Confirmed: 2D Final Exam:Thursday 18th March 11:30-2:30 PM WLH 2005

Quiz 5 will cover sections 4.1-4.5, emphasis on Uncertainty relations
Ignore optional stuff like section 4.4 & MS Desktop (pages157-161)

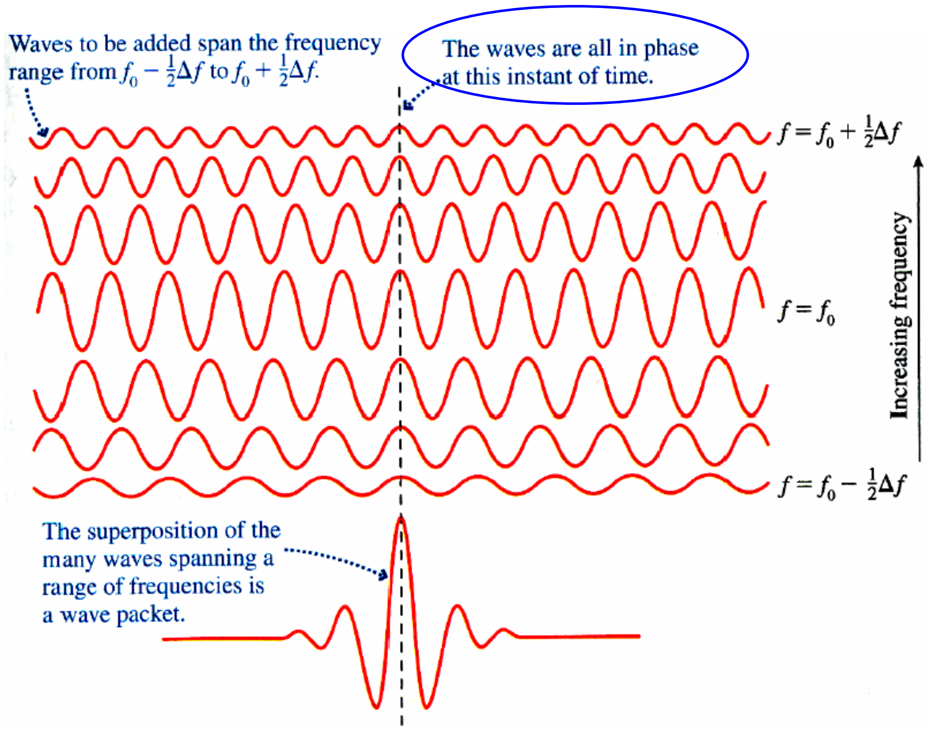
Week 6 Starts Monday Feb 9nd 2004

Date	Time	Read	Topic	HW problems for the week	Location
Monday	11:00 am	Ch 4	Matter Waves		WLH 2005
Tuesday	8:00 pm	Ch 4	Particle Nature of Matter	Ch 4: 4, 6 ,11,13, 17	WLH 2005
Wednesday	11:00 am	Ch 4	Particle Nature of Matter	Ch 4 : 23, 26, 28, 29, 30,36 22, 24, 25, 31, 32, 33	WLH 2005
Wednesday	3:00 pm	Ch 4	Discussion	-	WLH 2005
Thursday	5:00-6:20 pm	-	Problem Session		Peterson 108
Friday	11:00am	-	Quiz	Matter Waves: 4.1-4.5	WLH 2005

Physics 2D Lecture Slides
Lecture 18: Feb 11th

Vivek Sharma
UCSD Physics

Non-repeating wave packet can be created thru superposition
Of many waves of similar (but different) frequencies and wavelengths



Wave Packets & Uncertainty Principles of Subatomic Physics

in space x: $\boxed{\Delta k.\Delta x = \pi}$ $\Rightarrow$ since $k = \frac{2\pi}{\lambda}$, $p = \frac{h}{\lambda}$

$\Rightarrow$ $\boxed{\Delta p.\Delta x = h/2}$

usually one writes $\boxed{\Delta p.\Delta x \geq \hbar/2}$ approximate relation

In time t : $\boxed{\Delta w.\Delta t = \pi}$ $\Rightarrow$ since $\omega = 2\pi f, E = hf$

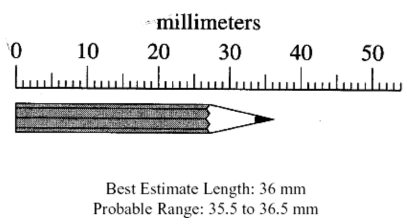
$\Rightarrow$ $\boxed{\Delta E.\Delta t = h/2}$

usually one writes $\boxed{\Delta E.\Delta t \geq \hbar/2}$ approximate relation

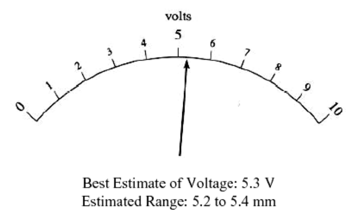
What do these inequalities mean physically?

Know the Error of Thy Ways: Measurement Error →Δ

- Measurements are made by observing something : length, time, momentum, energy
- All measurements have some (limited) precision.…no matter the instrument used
- Examples:
  - How long is a desk ? L = (5 ± 0.1) m = L ± ΔL   (depends on ruler used)
  - How long was this lecture ? T = (50 ± 1)minutes = T ± ΔT (depends on the accuracy of your watch)
  - How much does Prof. Sharma weigh ? M = (1000 ± 700) kg = m ± Δm
    - Is this a correct measure of my weight ?
      - Correct (because of large error reported) but imprecise
      - My correct weight is covered by the (large) error in observation



Length Measure

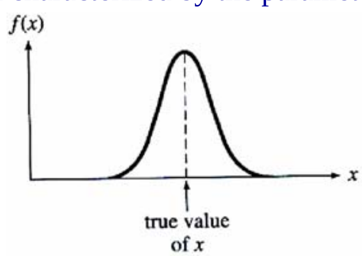


Voltage (or time) Measure

Measurement Error : x ± Δx

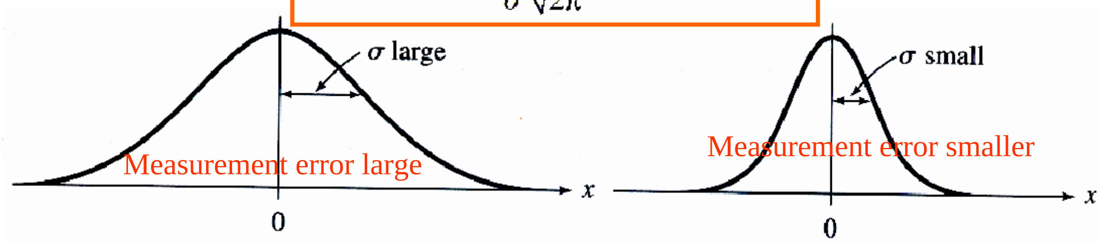
- Measurement errors are unavoidable since the measurement procedure is an experimental one
- True value of an measurable quantity is an abstract concept
- In a set of repeated measurements with random errors, the distribution of measurements

resembles a Gaussian distribution characterized by the parameter σ or Δ characterizing the width of the distribution

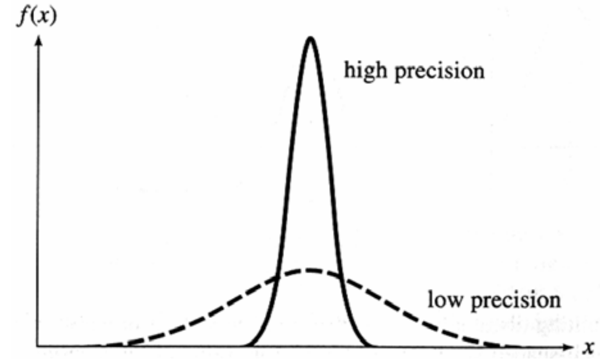


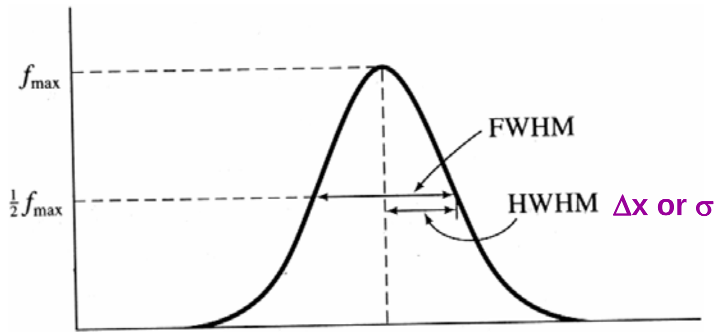
The Gauss, or Normal, Distribution

$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2/2\sigma^2}.$$

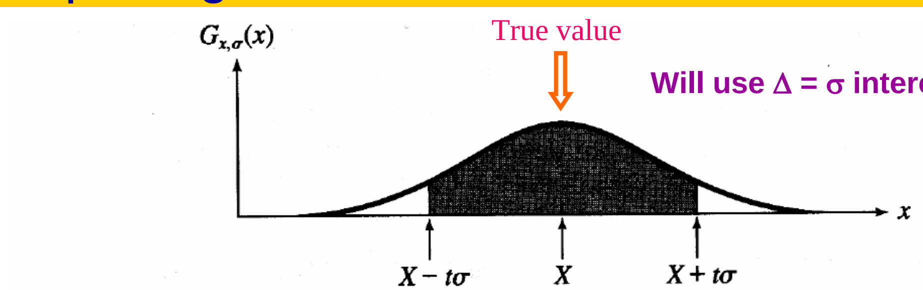


Measurement Error : x ± Δx



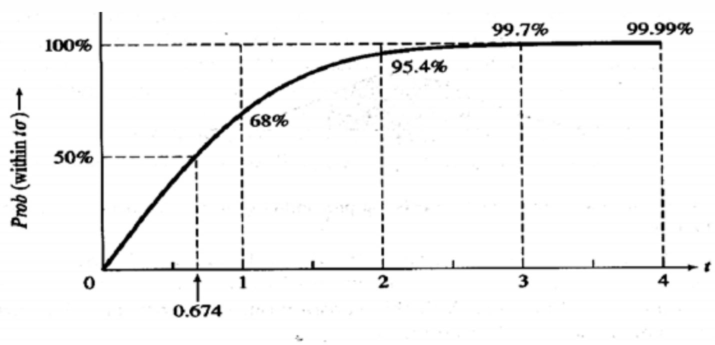


Interpreting Measurements with random Error : Δ



Will use Δ = σ interchangeably

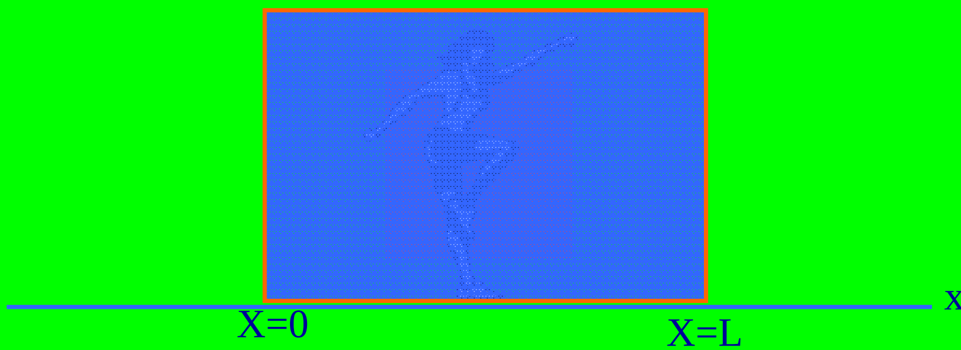
Figure 5.12. The shaded area between $X \pm t\sigma$ is the probability of a measurement within t standard deviations of X.



t	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.5	3.0	3.5	4.0
Prob (%)	0	20	38	55	68	79	87	92	95.4	98.8	99.7	99.95	99.99

Where in the World is Carmen San Diego?

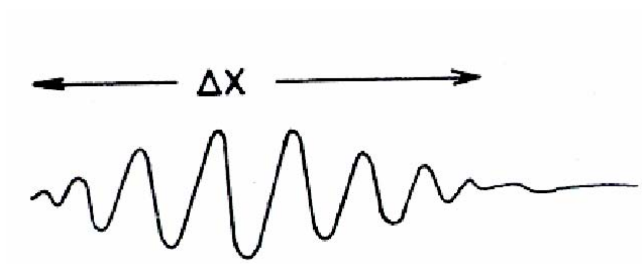
- Carmen San Diego hidden inside a big box of length L
- Suppose you can't see thru the (blue) box, what is you best estimate of her location inside box (she could be anywhere inside the box)



Your best unbiased measure would be x = L/2 ± L/2

There is no perfect measurement, there are always measurement error

Wave Packets & Matter Waves



- What is the Wave Length of this wave packet?
  - made of waves with $\lambda-\Delta\lambda < \lambda < \lambda+\Delta\lambda$
  - De Broglie wavelength $\lambda = h/p$
    - → Momentum Uncertainty: $p-\Delta p < p < p+\Delta p$
- Similarly for frequency ω or f
  - made of waves with $\omega-\Delta\omega < \omega < \omega+\Delta\omega$

Planck's condition E= hf = hω/2

→ Energy Uncertainty: $E-\Delta E < E < E + \Delta E$

Back to Heisenberg's Uncertainty Principle & Δ

- $\Delta x. \Delta p \geq h/4\pi \Rightarrow$
  - If the measurement of the position of a particle is made with a precision Δx and a SIMULTANEOUS measurement of its momentum p_x in the X direction , then the product of the two uncertainties (measurement errors) can never be smaller than $\cong h/4\pi$ irrespective of how precise the measurement tools

- $\Delta E. \Delta t \geq h/4\pi \Rightarrow$
  - If the measurement of the energy E of a particle is made with a precision ΔE and it took time Δt to make that measurement, then the product of the two uncertainties (measurement errors) can never be smaller than $\cong h/4\pi$ irrespective of how precise the measurement tools

These rules arise from the way we constructed the Wave packets describing Matter "pilot" waves

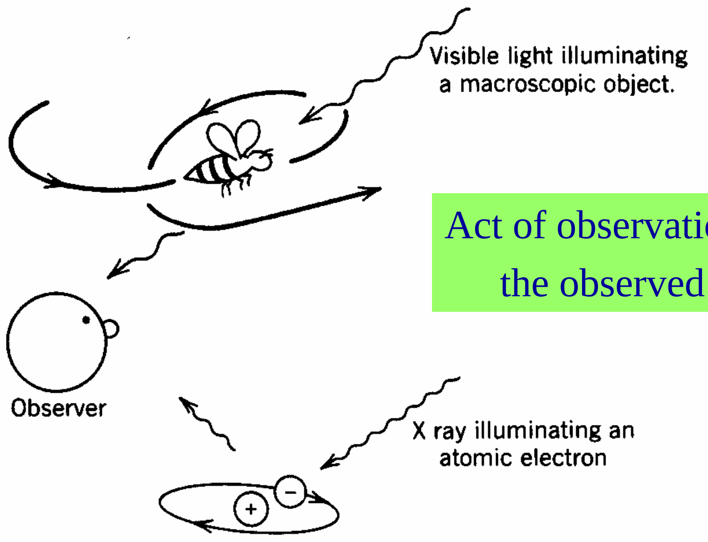
Perhaps these rules Are bogus, can we verify this with some physical picture ??

Are You Experienced ?

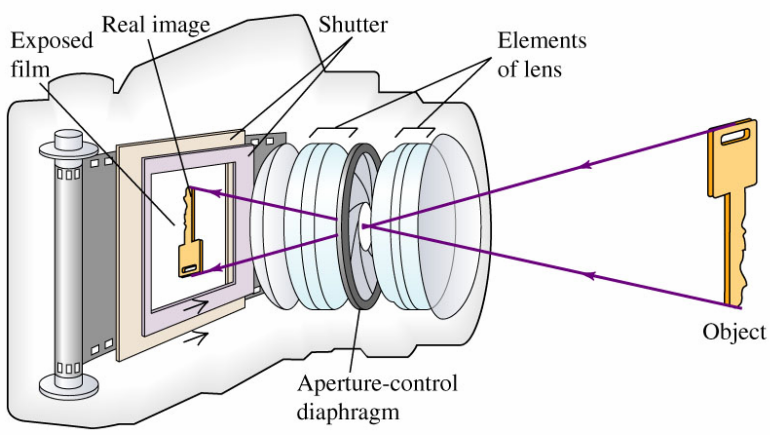
- What you experience is what you observe
- What you observe is what you measure
- No measurement is perfect, they all have measurement error: question is of the degree
  - Small or large Δ
- Uncertainty Principle and Breaking of Conservation Rules
  - Energy Conservation
  - Momentum Conservation

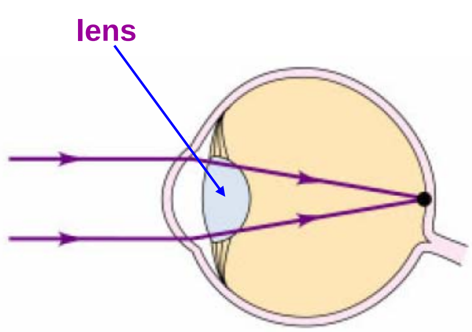
The Act of Observation (Compton Scattering)

Observations of particle motion by means of scattered illumination. When the incident wavelength is reduced to accommodate the size of the particle, the momentum transferred by the photon becomes large enough to disturb the observed motion.



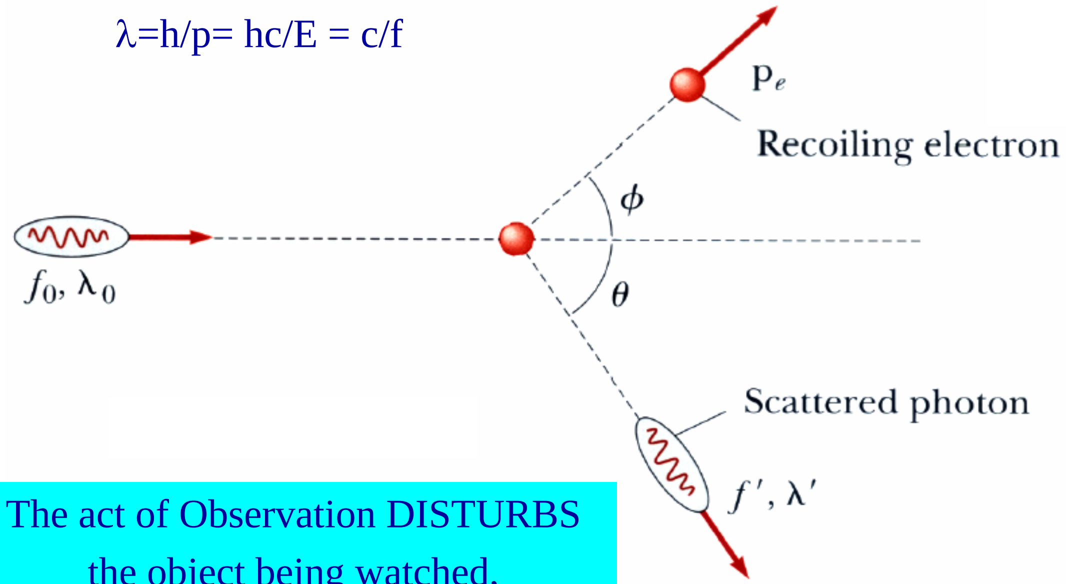
Act of observation disturbs the observed system



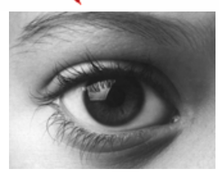


Compton Scattering: Shining light to observe electron

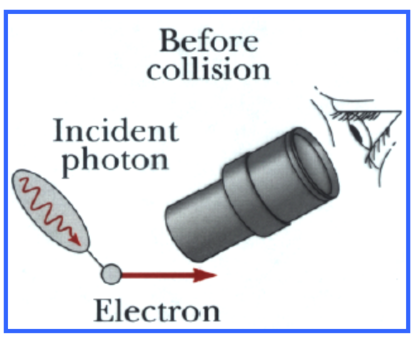
$\lambda = h/p = hc/E = c/f$

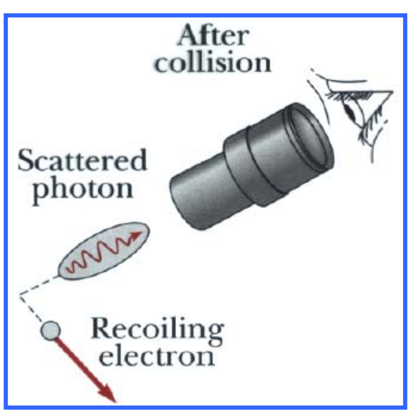


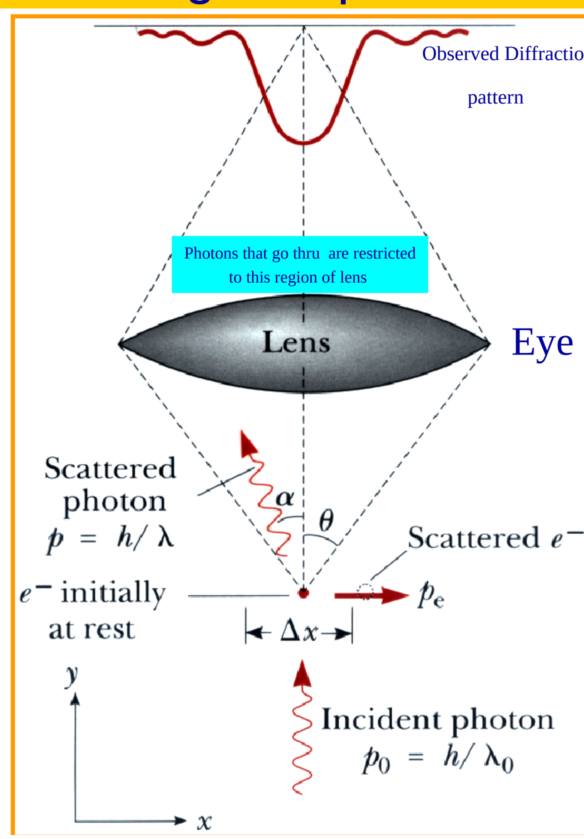
The act of Observation DISTURBS the object being watched, here the electron moves away from where it was originally



Act of Watching: A Thought Experiment







Diffraction By a Circular Aperture (Lens)

See Resnick, Halliday Walker 6th Ed (on S.Reserve), Ch 37, pages 898-900

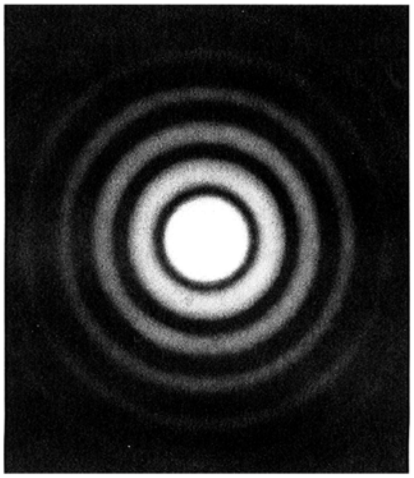


Fig. 37-9 The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

Diffracted image of a point source of light thru a lens (circular aperture of size d)

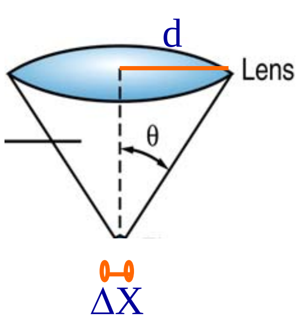
First minimum of diffraction pattern is located by

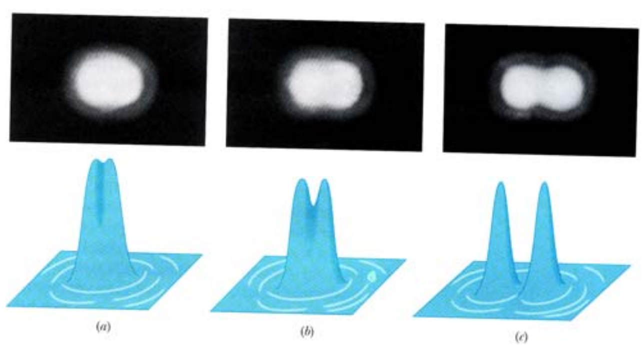
$$\sin\theta = 1.22\frac{\lambda}{d}$$

See previous picture for definitions of ϑ, λ, d

Resolving Power of Light Thru a Lens

Image of 2 separate point sources formed by a converging lens of diameter d, ability to resolve them depends on λ & d because of the Inherent diffraction in image formation



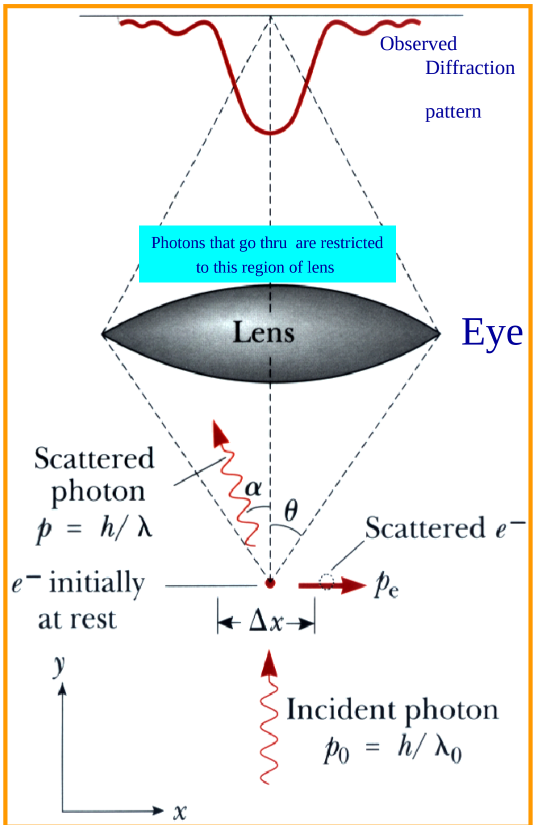


Not resolved | barely resolved | resolved

Resolving power $\Delta x \simeq \dfrac{\lambda}{2\sin\theta}$

ϑ Depends on d

Putting it all together: act of Observing an electron



- Incident light (p,λ) scatters off electron
- To be collected by lens → γ must scatter thru angle α
  - $-\vartheta \leq \alpha \leq \vartheta$
- Due to Compton scatter, electron picks up momentum
  - P_X , P_Y

$$-\frac{h}{\lambda}\sin\theta \leq P_x \leq \frac{h}{\lambda}\sin\theta$$

electron momentum uncertainty is

$$\Delta p \cong \frac{\sim 2h}{\lambda}\sin\theta$$

- After passing thru lens, photon diffracts, lands somewhere on screen, image (of electron) is fuzzy
- How fuzzy ? Optics says shortest distance between two resolvable points is :

$$\Delta x = \frac{\lambda}{2\sin\theta}$$

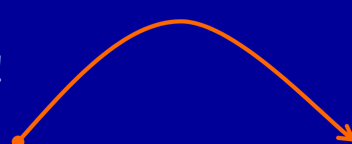
- Larger the lens radius, larger the $\vartheta \Rightarrow$ better resolution

$$\Rightarrow \Delta p.\Delta x \simeq \left(\frac{2h\sin\theta}{\lambda}\right)\left(\frac{\lambda}{2\sin\theta}\right) = h$$

$$\Rightarrow \Delta p.\Delta x \geq \hbar/2$$

Pseudo-Philosophical Aftermath of Uncertainty Principle

- Newtonian Physics & Deterministic physics topples over
  - Newton's laws told you all you needed to know about trajectory of a particle
    - Apply a force, watch the particle go !
      - Know every thing ! X, v, p , F, a
      - Can predict exact trajectory of particle if you had perfect device



- No so in the subatomic world !
  - Of small momenta, forces, energies
  - Cant predict anything exactly
    - Can only predict probabilities
      - There is so much chance that the particle landed here or there
      - Cant be sure !....cognizant of the errors of thy observations

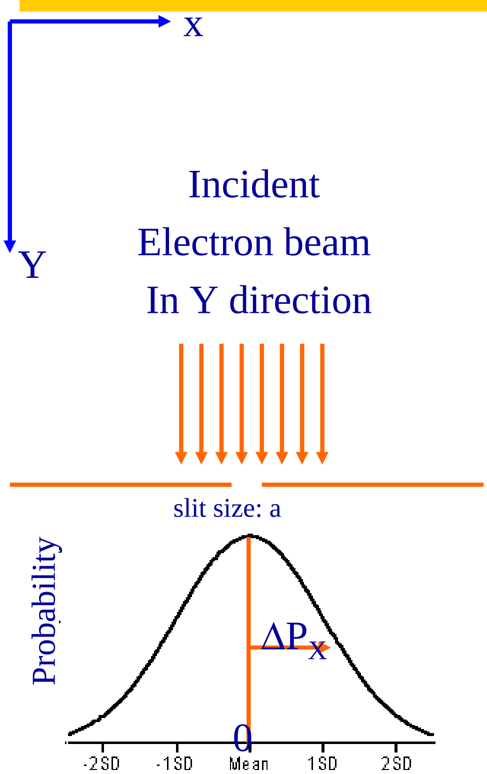
Philosophers went nuts !...what has happened to nature

Nothing is CERTAIN any more… life, job….nothing !

All Measurements Have Associated Errors

- If your measuring apparatus has an intrinsic inaccuracy (error) of amount Δp
- Then results of measurement of momentum p of an object at rest can easily yield a range of values accommodated by the measurement imprecision :
  - $-\Delta p \leq p \leq \Delta p$: you will measure any of these values for the momentum of the particle
- Similarly for all measurable quantities like x, t, Energy !

Matter Diffraction & Uncertainty Principle



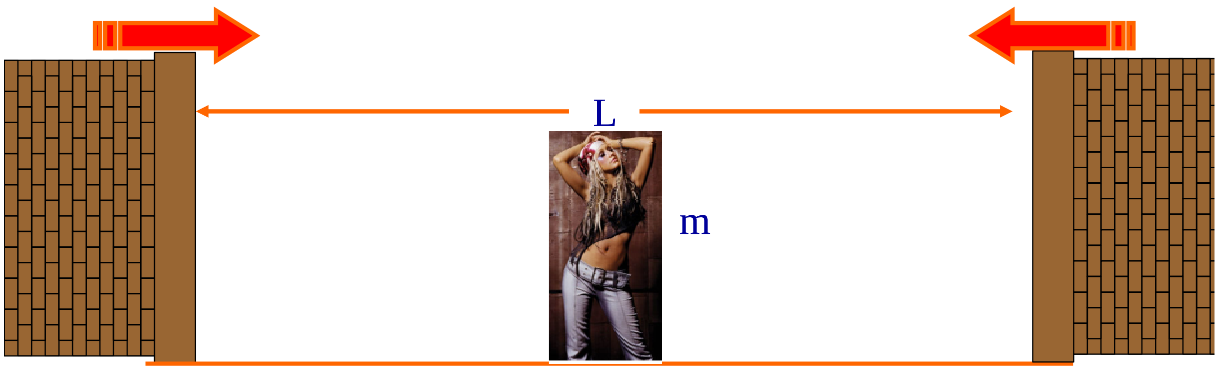
X component P_X of momentum

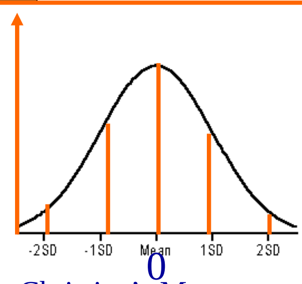
Momentum measurement beyond slit show particle not moving exactly in Y direction, develops a X component Of motion $-\Delta p_x \leq p_x \leq \Delta p_x$ **with**

$$\Delta p_X = h/(2\pi\, a)$$

Making Christina Dance !

Object of mass M at rest between two walls originally at infinity
What happens to our perception of Christina’s momentum as the walls are brought in ?





Christina’s Momentum p

On average, measure <p> = 0
but there are quite large fluctuations!
Width of Distribution = ΔP

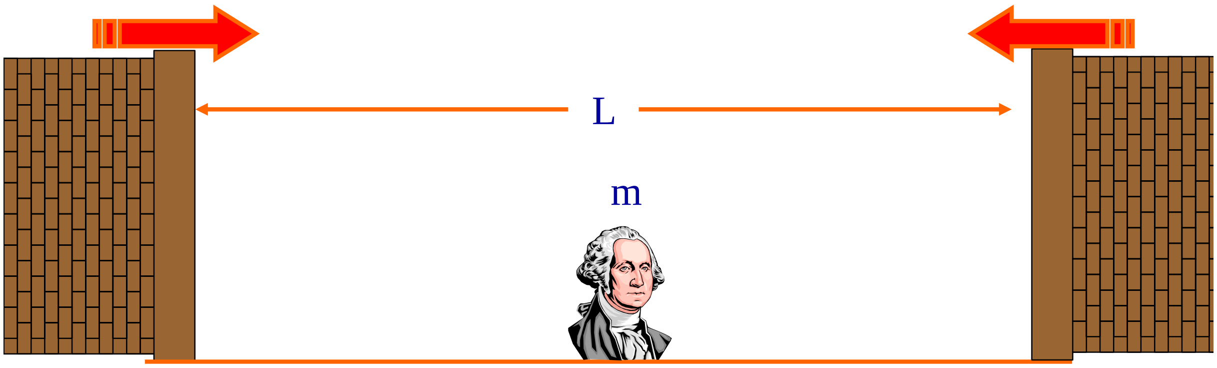
$$\Delta P = \sqrt{(P^2)_{ave} - (P_{ave})^2}; \quad \Delta P \sim \frac{\hbar}{L}$$

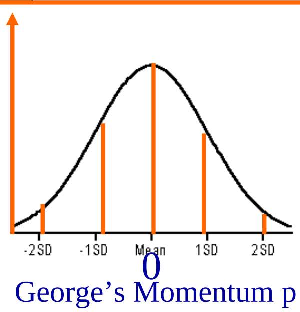
Discuss example problems from book

4.10, 4.11, 14.12

Particle at Rest Between Two Walls

Object of mass M at rest between two walls originally at infinity
What happens to our perception of George's momentum as the walls are brought in ?





On average, measure <p> = 0
but there are quite large fluctuations!
Width of Distribution = ΔP

$$\Delta P = \sqrt{(P^2)_{ave} - (P_{ave})^2}; \quad \Delta P \sim \frac{\hbar}{L}$$

Somewhere (ΔX= ∞) Christina is originally at rest (Δv=0)
And in no mood to dance !

wall

wall