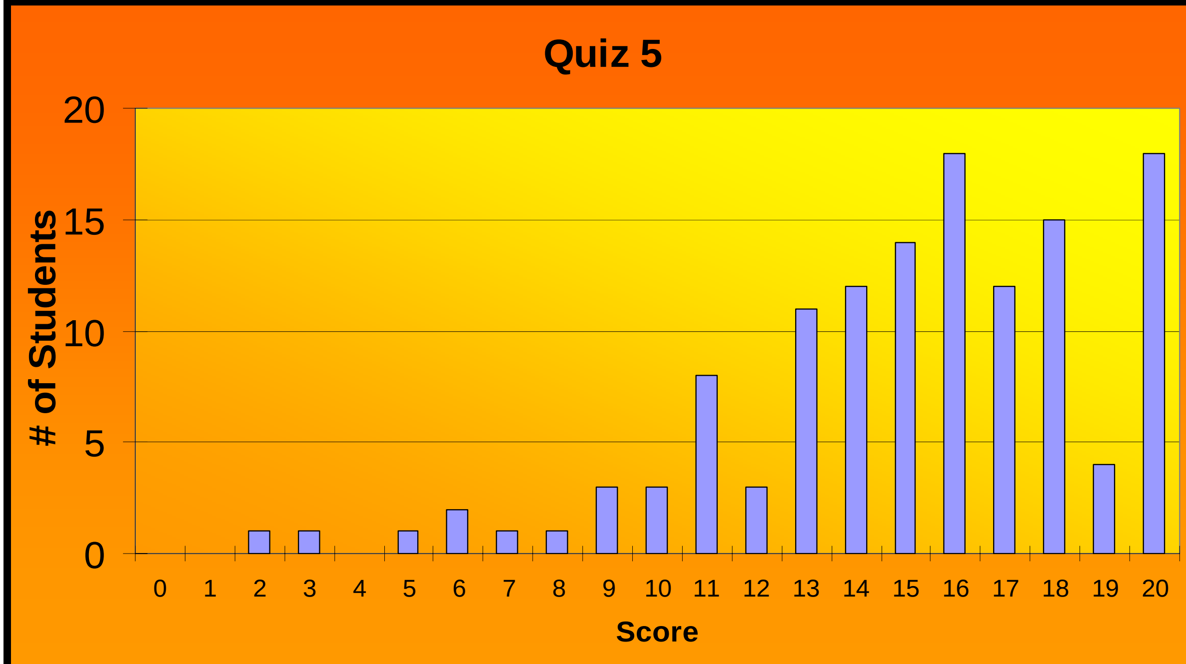
Confirmed: 2D Final Exam:Thursday 18th March 11:30-2:30 PM   WLH 2005

| Date | Time | Read | Topic | HW problems for the week | Location |
|---|---|---|---|---|---|
| Monday | 11:00 am | Ch 4 | President Day Holiday | Read sections 4.6-4.8 | WLH 2005 |
| Tuesday | 8:00 pm | Ch 4 | Matter Waves | Ch 4: 22, 24, 25, 31, 32, 33 | WLH 2005 |
| Wednesday | 11:00 am | Ch 5 | Quantum Mechanics in 1 Dim | Ch 5 : 1, 2, 3, 5, 7, 9 | WLH 2005 |
| Wednesday | 3:00 pm | Ch 5 | Discussion | - | WLH 2005 |
| Thursday | 5:00-6:20 pm | - | Problem Session | | Peterson 108 |
| Friday | 11:00am | - | Quiz | Uncertainty Principle + Ch 5.1-5.4 | WLH 2005 |

# Physics 2D Lecture Slides
# Lecture 19: Feb 17th


Vivek Sharma
UCSD Physics




## Heisenberg's Uncertainty Principles

- $\Delta x.\ \Delta p \geq h/4\pi \Rightarrow$
  - If the measurement of the position of a particle is made with a precision $\Delta x$ and a SIMULTANEOUS measurement of its momentum $p_x$ in the X direction , then the product of the two uncertainties (measurement errors) can never be smaller than $\cong h/4\pi$ irrespective of how precise the measurement tools

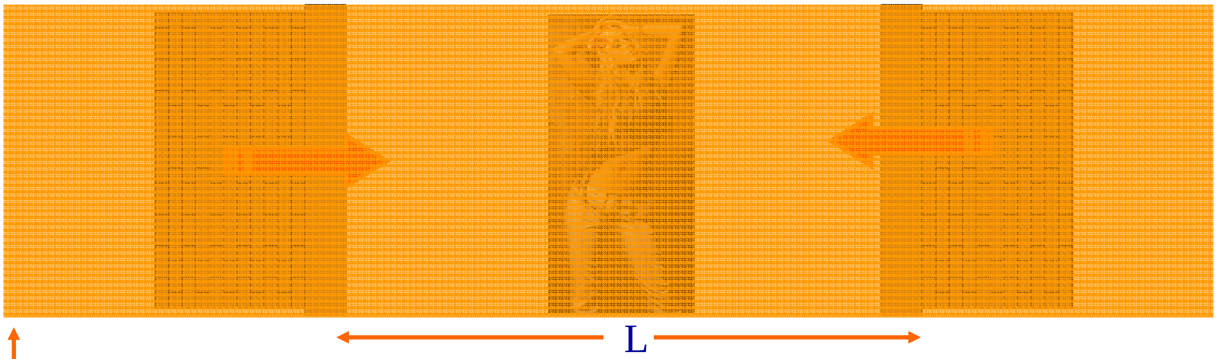
- $\Delta E.\ \Delta t \geq h/4\pi \Rightarrow$
  - If the measurement of the energy E of a particle is made with a precision $\Delta E$ and it took time $\Delta t$ to make that measurement, then the product of the two uncertainties (measurement errors) can never be smaller than $\cong h/4\pi$ irrespective of how precise the measurement tools

**What do these simple equations mean ?**

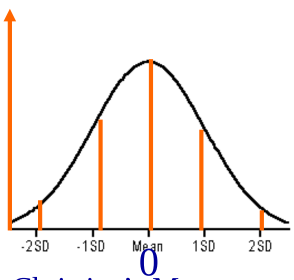
# The Quantum Mechanics of Christina Aguilera!

Christina at rest between two walls originally at infinity: Uncertainty in her location $\Delta X = \infty$ .
At rest means her momentum P=0 , $\Delta P=0$ (Uncertainty principle)

**Slowly two walls move in from infinity on each side, now** $\Delta X = L$ , so $\Delta p \neq 0$
She is not at rest now, in fact her momentum P $\approx \pm$ (h/2$\pi$ L)




**X Axis**




Christina's Momentum p

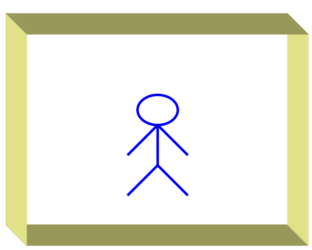
On average, measure <p> = 0
but there are quite large fluctuations!
Width of Distribution = $\Delta P$

$$\Delta P = \sqrt{(P^2)_{ave} - (P_{ave})^2}; \quad \Delta P \sim \frac{\hbar}{L}$$

**Bottomline : Christina dances to the tune of Uncertainty Principle!**

# Implications of Uncertainty Principles

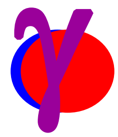
**A bound "particle" is one that is confined in some finite region of space.**



**One of the cornerstones of Quantum mechanics is that bound particles can not be stationary – <u>even at Zero absolute temperature !</u>**

**There is a non-zero limit on the kinetic energy of a bound particle**

## Matter-Antimatter Collisions and Uncertainty Principle




**Look at Rules of Energy and Momentum Conservation : Are they ?**

**$E_{before} = mc^2 + mc^2$ and $E_{after} = 2mc^2$**

**$P_{before} = 0$ but since photon produced in the annihilation → $P_{after} = 2mc$ !**

**Such violation are allowed but must be consumed instantaneously !**
**Hence the name "virtual" particles**

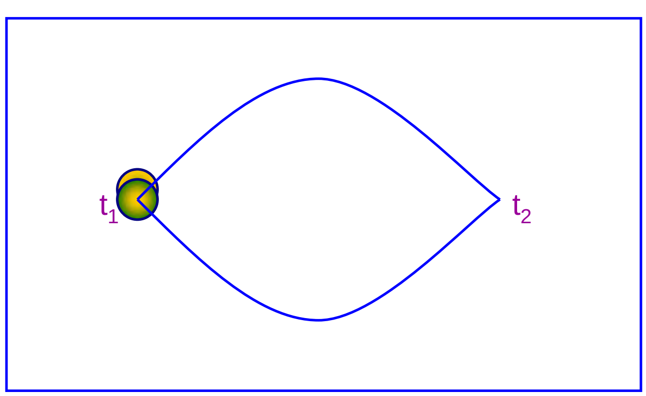
## Fluctuations In The Vacuum : Breaking Energy Conservation Rules

**Vaccum, at any energy, is bubbling with particle creation and annihilation**

$\Delta E \,.\, \Delta t \approx h/2\pi$ implies that you can (in principle) pull out an **<u>elephant + anti-elephant</u>** from NOTHING (Vaccum) but for a very very short time $\Delta t$ !!

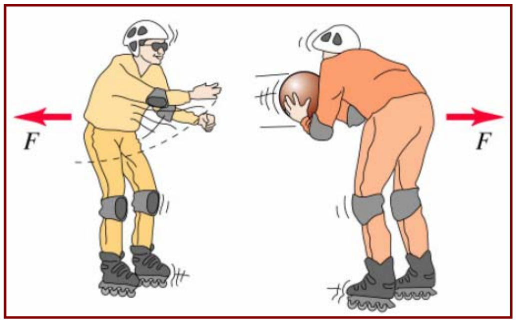
How Much Time : $\Delta t = \dfrac{\hbar}{2Mc^2}$

How cool is that !

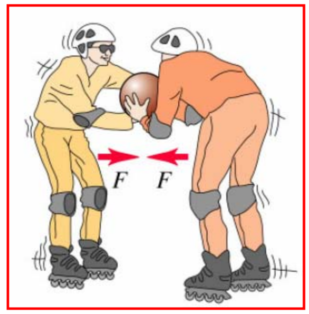



How far can the virtual particles propagate ? Depends on their mass

## Strong Force Within Nucleus ➔ Exchange Force and Virtual Particles

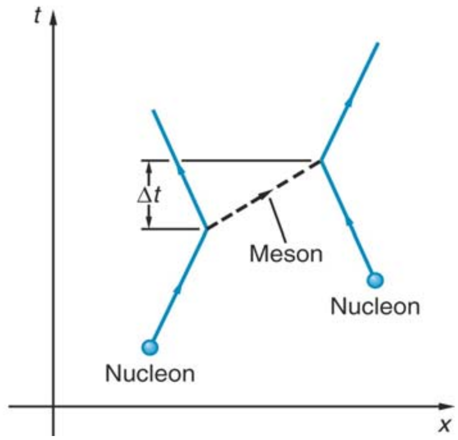



Repulsive force

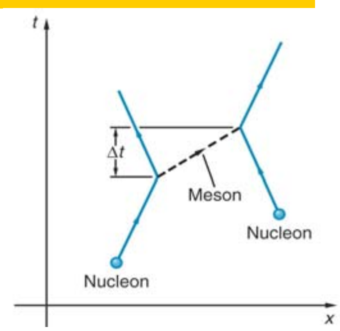



Attractive force

- Strong Nuclear force can be modeled as exchange of virtual particles called $\pi^{\pm}$ mesons by nucleons (protons & neutrons)
- $\pi^{\pm}$ mesons are emitted by proton and reabsorbed by a neutron
- The short range of the Nuclear force is due to the "large" mass of the exchanged meson
- $M_{\pi} = 140$ MeV/c$^2$




# Range of Nuclear Exchange Force




How long can the emitted virtual particle last?

$\Delta E \times \Delta t \geq \hbar$

The virtual particle has rest mass + kinetic energy

$\Rightarrow$ Its energy $\boxed{\Delta E \geq Mc^2}$

$\Rightarrow$ Particle can not live for more than $\boxed{\Delta t \leq \hbar / Mc^2}$

Range R of the meson (and thus the exchange force)

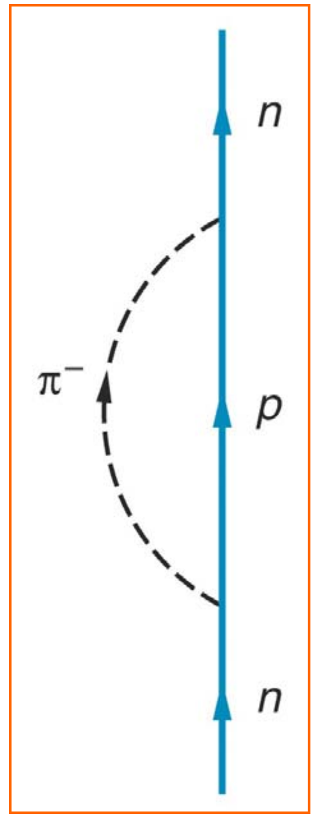
$R = c\Delta t = c\hbar / Mc^2 = \hbar / Mc$

For M=140 MeV/c$^2$ $\Rightarrow R \simeq \dfrac{1.06 \times 10^{-34}\, J.s}{(140 MeV / c^2) \times c^2 \times (1.60 \times 10^{-13}\, J / MeV)}$

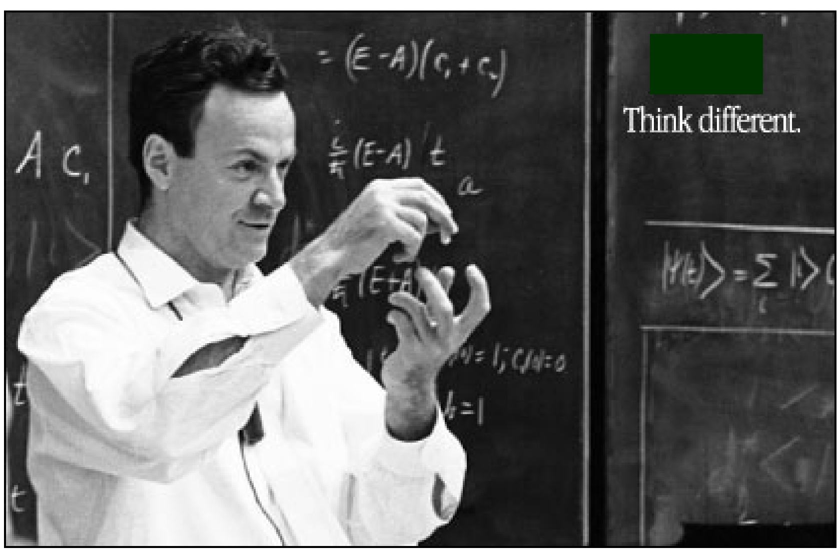
$\boxed{R \simeq 1.4 \times 10^{-15} m = 1.4\, fm}$

## Subatomic Cinderella Act

- Neutron emits a charged pion for a time $\Delta t$ and becomes a (charged) proton
- After time $\Delta t$ , the proton reabsorbs charged pion particle ($\pi^-$) to become neutron again
- But in the time $\Delta t$ that the positive proton and $\pi^-$ particle exist, they can interact with other charged particles
- After time $\Delta t$ strikes , the Cinderella act is over !




## Quantum Behavior : Richard Feynman

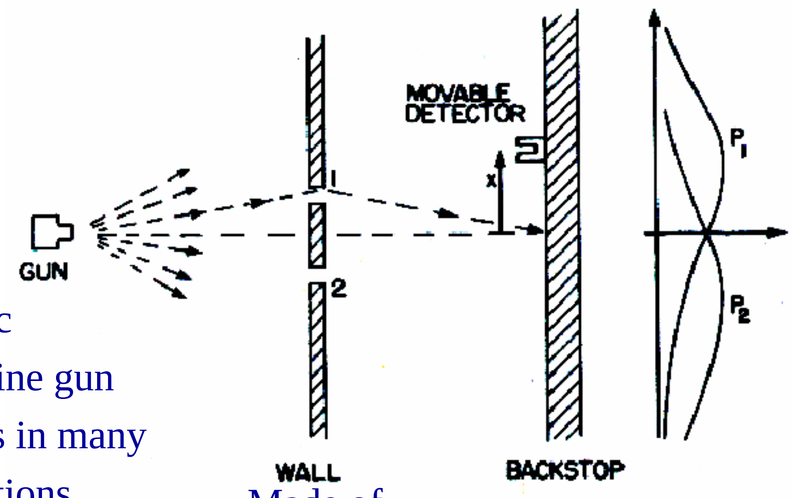



See Chapters 1 & 2 of Feynman Lectures in Physics Vol III

Or Six Easy Pieces by Richard Feynman : Addison Wesley Publishers

# An Experiment with Indestructible Bullets

Probability $P_{12}$ when Both holes open




Erratic Machine gun sprays in many directions

Made of Armor plate

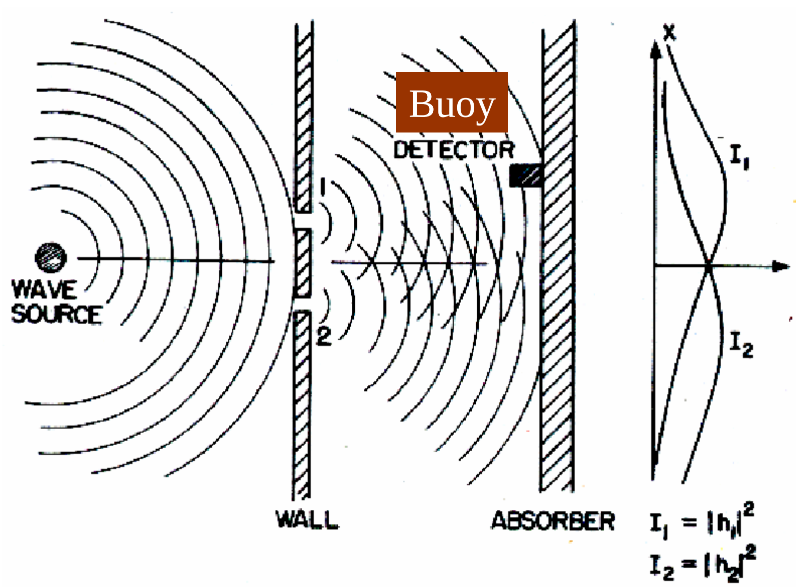
$P_{12} = P_1 + P_2$

# An Experiment With Water Waves

Measure Intensity of Waves
(by measuring amplitude of displacement)

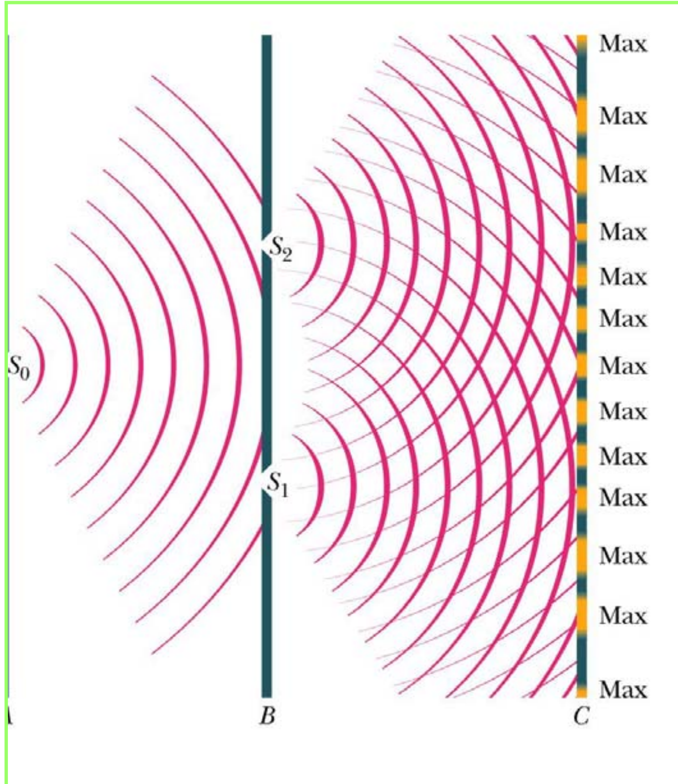
Intensity $I_{12}$ when Both holes open

Buoy

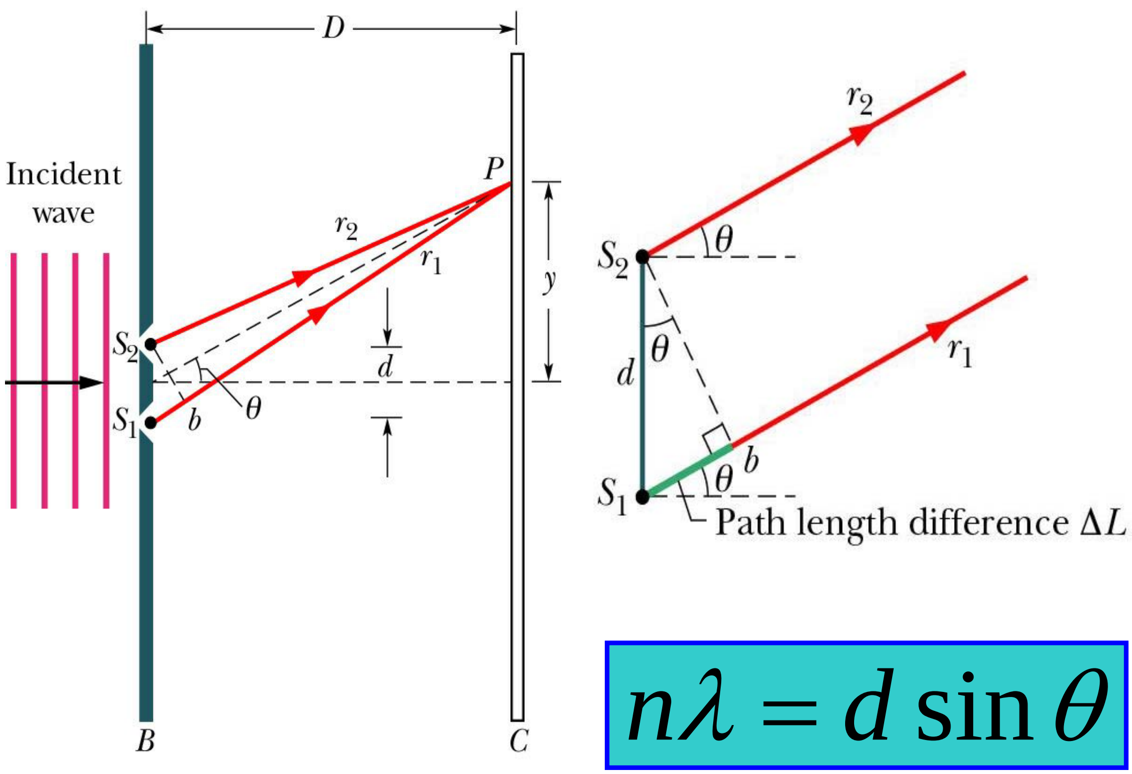



$$I_{12} = |h_1 + h_2|^2 = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

## Interference and Diffraction: Ch 36 & 37, RHW



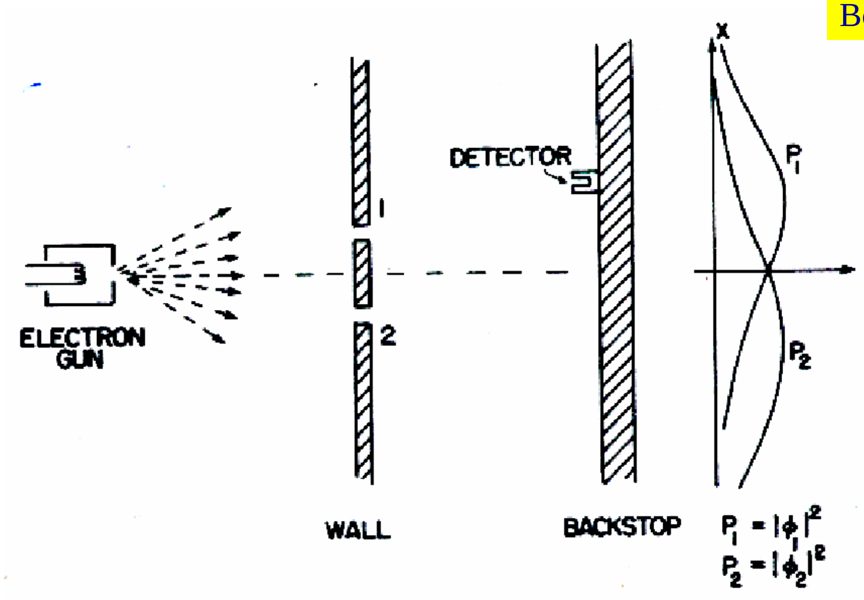

## Interference Phenomenon in Waves




$$n\lambda = d\sin\theta$$

# An Experiment With (indestructible) Electrons

Probability $P_{12}$ when Both holes open

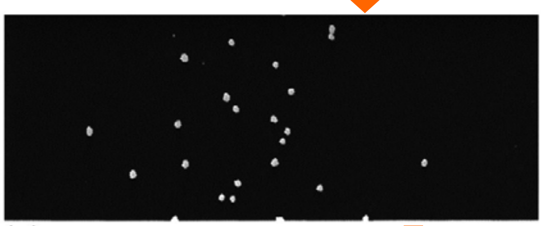



$P_{12} \neq P_1 + P_2$

# Interference Pattern of Electrons When Both slits open

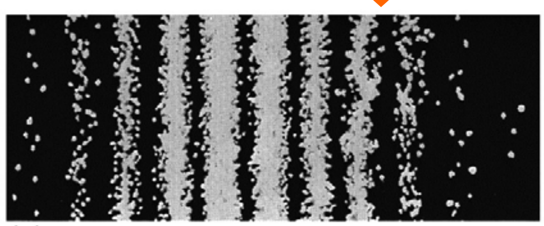
Growth of 2-slit Interference pattern thru different exposure periods
Photographic plate (screen) struck by:

28 electrons



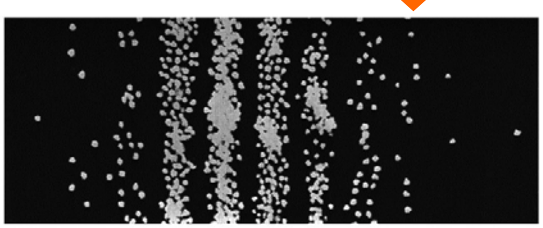
(a)

10,000 electron



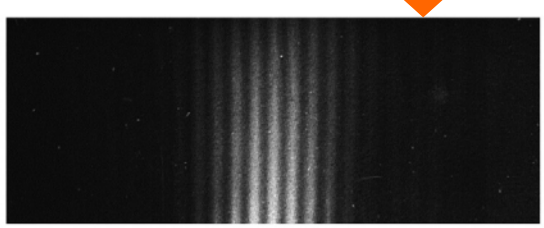
(c)

1000 electrons



(b)

$10^6$ electrons

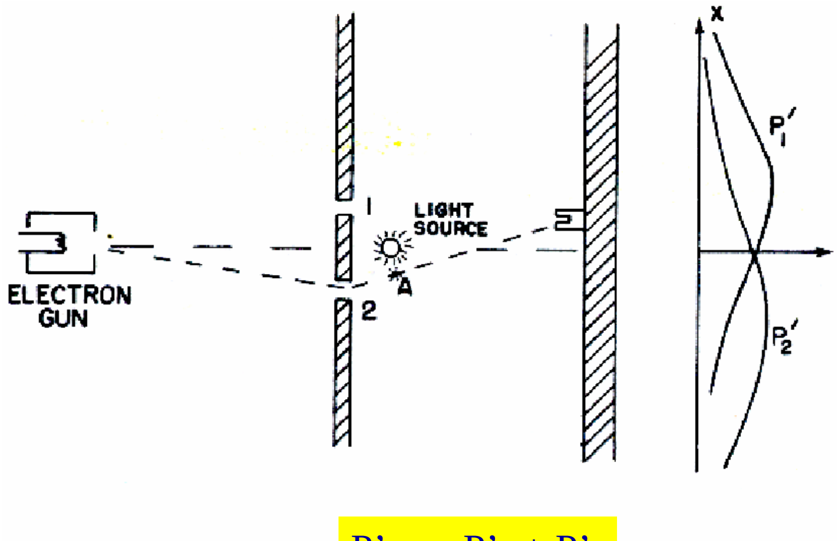


(d)

White dots simulate presence of electron
No white dots at the place of destructive Interference (minima)

## Watching The Electrons By Shining Intense Light

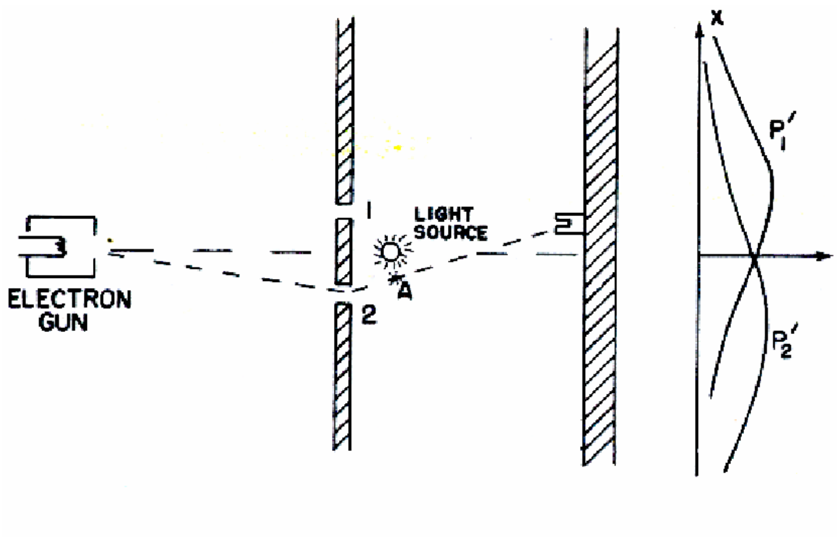
Probability $P_{12}$ when both holes open and I see which hole the electron came thru




$P'_{12} = P'_1 + P'_2$

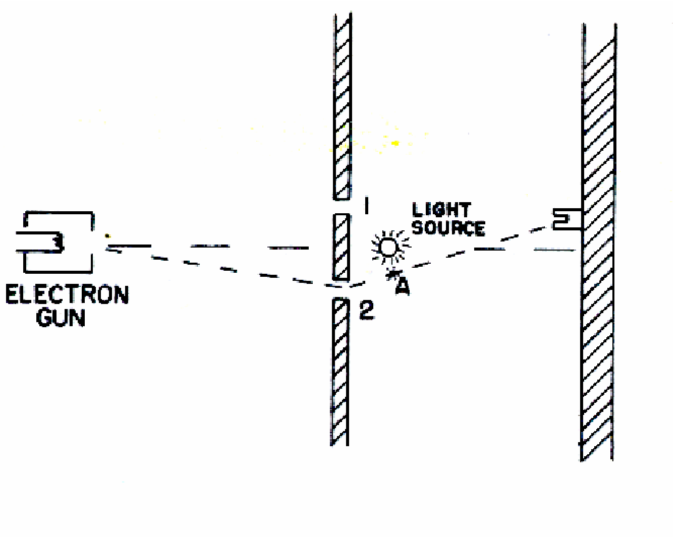
## Watching electrons with dim light: See flash of light & hear detector clicks

Probability $P_{12}$ when both holes open and I see which hole the electron came thru



Watching electrons with dim light: don't see flash of light but hear detector clicks

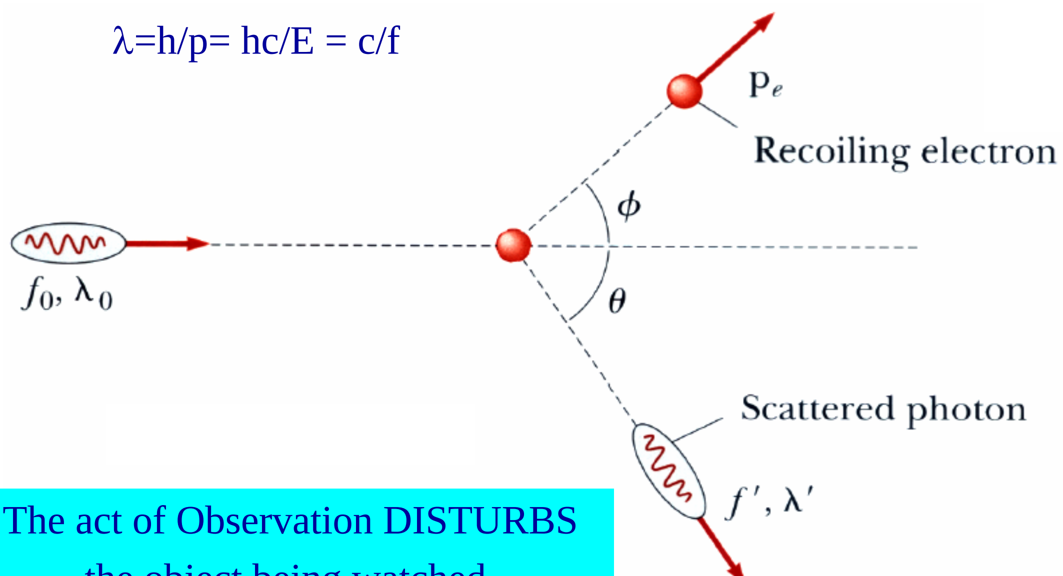
Probability $P_{12}$ when both holes open and I Don't see which hole the electron came thru



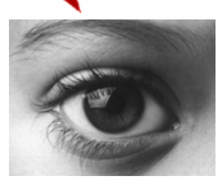

$P_{12} = |\phi_1 + \phi_2|^2$

Compton Scattering: Shining light to observe electron

$\lambda = h/p = hc/E = c/f$

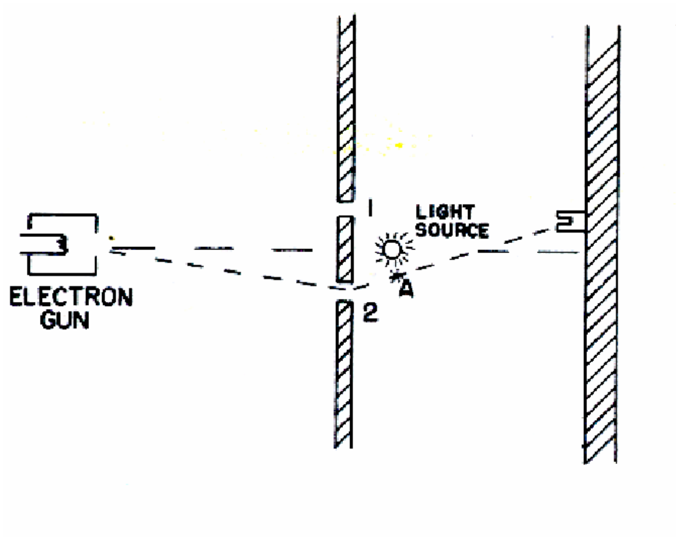



The act of Observation DISTURBS the object being watched, here the electron moves away from where it was originally



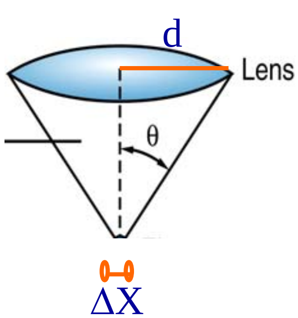
## Watching Electrons With Light of λ >> slit size but High Intensity

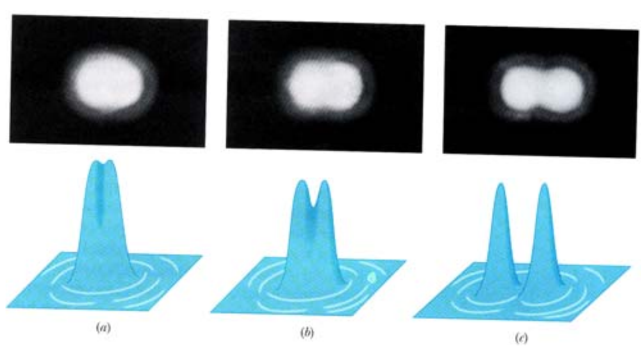
Probability $P_{12}$ when both holes open but can't tell, from the location of flash, which hole the electron came thru




## Why Fuzzy Flash? → Resolving Power of Light

Image of 2 separate point sources formed by a converging lens of diameter d, ability to resolve them depends on λ & d because of the Inherent diffraction in image formation




Not resolved    barely resolved    resolved

$$\text{Resolving power } \Delta x \simeq \frac{\lambda}{2\sin\theta}$$

## Summary of Experiments So Far

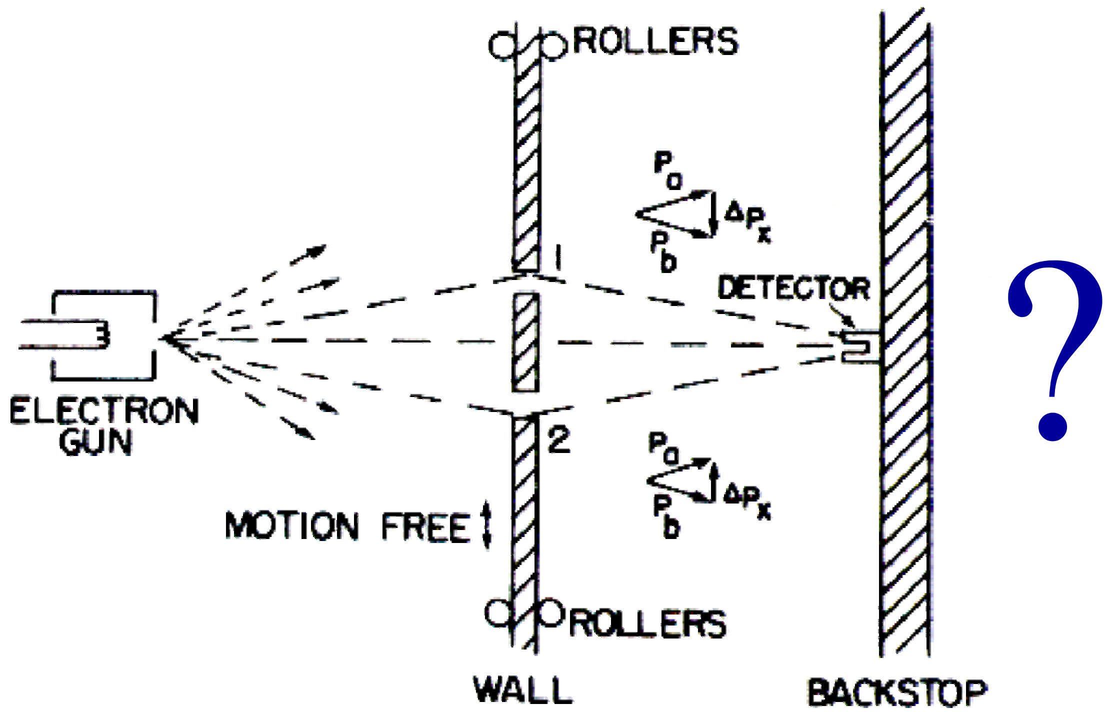
1. Probability of an event is given by the square of amplitude of a complex # Ψ: Probability Amplitude
2. When an event occurs in several alternate ways, probability amplitude for the event is sum of probability amplitudes for each way considered seperately. There is interference:
   - $\Psi = \Psi_1 + \Psi_2$

     $P_{12} = |\Psi_1 + \Psi_2|^2$
3. If an experiment is done which is capable of determining whether one or other alternative is actually taken, probability for event is just sum of each alternative
   - Interference pattern is LOST !

## Is There No Way to Beat Uncertainty Principle?

- How about NOT watching the electrons!
- Let's be a bit crafty !!
- Since this is a Thought experiment→ ideal conditions
  - Mount the wall on rollers, put a lot of grease → frictionless
  - Wall will move when electron hits it
  - Watch recoil of the wall containing the slits when the electron hits it
  - By watching whether wall moved up or down I can tell
    - Electron went thru hole # 1
    - Electron went thru hole #2
- Will my ingenious plot succeed?

## Measuring The Recoil of The Wall → Not Watching Electron !




## Losing Out To Uncertainty Principle

- To measure the RECOIL of the wall ⇒
  - must know the initial momentum of the wall before electron hit it
  - Final momentum after electron hits the wall
  - Calculate vector sum → recoil
- Uncertainty principle :
  - To do this ⇒ $\Delta P = 0$ → $\Delta X = \infty$ [can not know the position of wall exactly]
  - If don't know the wall location, then down know where the holes are
  - Holes will be in different place for every electron that goes thru
  - → The center of interference pattern will have different (random) location for each electron
  - Such random shift is just enough to Smear out the pattern so that no interference is observed !
- Uncertainty Principle Protects Quantum Mechanics !

## Summary

- Probability of an event in an ideal experiment is given by the square of the absolute value of a complex number $\Psi$ which is call probability amplitude
  - P = probability
  - $\Psi$= probability amplitude,
  - $P=|\Psi|^2$
- When an even can occur in several alternative ways, the probability amplitude for the event is the sum of the probability amplitudes for each way considered separately. There is interference:
  - $\Psi= \Psi_1+ \Psi_2$
  - $P=|\Psi_1+ \Psi_2|^2$
- If an experiment is performed which is capable of determining whether one or other alternative is actually taken, the probability of the event is the sum of probabilities for each alternative. The interferenence is lost: $P = P_1 + P_2$

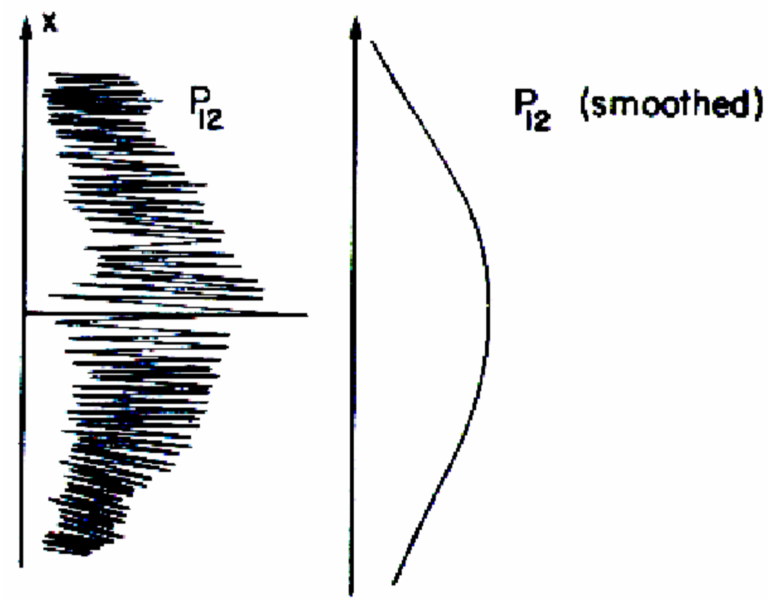
## The Lesson Learnt

- In trying to determine which slit the particle went through, we are examining particle-like behavior
- In examining the interference pattern of electron, we are using wave like behavior of electron

Bohr's Principle of Complementarity:

***It is not possible to simultaneously determine physical observables in terms of both particles and waves***

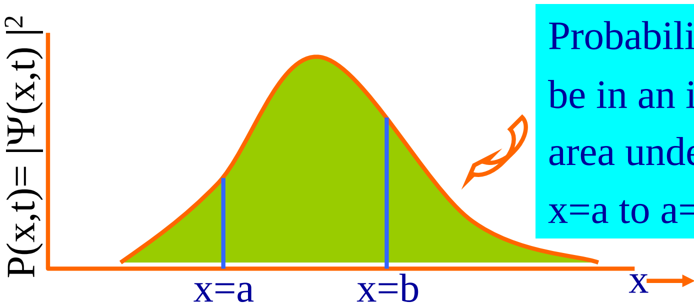
## The Bullet Vs The Electron: Each Behaves the Same Way




## Quantum Mechanics of Subatomic Particles

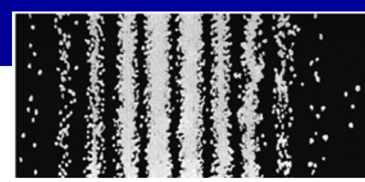
- Act of Observation destroys the system (No watching!)
- If can't watch then all conversations can only be in terms of Probability P
- Every particle under the influence of a force is described by a Complex wave function $\Psi(x,y,z,t)$
- $\Psi$ is the ultimate DNA of particle: contains all info about the particle under the force (in a potential e.g Hydrogen )
- Probability of per unit volume of finding the particle at some point (x,y,z) and time t is given by
  - $P(x,y,z,t) = \Psi(x,y,z,t) . \Psi^*(x,y,z,t) =| \Psi(x,y,z,t) |^2$
- When there are more than one path to reach a final location then the probability of the event is
  - $\Psi = \Psi_1 + \Psi_2$
  - $P = | \Psi^* \Psi| = |\Psi_1|^2 + |\Psi_2|^2 +2 |\Psi_1| |\Psi_2| \cos\phi$

## Wave Function of “Stuff” & Probability Density




Probability of a particle to be in an interval a ≤ x ≤b is area under the curve from x=a to a=b

- Although not possible to specify with certainty the location of particle, its possible to assign probability P(x)dx of finding particle between x and x+dx
- $P(x)\, dx = |\, \Psi(x,t)|^2\, dx$
- E.g intensity distribution in light diffraction pattern is a measure of the probability that a photon will strike a given point within the pattern



## Ψ: The Wave function Of A Particle

- The particle must be some where

$$\int_{-\infty}^{+\infty} |\psi(x,t)|^2\, dx = 1$$

- Any Ψ satisfying this condition is NORMALIZED
- Prob of finding particle in finite interval

$$P(a \le x \le b) = \int_a^b \psi^*(x,t)\ \psi(x,t)dx$$

- Fundamental aim of Quantum Mechanics
  - Given the wavefunction at some instant (say t=0) find Ψ at some subsequent time t
  - Ψ(x,t=0) → Ψ(x,t) …evolution
  - Think of a probabilistic view of particle’s “newtonian trajectory”
    - We are replacing Newton’s 2nd law for subatomic systems

The Wave Function is a mathematical function that describes a physical object → Wave function must have some rigorous properties :

- Ψ must be finite
- Ψ must be continuous fn of x,t
- Ψ must be single-valued
- Ψ must be smooth fn →

$$\frac{d\psi}{dx}$$ must be continuous

WHY ?

# The Anti-matter Sea