Department of Physics
University of
California San Diego

Modern Physics (2D)
Prof. V. Sharma
Quiz#2 (Jan 23 2004)

Some Relevant Formulae, Constants and Identities

$$x^{'} = \gamma(x - vt), \quad t^{'} = \gamma(t - \frac{vx}{c^2}), u_{x^{'}} = \frac{u_x - v}{1 - (\frac{u_x v}{c^2})}, \quad u_{y^{'}} = \frac{u_y}{\gamma\left[1 - (\frac{u_x v}{c^2})\right]}$$

$$p = \gamma m u, \quad K = \gamma mc^2 - mc^2, \ E = K + mc^2 = \gamma mc^2$$

Centripetal Acc.= $\frac{u^2}{r}$

For particle (q,m) in B field : $p = \gamma m u = qBR$

$$\int \frac{du}{(1-u^2)^{3/2}} = \frac{u}{(1-u^2)^{1/2}} + \text{ Constant}$$

$$E^2 = (pc)^2 + (mc^2)^2$$

1 eV = 1.602×10^{-19} J

Electron rest mass =9.11×10^{-31}Kg =8.2×10^{-14} J/c^2= 0.511 MeV/c^2

Proton rest mass = 1.673×10^{-27}Kg = 938.3MeV/c^2

Speed of Light in Vaccum c = 2.998×10^{8}m/s

Electron Charge = 1.602×10^{-19} C

Atomic mass unit u = 1.6605×10^{-27}kg = 931.49 MeV/c^2

Pl. write you answer in a Blue book in indelible ink. Make sure your code number is prominently displayed on each page

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Problem 1: Ultimate Fuel ! **[6 pts]** The fictional starship ***Enterprise*** obtains its power by combining matter and antimatter of equal mass (magnetically stored in the fuel tanks), achieving complete conversion of mass into energy (100% efficiency). If the mass of the ***Enterprise*** is 5×10^9 kg, how much of its net mass must be converted into kinetic energy to accelerate it from rest to a speed of u = 0.1c?

Problem 2: The Need For Speed ! **[14 pts]** A particle of charge q and mass m moves along a straight line in a uniform electric field **E** with speed v. If the motion and the electric field are both in the same direction (say x) (a) show that the magnitude of acceleration a of this particle is given by :

$$a = \frac{dv}{dt} = \frac{qE}{m}\left(1 - \left(\frac{v}{c}\right)^2\right)^{\frac{3}{2}}$$

(b) If the particle starts from rest at x=0 at t=0 find the expression for the speed of the particle after a time t has elapsed. (c) What is the limiting value of its speed v as $t\to\infty$?

For an electric field **E** = 10^7 V/m, find the speed of an electron which is accelerated from rest for a time period of just (c) 0.10ns and (d) 10.0 ns in Prof. Sharma's linear accelerator. Express your answer as a fraction of c. What would be the speed of the electron if you had used the Newtonian expression for acceleration instead of the relativistic one for (e) 0.10ns and (f) 10.0ns ?

2D Quiz 2 Solns

1] First, find the change in energy. We can assume here that the mass doesn't change much, since the final speed is fairly slow (0.1c).

So assuming no change in mass, the final KE is $KE = (\gamma - 1)mc^2$. This corresponds to a mass of $(\gamma-1)m$, just by using $\frac{E}{c^2} = \text{mass}$.

$$\text{So, } (\gamma - 1) = \frac{1}{\sqrt{1-(.1)^2}} - 1 = 5\times10^{-3}$$

$$\Rightarrow (\gamma-1)m = (5\times10^{-3})(5\times10^{9}) = \boxed{2.5\times10^{7}\,\text{kg}}$$

This means our approximation was OK - the mass didn't change that much, only by 0.5%.

2a] $F = qE = \frac{dp}{dt} \Rightarrow qE = \frac{d}{dt}\left(\frac{mv}{\sqrt{1-v^2/c^2}}\right)$

$$\Rightarrow \frac{qE}{m} = \frac{dv}{dt}\left(\frac{1}{\sqrt{1-v^2/c^2}}\right) + v\frac{1}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\left(-\cancel{\frac{1}{2}}\right)\left(-\frac{\cancel{2}v}{c^2}\right)\frac{dv}{dt}$$

$$\Rightarrow \frac{qE}{m} = \frac{dv}{dt}\left(\frac{1}{(1-v^2/c^2)^{1/2}} + \frac{v^2/c^2}{(1-v^2/c^2)^{3/2}}\right)$$

$$\Rightarrow \frac{qE}{m} = \frac{dv}{dt}\frac{1}{(1-v^2/c^2)^{3/2}} \Rightarrow \boxed{a \equiv \frac{dv}{dt} = \frac{qE}{m}\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

b) Now, we want v.

$$\frac{dv}{dt} = \frac{qE}{m}\left(1-\frac{v^2}{c^2}\right)^{3/2} \Rightarrow \frac{dv}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} = \frac{qE}{m}\,dt$$

Integrate both sides $\Rightarrow \displaystyle\int \frac{dv}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} \overset{u=\frac{v}{c}}{=} \int \frac{c\,du}{(1-u^2)^{3/2}}$

$$= \frac{cu}{(1-u^2)^{1/2}} = \frac{v}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}$$

and $\displaystyle\int \frac{qE}{m}dt = \frac{qEt}{m}$ (Note that we used $v=0$ at $t=0$ when doing the integral)

$$\Rightarrow \frac{v}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} = \frac{qEt}{m} \Rightarrow v^2 = \left(\frac{qEt}{m}\right)^2\left(1-\frac{v^2}{c^2}\right)$$

$$\Rightarrow v^2\left(1+\left(\frac{qEt}{mc}\right)^2\right) = \left(\frac{qEt}{m}\right)^2$$

$$\Rightarrow \boxed{v = \frac{qE}{m}\frac{t}{\left(1+\left(\frac{qEt}{mc}\right)^2\right)^{1/2}}}$$

c] As $t \to \infty$, $v \to \frac{qE}{m} \frac{t}{\left(\frac{qEt}{mc}\right)} = \boxed{c}$

d] Plug in $E = 10^7$ V/m, $q = 1.6 \times 10^{-19}$,
$m = 9.11 \times 10^{-31}$ kg, and $t = .1 \times 10^{-9}$ s

$\Rightarrow \boxed{v = 0.51c}$

e] $t = 10 \times 10^{-9}$ s $\Rightarrow \boxed{v = 0.9999c}$

f] Here, $a = \frac{qE}{m}$, so $v = \frac{qEt}{m}$,
$t = .1 \times 10^{-9}$ s $\Rightarrow \boxed{v = 0.57c}$

g] $t = 10 \times 10^{-9}$ s $\Rightarrow \boxed{v = 58.6c}$ WOW!