standing waves on a plucked string to explain the quantization condition of Bohr orbits**

Matter Waves : How big, how small

1.Wavelength of baseball, m=140g, v=27m/s

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\,J.s}{(.14kg)(27m/s)} = 1.75\times10^{-34}\,m$$

$\Rightarrow$ $\lambda_{baseball}$ <<< size of nucleus

$\Rightarrow$ Baseball "looks" like a particle

2. Wavelength of electron K=120eV (assume NR)

$$K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$$

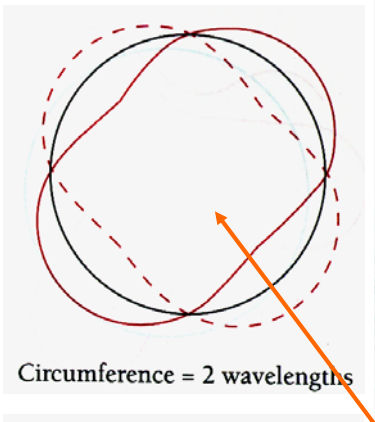
$$= \sqrt{2(9.11\times10^{-31})(120eV)(1.6\times10^{-19})}$$

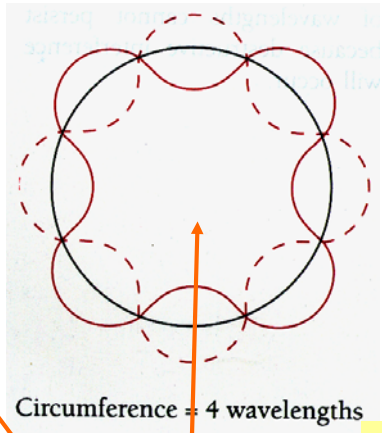
$$= 5.91\times 10^{-24}\,Kg.m/s$$

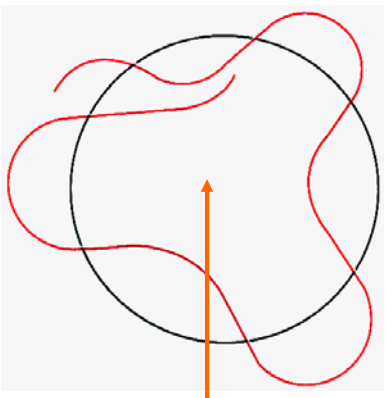
$$\lambda_e = \frac{h}{p} = \frac{6.63\times10^{-34}\,J.s}{5.91\times10^{-24}\,kg.m/s} = 1.12\times10^{-10}\,m$$

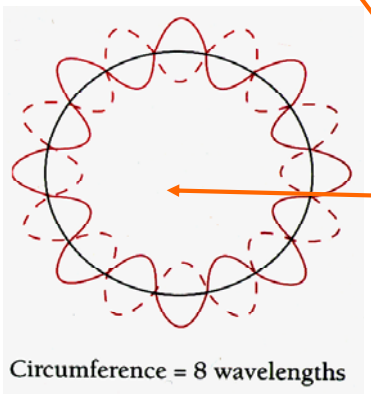
$\Rightarrow$ $\lambda_e \simeq$ Size of atom !!

Models of Vibrations on a Loop: Model of e in atom









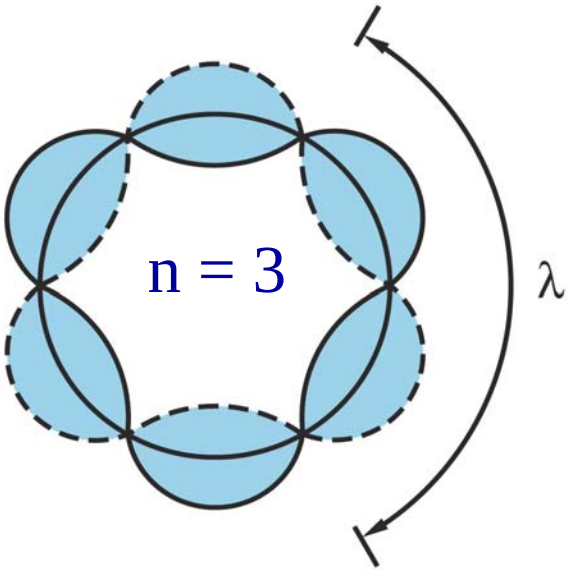
Modes of vibration when a integral # of λ fit into loop

(Standing waves)

vibrations continue Indefinitely

Fractional # of waves in a loop can not persist due to destructive interference

De Broglie's Explanation of Bohr's Quantization



Standing waves in H atom:

Constructive interference when

$n\lambda = 2\pi r$

since $\lambda = \frac{h}{p} = \frac{h}{mv}$ .....(*NR*)

$$\Rightarrow \frac{nh}{mv} = 2\pi r$$

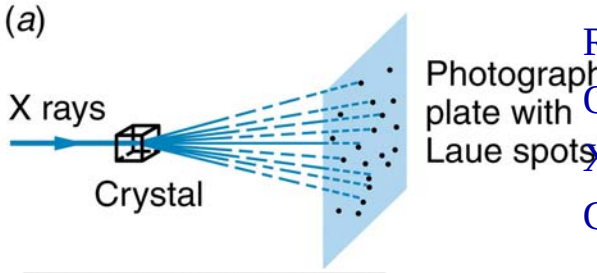
$$\Rightarrow \boxed{n\hbar = mvr}$$

Angular momentum

Quantization condition!

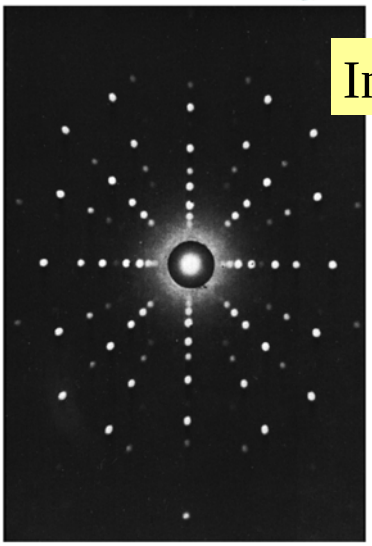
This is too intense ! Must verify such "loony tunes" with experiment

Reminder: Light as a Wave : Bragg Scattering Expt

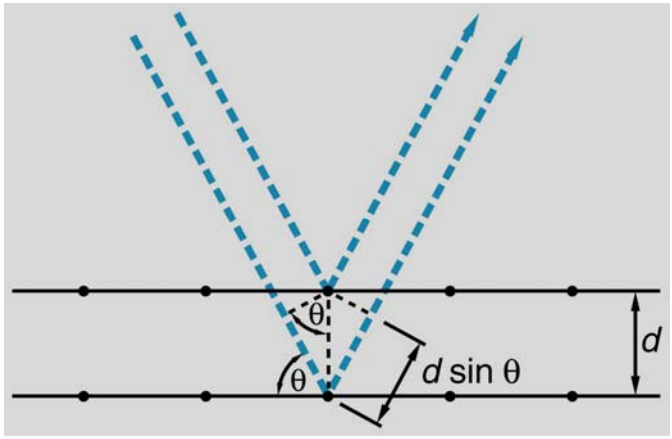


Range of X-ray wavelengths scatter Off a crystal sample

X-rays constructively interfere from Certain planes producing bright spots



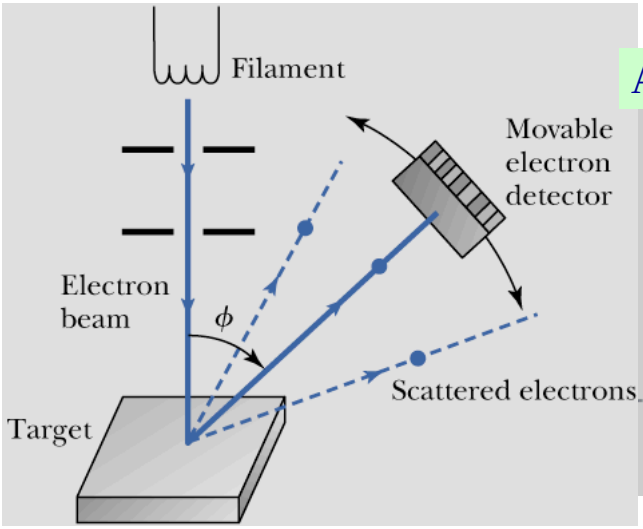
Interference → Path diff = $2d\sin\vartheta = n\lambda$



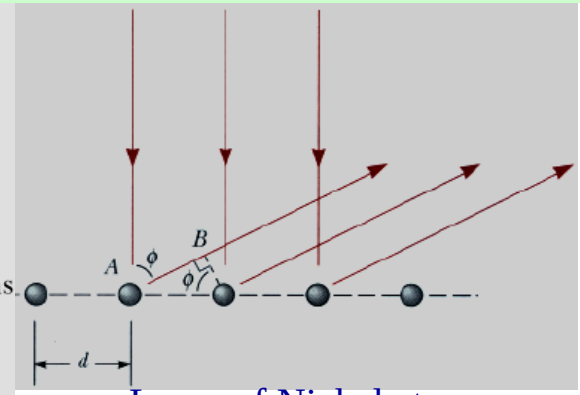
Verification of Matter Waves: Davisson & Germer Expt

If electrons have associated wave like properties → expect interference pattern when incident on a layer of atoms (reflection diffraction grating) with inter-atomic separation d such that

path diff AB= $d\sin\vartheta = n\lambda$

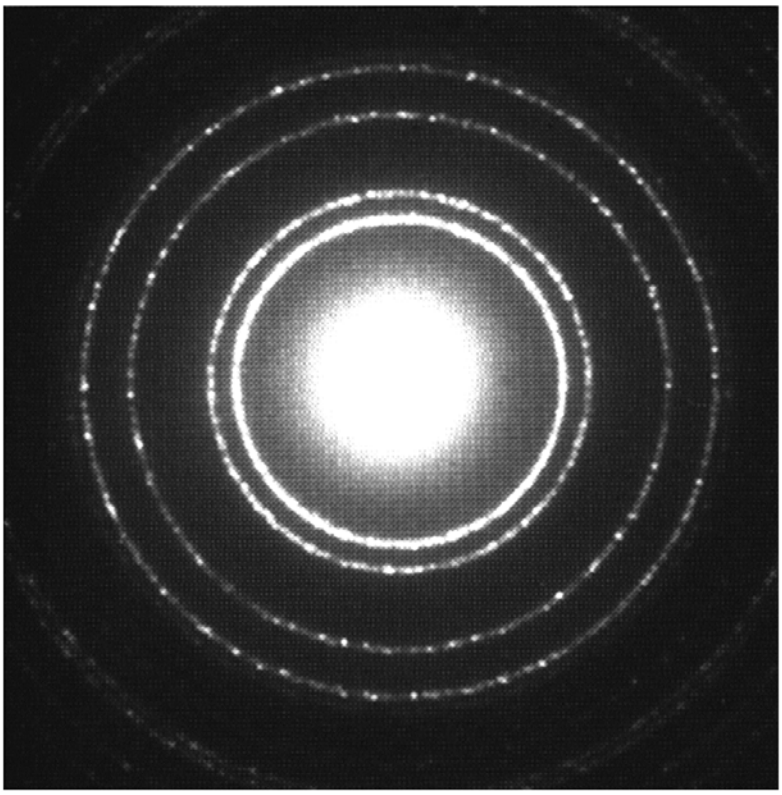


Atomic lattice as diffraction grating



Layer of Nickel atoms

Electrons Diffract in Crystal, just like X-rays

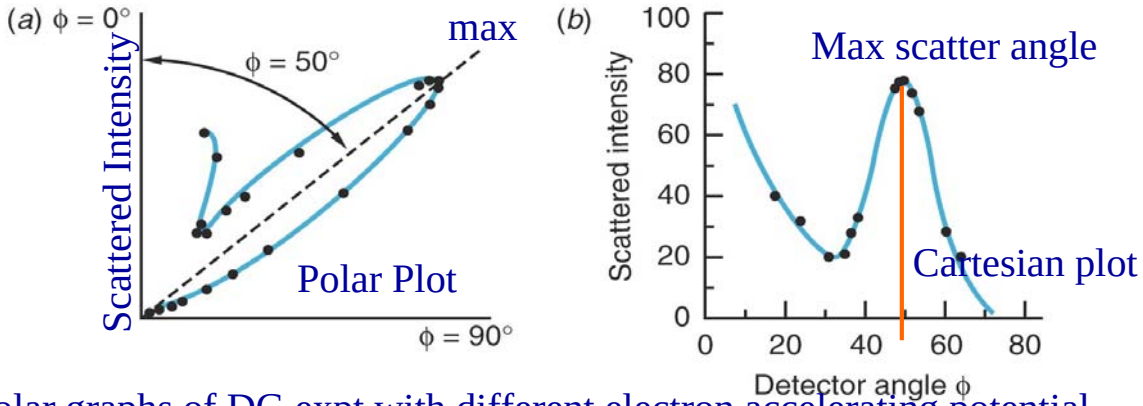


Diffraction pattern produced by 600eV electrons incident on a Al foil target

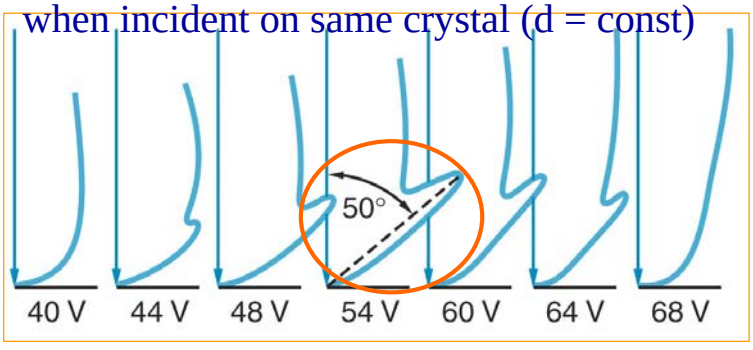
Notice the waxing and waning of scattered electron Intensity.

What to expect if electron had no wave like attribute

Davisson-Germer Experiment: 54 eV electron Beam



Polar graphs of DG expt with different electron accelerating potential when incident on same crystal (d = const)



Peak at $\Phi=50^{o}$ when $V_{acc} = 54$ V

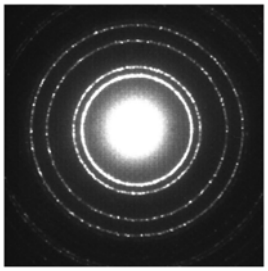
Analyzing Davisson-Germer Expt with de Broglie idea

de Broglie λ for electron accelerated thru V_{acc} =54V

- $\frac{1}{2}mv^2 = K = \frac{p^2}{2m} = eV \Rightarrow v = \sqrt{\frac{2eV}{m}}$; $p = mv = m\sqrt{\frac{2eV}{m}}$

If you believe de Broglie

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \boxed{\frac{h}{\sqrt{2meV}} = \lambda^{predict}}$$



For V_{acc} = 54 Volts $\Rightarrow \lambda = 1.67\times10^{-10} m$ (de Broglie)

Exptal data from Davisson-Germer Observation:

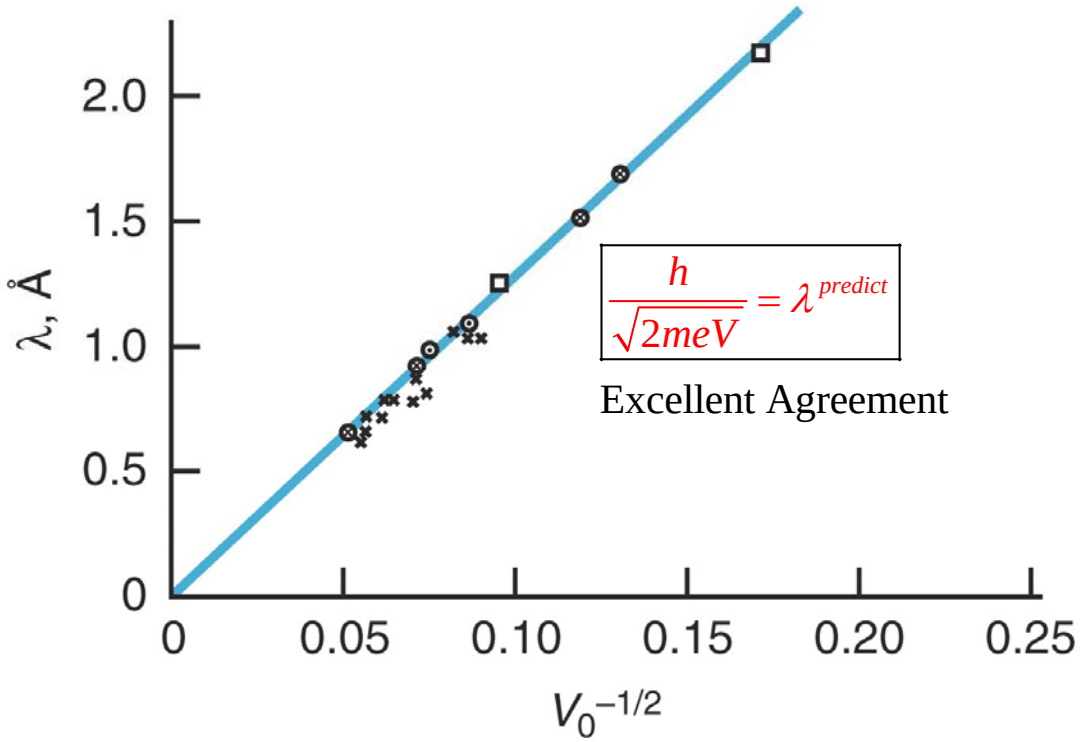
d_{nickel} =2.15 Å =$2.15\times10^{-10} m$ (from Bragg Scattering)

$\theta_{diff}^{max} = 50^o$ (observation from scattering intensity plot)

Diffraction Rule : $d \sin\phi = n\lambda$

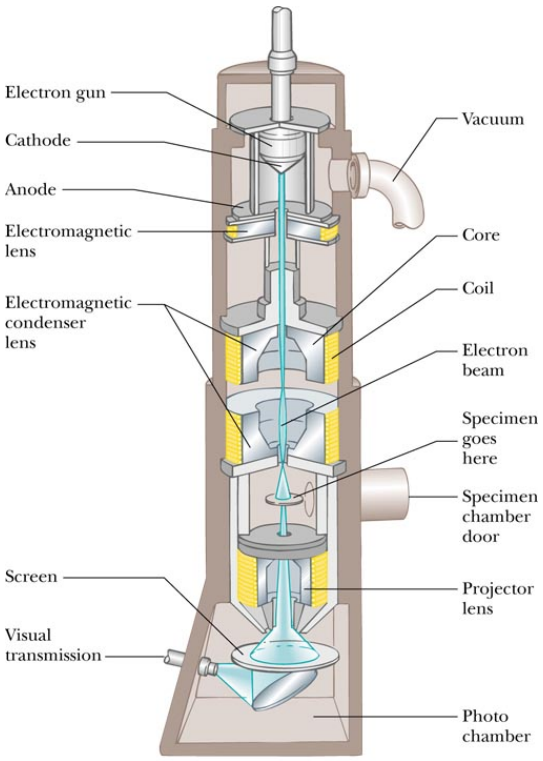
$\Rightarrow$ For Principal Maxima (n=1); λ^{meas} =(2.15 Å)(sin 50°) $\boxed{\lambda^{predict} = 1.67\,\text{Å}}$ $\boxed{\lambda^{observ} = 1.65\,\text{Å}}$ *Excellent* agreement

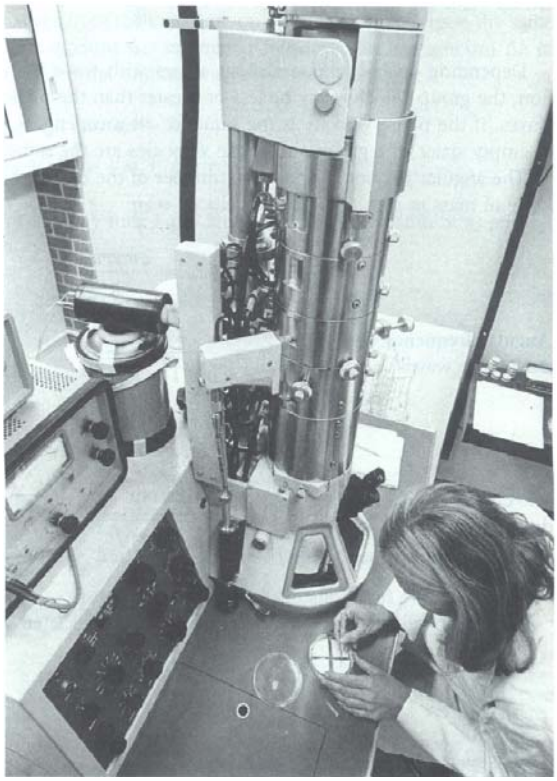
Davisson Germer Experiment: Matter Waves !



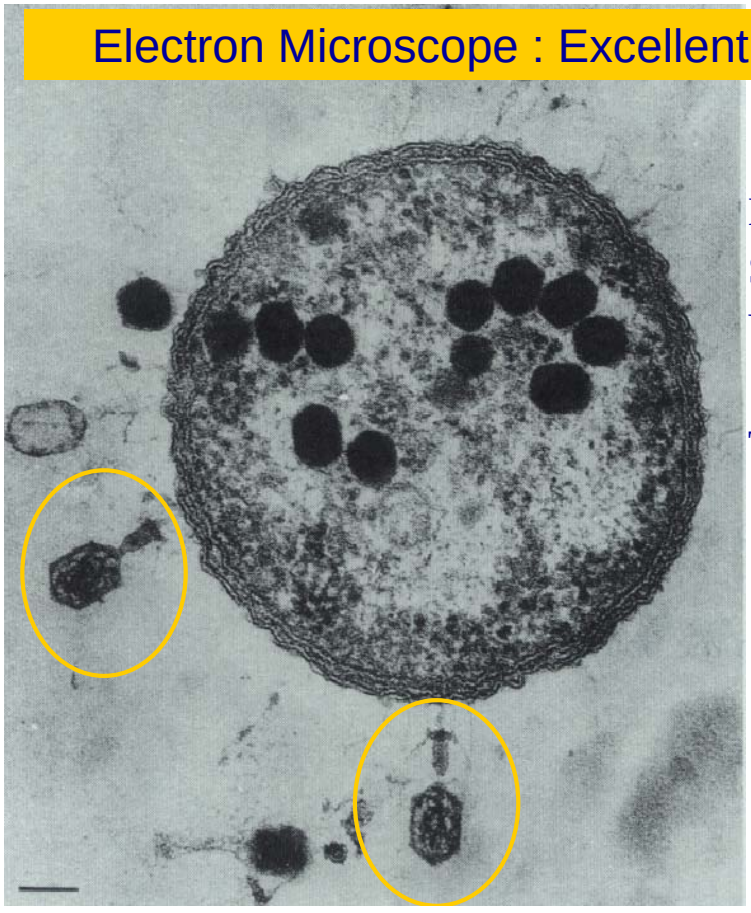
Excellent Agreement

Practical Application : Electron Microscope





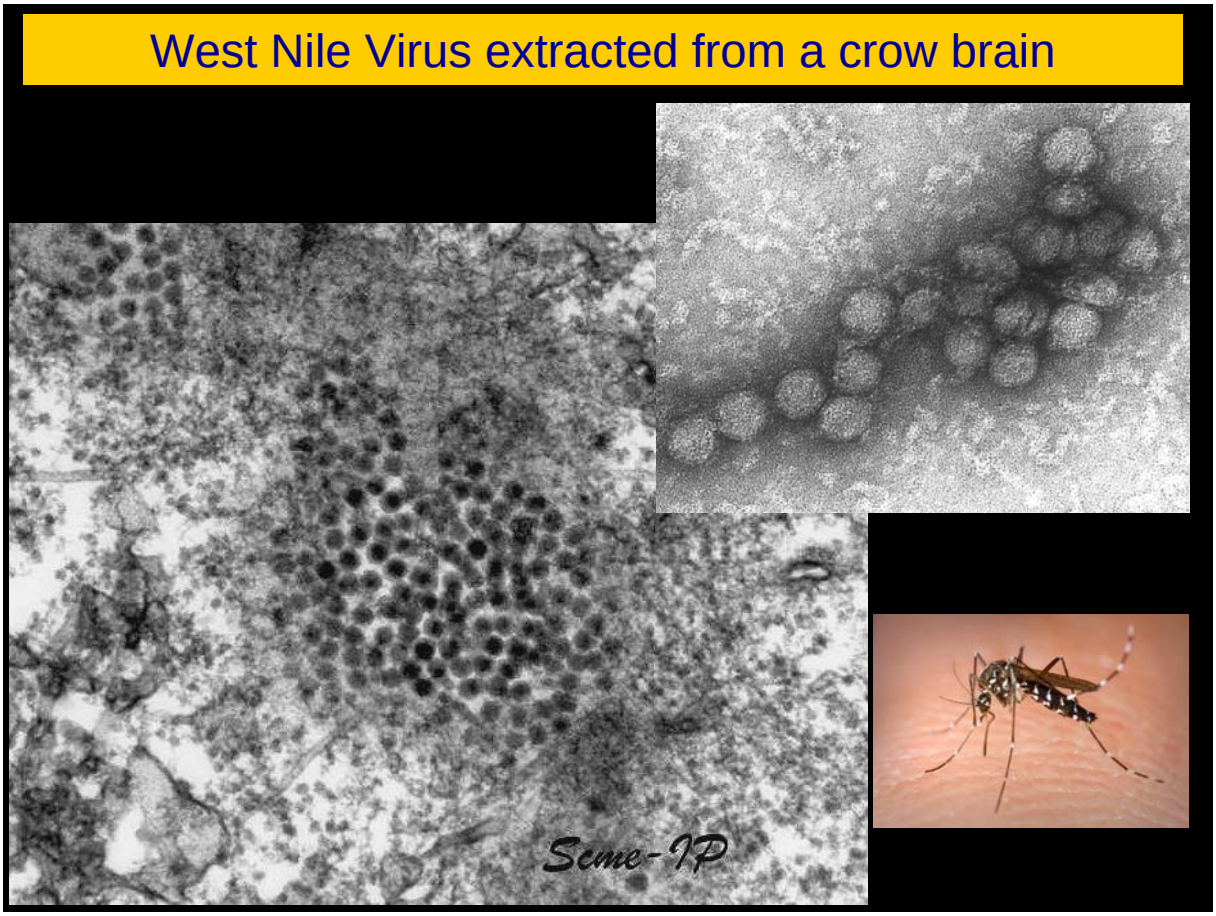
Electron Microscope : Excellent Resolving Power



Electron Micrograph
Showing Bacteriophage
Viruses in E. Coli bacterium

The bacterium is ≅ 1μ size

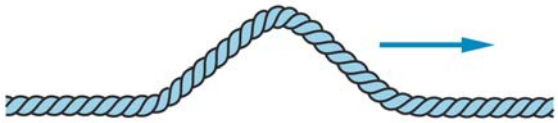
West Nile Virus extracted from a crow brain



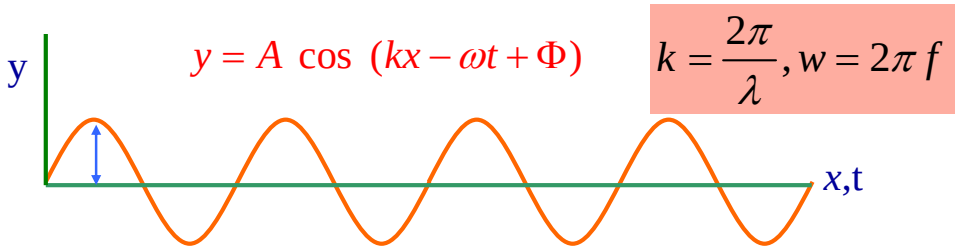
Just What is Waving in Matter Waves ?

- For waves in an ocean, it's the water that "waves"
- For sound waves, it's the molecules in medium
- For light it's the **E** & **B** vectors
- What's waving for matter waves ?
  - **It's the PROBABLILITY OF FINDING THE PARTICLE that waves !**
  - **Particle can be represented by a wave packet in**
    - **Space**
    - **Time**
    - **Made by superposition of many sinusoidal waves of different λ**
    - **It's a "pulse" of probability**

Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)



What Wave Does Not Describe a Particle



- What wave form can be associated with particle's pilot wave?
- A traveling sinusoidal wave? $y = A\ \cos\ (kx - \omega t + \Phi)$
- Since de Broglie "pilot wave" represents particle, it must travel with same speed as particle ……(like me and my shadow)

Phase velocity (v_p) of sinusoidal wave: $v_p = \lambda f$

In Matter:

(a) $\lambda = \frac{h}{p} = \frac{h}{\gamma m v}$

(b) $f = \frac{E}{h} = \frac{\gamma m c^2}{h}$

$\Rightarrow v_p = \lambda f = \frac{E}{p} = \frac{\gamma m c^2}{\gamma m v} = \frac{c^2}{v} > c!$

Conflicts with Relativity → Unphysical

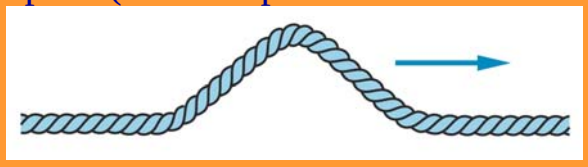
Single sinusoidal wave of infinite extent does not represent particle localized in space

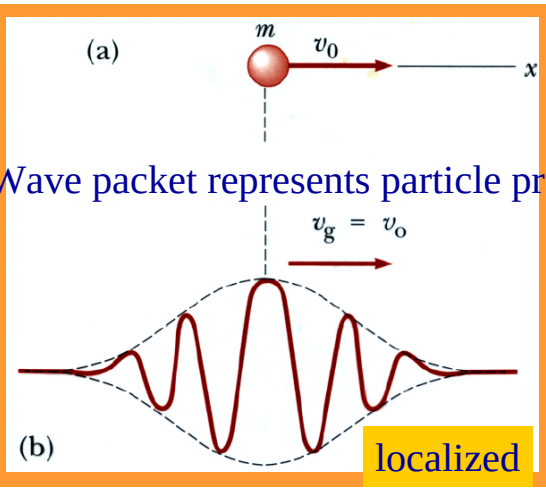
Need "wave packets" localized Spatially (x) and Temporally (t)

Wave Group or Wave Pulse

- Wave Group/packet:
  - Superposition of many sinusoidal waves with different wavelengths and frequencies
  - Localized in space, time
  - Size designated by
    - Δx or Δt
  - Wave groups travel with the speed $v_g = v_0$ of particle
- Constructing Wave Packets
  - Add waves of diff λ,
  - For each wave, pick
    - Amplitude
    - Phase
  - Constructive interference over the space-time of particle
  - Destructive interference elsewhere !

Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)





Wave packet represents particle prob

localized