Department of Physics
University of California
San Diego

Modern Physics (2D)
Prof. V. Sharma
Quiz # 4 (Feb 6 2004)

Some Relevant Formulae, Constants and Identities

Reduced Mass of a two-body system : $\mu = \dfrac{M_1 M_2}{M_1 + M_2}$

Charged particle q in a B Field experiences $\vec{F} = q(\vec{v} \times \vec{B})$

Centripetal Acc. $= \dfrac{u^2}{r}$

Bohr's Angular Momentum Quantization: $mvr = n\hbar$

Bohr Radius for Ground state H atom, $a_0 = \dfrac{\hbar^2}{mke^2} = 0.529\ \dot{A}$

Quantized Orbit in Hydrogenlike atom $r_n = \dfrac{n^2 a_0}{Z}$

Energy in Hydrogen atom $E_n = \dfrac{-ke^2}{2a_0}\left(\dfrac{1}{n^2}\right) = \left(\dfrac{-13.6\text{ eV}}{n^2}\right)$

Coulomb's Constant $k = 8.988 \times 10^9\ \text{N.m}^2/\text{C}^2$

Planck's Constant $h = 6.626 \times 10^{-34}$ J.s $= 4.136 \times 10^{-15}$ eV.s

1 eV = 1.60×10^{-19} J, Electron Mass $= 9.1 \times 10^{-31}$ Kg = 0.511 MeV/c^2

Proton Mass = 938.3 MeV/c^2, Muon Mass = $1.8835326 \times 10^{-28}$ Kg

Speed of Light in Vaccum $c = 2.998 \times 10^8$ m/s

Electron Charge = 1.602×10^{-19} C

Pl. write you answer in the Blue Book in indelible ink. Make sure your code number is prominently displayed on each page.

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Problem 1 : Reachout and Touch Some One [10 pts]

The nuclear radius of Gold (Atomic number Z= 79) is 7.0 fm. The nuclear radius of Aluminum (Z=13) is 3.6 fm. The radius of an alpha particle (He^{++} ion) is 2.6 fm. Alpha particles of what kinetic energy are needed for a head on collision in which the two nuclear surfaces just touch each other [This is the distance at which the Strong Nuclear force becomes effective] for the (a) Aluminum and (b) Gold nucleus.

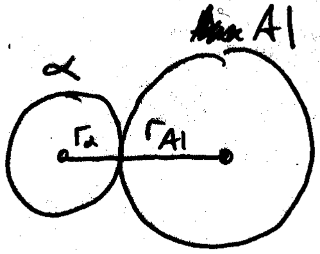
Hint: Start by drawing a clear sketch of the scattering process. In case you need it, 1fm = 10^{-15} m.

Problem 2: A Not So "Bohring" Atom [10 pts]

A muon is a particle with same charge as an electron and a rest mass 207 times larger. "Designer" atoms can be made of a muon particle around a deuteron (^2H) nucleus of charge Q = +1. The mass of the muon is not insignificant compared to the mass of the nucleus. Using the concepts of reduced mass, Energy conservation and Newton's second law (Coulombic attraction = Centripetal force) (a) find the expressions for the allowed orbit radii (relative to the center of mass of the two particles) and (b) the ground state energy of this muonic atom.

2D Quiz 4 Sols

1]



The distance between the two nuclei (or, more specifically, the centers of the two) is $r_\alpha + r_{Al}$.

radius of α radius of Al.

The potential energy is thus $K\dfrac{(2e)(Z_{Al}e)}{r_\alpha + r_{Al}}$

(The reason we wanted the dist. between the centers is b/c that's what matters in Coulomb's law)

The KE of the α must get entirely converted to this PE, so

$$KE_\alpha = K\frac{(2e)(Z_{Al}e)}{r_\alpha + r_{Al}} = K\frac{(2e)(13e)}{2.6+3.6\text{ fm}}$$

$$= 9.65\times10^{-13}\text{ J} = \boxed{6.03\text{ MeV}}$$

a]

b] And for gold, it's $3.79\times10^{-12}\text{ J} = \boxed{23.7\text{ MeV}}$

2] Ang. Mom. Quantization $\mu v r = n\hbar$

Newton's 2nd Law $\dfrac{\mu v^2}{r} = k\dfrac{e^2}{r^2}$ b/c charge of $\mu = -e$, charge of $^4H = e$

Energy Cons $E = \frac{1}{2}\mu v^2 - \dfrac{ke^2}{r}$

a] $\mu v r = n\hbar \Rightarrow v = \dfrac{n\hbar}{\mu r}$

$$\frac{\mu v^2}{r} = \frac{ke^2}{r^2} \Rightarrow \mu v^2 = \frac{ke^2}{r} \Rightarrow \mu\left(\frac{n\hbar}{\mu r}\right)^2 = \frac{ke^2}{r}$$

So $\dfrac{\mu n^2\hbar^2}{\mu^2 r^2} = \dfrac{ke^2}{r} \Rightarrow r_n = \dfrac{n^2\hbar^2}{\mu k e^2}$

Now, $$\mu = \frac{m_{muon} m_{2H}}{m_{muon} + m_{2H}} = \frac{(207 m_e)(m_n + m_p)}{207 m_e + m_n + m_p}$$

$$= 1.78 \times 10^{-28}$$ (different than m_{muon}!)

So $$\boxed{r_n = (2.71 \times 10^{-13}\, m)\, n^2 = (2.71 \times 10^{-3}\, Å)\, n^2}$$

b) Now,

$$E = \frac{1}{2}\mu v^2 - \frac{ke^2}{r}$$

We know $\frac{\mu v^2}{r} = \frac{ke^2}{r^2} \Rightarrow \mu v^2 = \frac{ke^2}{r}$, so

$$E = -\frac{1}{2}\frac{ke^2}{r}$$

$$\Rightarrow E_n = -\frac{1}{2}\frac{ke^2}{r_n} = -\frac{1}{2}ke^2 \frac{1}{(2.71\times10^{-13}\,\mathrm{m})n^2}$$

$$= \frac{-4.24\times10^{-16}\,\mathrm{J}}{n^2} = \frac{-2.6\,\mathrm{keV}}{n^2}$$

So the ground state energy is $\boxed{E_1 = -2.6\,\mathrm{keV}}$