Department of Physics
University of California
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Modern Physics (2D)
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Final Exam
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$$\text{Speed of Light, } c = 2.998 \times 10^8 \text{m/s}; \ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$x' = \gamma(x - vt); \quad t' = \gamma(t - \frac{xv}{c^2})$$

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}; \ u'_z = \frac{u_z}{\gamma(1 - \frac{u_z v}{c^2})}$$

$$p = \frac{mu}{\sqrt{1 - u^2/c^2}}; \ E = \frac{mc^2}{\sqrt{1 - u^2/c^2}} = K + mc^2$$

$$E^2 = p^2c^2 + m^2c^4$$

$$f_{obs} = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} f_{source}$$

$$\text{For } x < 1; (1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{2}x^2 \pm \frac{n(n-1)(n-2)}{3}x^3 + ..$$

$$\text{Planck's Constant, } h = 6.626 \times 10^{-34} \text{J} \times \text{S} = 4.136 \times 10^{-15} \text{eV} \times \text{S}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{J}; \quad 1 \text{ MeV/c} = 5.344 \times 10^{-22} \text{Kg.m/s}$$

$$\text{Coulomb's Constant, } k = 8.99 \times 10^9 \text{N} \times \text{m}^2/\text{C}^2$$

$$\text{Gravitational Constant, } G = 6.67 \times 10^{-11} \text{N} \times \text{m}^2/\text{kg}^2$$

$$\text{Stefan} - \text{Boltzmann's Constant, } \sigma = 5.670399 \times 10^{-8} \text{W/m}^2 \times \text{K}^4$$

$$\text{Wien's Wavelength Displacement Constant} = 2.898 \times 10^{-3} \text{m} \times \text{K}$$

$$\text{Boltzmann's Constant, } k = 1.381 \times 10^{-23} \text{J/K}$$

$$\text{Electron Mass} = 9.11 \times 10^{-31} \text{Kg}; \ \text{Electron Charge} = 1.602 \times 10^{-19} \text{C}$$

$$\text{Atomic Mass Unit } u = 1.6606 \times 10^{-27} \text{Kg or } 931.5 \text{ MeV/c}^2$$

$$\text{Proton Mass} = 1.673 \times 10^{-27} \text{Kg or } 1.0073\text{u}$$

$$\text{Neutron Mass} = 1.675 \times 10^{-27} \text{Kg or } 1.0087\text{u}$$

$$\text{Electron Rest Energy} = 0.511 \text{ MeV/c}^2; \quad \text{Proton Rest Energy} = 938 \text{ MeV/c}^2$$

$$1\text{kW} \times \text{Hour} = 3.6 \times 10^6 \text{J}$$

$$\text{Intensity of Black Body Radiation } I = \sigma \times T^4$$

$$\text{Force on a charged particle in B field : } \vec{F} = q\vec{v} \times \vec{B}$$

$$\text{Centripetal Acc.} = v^2/R \text{ where v and R are velocity and radius of orbit}$$

$$\text{Momentum of a charged particle in B field : } p = qBR$$

$$\text{Photoelectric Effect : } K_{max} = hf - \phi$$

$$\text{Compton Wavelength } \lambda_c \text{ for scattering off electron} = 0.00243 \text{ nm}$$

$$\text{In Compton Scattering } \Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\text{Construct. interfer. when path diff. between two adjacent rays is } d\sin\phi = n\lambda$$

$$\text{TDSE : } -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

$$\text{TISE : } -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x) = E\psi(x)$$

$$\text{Ground state energy } E_1 \text{ for particle in 1D rigid box}(0 < x < L) = \frac{\pi^2\hbar^2}{2mL^2}$$

$$\text{Normalized wavefunction of particle in 1D rigid box}(0 < x < L) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi}{L}x)$$

$$\text{Operator : } [\hat{p}] = \frac{\hbar}{i}\frac{\partial}{\partial x}, \quad [\hat{p^2}] = -\hbar^2\frac{\partial^2}{\partial x^2}, \; [\hat{E}] = i\hbar\frac{\partial}{\partial t}$$

Time dependent form of Wave Function: $\Psi(\vec{r},t) = \psi(\vec{r})e^{-i\omega t}\,;\, \omega = E/\hbar$

$$\text{Uncertainty on Observable Q : } \Delta Q = \sqrt{\langle Q^2\rangle - \langle Q\rangle^2}$$

$$\text{ExpectationValue} < Q > = \int \psi(x)^*[Q]\psi(x)dx$$

$$< f(r) > = \int_0^\infty f(r)P(r)dr$$

$$\int u \cdot dv = uv - \int v \cdot du$$

$$\int \sin x \, dx = -cosx$$

$$\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4}; \quad \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4}$$

$$\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}$$

$$\int x \cos^2 x \, dx = \frac{x^2}{2} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8}$$

$$\int x^2 \sin^2 x \, dx = \frac{x^3}{6} - (\frac{x^2}{4} - \frac{1}{8}) \sin 2x - \frac{x \cos 2x}{4}$$

$$\sin(\theta_1)\sin(\theta_2) = (1/2)[\cos(\theta_1 - \theta_2) - \cos(\theta_1 + \theta_2)]$$

$$\cos(\theta_1)\cos(\theta_2) = (1/2)[\cos(\theta_1 - \theta_2) - \cos(\theta_1 + \theta_2)]$$

$$\sin(\theta_1)\cos(\theta_2) = (1/2)[\sin(\theta_1 - \theta_2) - \sin(\theta_1 + \theta_2)]$$

$$\int_{-1}^{+1} e^{-ax} \, dx = \frac{1}{a}[e^a - e^{-a}]$$

$$\int_{-1}^{+1} x e^{-ax} \, dx = \frac{1}{a^2}[e^a - e^{-a} - a(e^a + e^{-a})]$$

$$\int_{0}^{+\infty} e^{-ax^2} \, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}, \; a > 0$$

$$\int_{0}^{+\infty} x^2 e^{-ax^2} \, dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}, \; a > 0$$

$$\int_{0}^{+\infty} x^{(2n)} e^{-ax^2} \, dx = \frac{1.3..(2n-1)}{2^{(n+1)}}\sqrt{\frac{\pi}{a^{2n+1}}}, \; a > 0$$

$$\int_{0}^{+\infty} x^{(2n+1)} e^{-ax^2} \, dx = \frac{n!}{2a^{(n+1)}}, \; a > 0$$

$$\int_{0}^{+\infty} x^n e^{-x} \, dx = n!$$

$$\int_{0}^{+\infty} x^n e^{-x/\alpha} \, dx = n!\alpha^{n+1}$$

$$\int_1^{+\infty} e^{-ax}\,dx = \frac{e^{-a}}{a}$$

$$\int_1^{+\infty} xe^{-ax}\,dx = \frac{e^{-a}}{a^2}(1+a)$$

$$\int_0^b x^2 e^{-x}\,dx = 2 - (2 + 2b + b^2)e^{-b}$$

$$\int_0^b x^3 e^{-x}\,dx = 6 - (6 + 6b + 3b^2 + b^3)e^{-b}$$

$$\int_0^b x^4 e^{-x}\,dx = 24 - (24 + 24b + 12b^2 + 4b^3 + b^4)e^{-b}$$

$$\int_0^b x^2 e^{-x}\,dx = 2 - (2 + 2b + b^2)e^{-b}$$

The ground state wavefunction for 1D oscillator under $U(x) = \frac{1}{2}m\omega^2x^2$ has the form:

$$\psi_0(x) \propto e^{-\frac{m\omega x^2}{2\hbar}}$$

The wavefunction for Oscillator's first excited state:

$$\psi_1(x) \propto \sqrt{\frac{m\omega}{\hbar}}xe^{-\frac{m\omega x^2}{2\hbar}}$$

Next excited state:

$$\psi_2(x) \propto (1 - \frac{2m\omega x^2}{\hbar})e^{-\frac{m\omega x^2}{2\hbar}}$$

The energy of the nth Oscillator state $E_n = (n + \frac{1}{2})\hbar\omega$.

Volume element in Sph. polar coordinates is $dV = r^2 dr sin\theta d\theta d\phi$

<u>Information about Hydrogenic atom with Z protons in the nucleus:</u>

$$\text{Reduced Mass } \mu = \frac{M_1.M_2}{M_1 + M_2}$$

$$\mathrm{R_{1,0}(r) = (\frac{Z}{a_o})^{3/2} 2e^{-\frac{Zr}{a_o}}}$$

$$\mathrm{R_{2,0}(r) = (\frac{Z}{2a_o})^{3/2} (2 - \frac{Zr}{a_0})e^{-\frac{Zr}{2a_o}}}$$

$$\mathrm{R_{2,1}(r) = (\frac{Z}{2a_o})^{3/2} \frac{Zr}{\sqrt{3}a_0}e^{-\frac{Zr}{2a_o}}}$$

$$\text{Radial Prob. Density } \mathrm{P(r) \ = \ r^2|R(r)|^2}$$

$$\mathrm{Y_o^o}(\theta, \phi) = \Theta(\theta)\Phi(\phi) = \frac{1}{2\sqrt{\pi}}$$

$$\mathrm{Y_1^o}(\theta, \phi) = \frac{1}{2}\sqrt{\frac{3}{\pi}} \cos\theta \text{ and } \mathrm{Y_1^{\pm 1}}(\theta, \phi) = \mp\frac{1}{2}\sqrt{\frac{3}{2\pi}} \sin\theta \mathrm{e}^{\pm i\phi}$$

$$\text{Orbit radius } \mathrm{r_n \ = \ \frac{n^2\hbar^2}{Zm_eke^2}}$$

$$\text{bohr radius } \mathrm{a_0 = \frac{\hbar^2}{m_eke^2} = 0.0529 \, nm}$$

$$\text{Energy } \mathrm{E_n = \frac{-kZ^2e^2}{2a_o}\frac{1}{n^2}} \text{ for } \mathrm{n = 1, 2, 3, 4, ...}$$

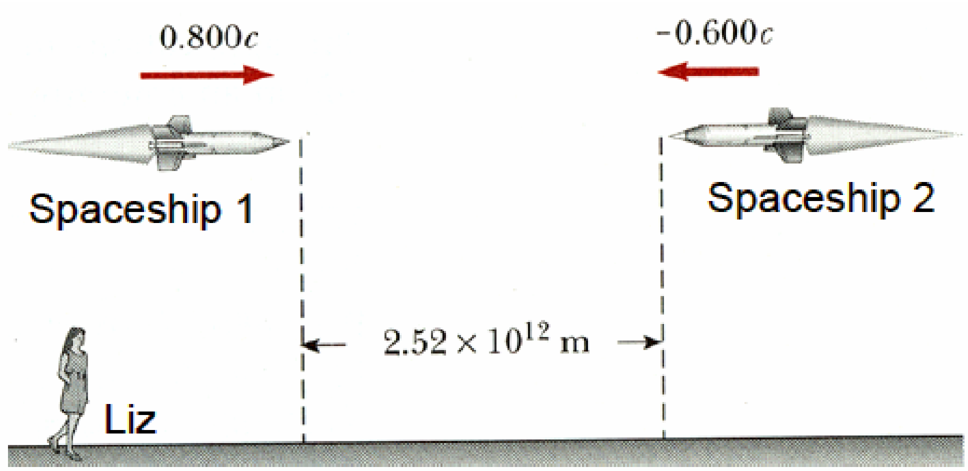
$$\text{Rydberg Constant } \mathrm{R = 1.0973732 \times 10^7 m^{-1}}$$

Problem 1: Weapons of Mass Destruction [20 pts]

A Nuclear bomb containing 8.0 kg of Plutonium explodes. The sum of the rest masses of the products of the explosion is less than the original rest mass by 0.01% (a) How much energy is released in the explosion (b) If the explosion takes place in a time period of 4.0μs, what is the average power deployed by the bomb? (c) What mass of water could the released energy lift to a height of 1.0 km.

Problem 2: A Game of Relativistic "Chicken" [30 pts]

In an exchange of prior gunfire two attacking spaceships are rendered powerless. They are on a collision course. The spaceships are moving with speeds of 0.80c (**spaceship 1)** and 0.60c (**spaceship 2)** and are initially 2.52×10^{12} m apart as measured by Liz, an Earth observer, as shown in the figure below. Both spaceships are 500m in length as measured by Liz. (a) What are their respective proper lengths? (b) what is the length of the each spaceship as measured by an observer in the other spaceship? (c) According to Liz, how long will it be before the spaceships collide? (d) According to Spaceship 1, how long before they collide? (e) According to Spaceship 2, how long before they collide? (f) If both spaceships are capable of evacuation within 90 minutes (by their own clock), will there be any casualties?

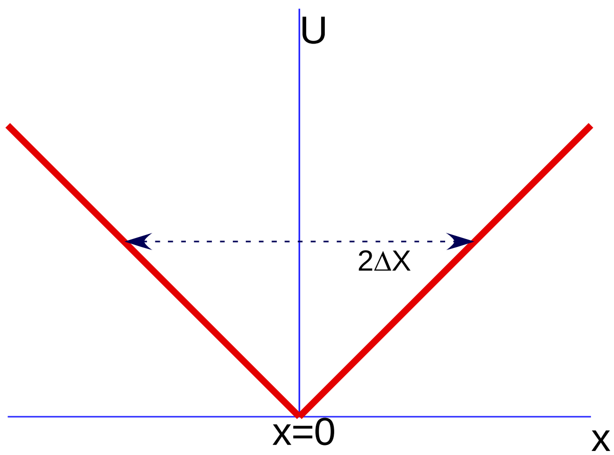


Problem 3: Mystery Metal : [20 pts]

Photons of wavelength 450nm are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular arc of radius 20cm in a magnetic field whose strength is equal to 2.0×10^{-5}T. What is the work function of the metal?

Problem 4: Quantum Pool: **[20 pts]**
An x-ray photon of wavelength 0.050nm strikes a free stationary electron. The photon scatters off at 90^o with respect to the direction of incidence. Determine the momentum of (a) the incident photon (b) the scattered photon and (c) the scattered electron.

Problem 5: The Wedge Function : **[20 pts]**



Consider a particle of mass m under a potential of the form

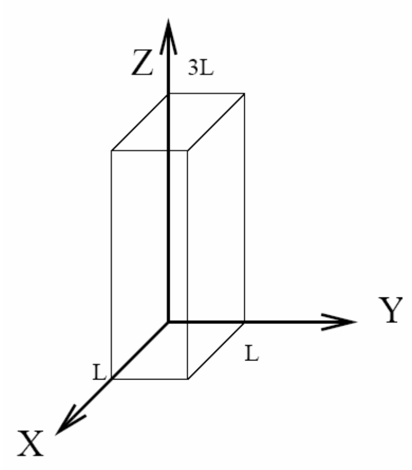
$\mathbf{U = -\ U_0\,(x/a)}$ for x < 0 and

$\mathbf{U = +\ U_0\,(x/a)}$ for x > 0.

(a) Use the uncertainty relations to write the expression for the total energy this particle as a function of x. (b) What is the lowest (***minimum***) energy of this particle. Assume the situation is non-relativistic.

Problem 6: Déjà vu All Over Again: [30pts]
Consider a particle of mass m moving in a one-dimensional box with rigid walls (infinite potential) at $\mathbf{x = -\ L/2}$ and $\mathbf{x = L/2}$. (a) Write down the expression for the normalized wavefunction for the ground state and sketch the wavefunction. Indicate the locations of the walls. Calculate (b) $\langle x \rangle$, (c) $\langle p \rangle$, (d) $\langle x^2 \rangle$, (e) $\langle p^2 \rangle$ and (f) calculate the product $\mathbf{\Delta x . \Delta p}$ in this case and compare it with the expectation from the Uncertainty principle.

Problem 7: Rapping About a Electron in a 3-D Rigid Box: **[30 pts]**
Suppose one puts an electron in a three-dimensional box of size L in the x and y direction but 3L in the z direction.



(a) What is the ground state energy of the electron in terms of $\hbar$, L and the electron mass m. (b) what is the energy of the first excited state ? What is the degeneracy of this energy level? (c) What is the probability of finding the electron between y=0.15L and y=0.35L in its ground state?

Problem 8: An Excited Hydrogen Atom: **[30 pts]**

The radial part of the wavefunction for the Hydrogen atom in n = 3, l =2 and m_l = 0 state is given by:

$$R_{32}(r) = \left(\frac{1}{3a_0}\right)^{3/2} \left(\frac{2\sqrt{2}}{27\sqrt{5}}\right)\left(\frac{r}{a_0}\right)^2 e^{-\frac{r}{3a_0}}$$

Calculate the average (a) potential and (b) kinetic energy for this configuration. [**Hint**: try to avoid needless computation when possible. Remember Energy Conservation rule.]

GOOD LUCK !

Physics 2D Final

1] a] The difference in mass is 0.01% of 8 kg, which is 8×10^{-4} kg.

$$E = mc^2 = (8\times10^{-4}\,\text{kg})c^2 = \boxed{7.2\times10^{13}\,\text{J}}$$

b] $$\text{Power} = \frac{\text{Energy}}{\text{Time}} = \boxed{1.8\times10^{19}\,\text{W}}$$

c] Use 7.2×10^{13} J to lift mass m of H_2O:

$$7.2\times10^{13} = mgh \Rightarrow m = \frac{7.2\times10^{13}}{9.8\cdot10^{3}}$$

$$\boxed{m = 7.35\times10^{9}\,\text{kg}}$$

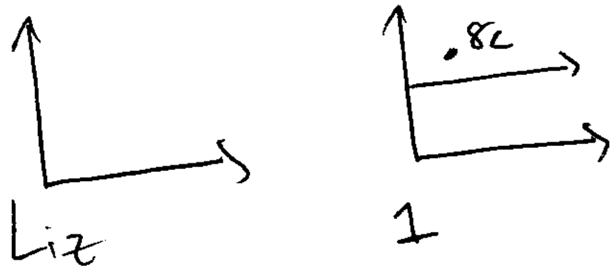
2] a] $$L_{Liz} = \sqrt{1-\frac{v^2}{c^2}}\,L_{prop}$$

So $$L_{prop} = \frac{L_{Liz}}{\sqrt{1-v^2/c^2}}$$

Plugging in v = .8c and .5c gives

$$\boxed{L_1 = 833\,\text{m},\quad L_2 = 625\,\text{m}}$$

b] Do a velocity transformation: First find how fast 2 moves in 1's frame:



$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\Rightarrow U_{2 \text{ in } 1} = \frac{-.6c - .8c}{1 - \frac{(-.6c)(.8c)}{c^2}}$$

$$= -0.946c$$

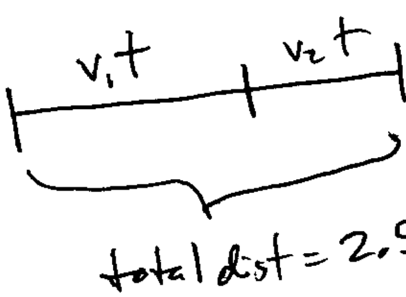
So: just contract proper length w/ this velocity:

$$L_{2 \text{ by } 1} = \sqrt{1 - (.946)^2}\; L_{2,\text{prop}} = \boxed{203\,m}$$

$$L_{1 \text{ by } 2} = \sqrt{1 - (.946)^2}\; L_{1,\text{prop}} = \boxed{270\,m}$$

b/c $U_{1 \text{ in } 2} = -U_{2 \text{ in } 1}$

c] After a time t, each ship has moved a distance vt:



So $0.8ct + 0.6ct = 2.52 \times 10^{12}\,m$

$$\Rightarrow t = \boxed{6000\,s} = 100\text{ min.}$$

d] Spaceship 1 sees a distance of

$$\sqrt{1-(.8)^2}\,(2.52\times10^{12}\,m) = 1.51\times10^{12}\,m$$

and since Spaceship 2 moves w/ speed $0.946c$,

it takes $\dfrac{1.51\times10^{12}\,m}{.946c} = \boxed{5321\,s} = 88.7$ min to hit.

e]. Same logic: #2 sees a dist. of

$$\sqrt{1-(.6)^2}\,(2.52\times10^{12}\,m) = 2.016\times10^{12}\,m$$

So it takes $\dfrac{2.016\times10^{12}\,m}{.946c} = \boxed{7104\,s} = 118$ min.

f] The occupants of Spaceship 1 all die.
The people in Spaceship 2 live!

3] $p_{max} = qBR = 6.4 \times 10^{-25}\ \frac{kg \cdot m}{s}$

So $KE_{max} = \frac{p^2}{2m_e} = 2.25 \times 10^{-19}\ J = 1.4\,eV$

$E_{photon} = \frac{hc}{\lambda} = 2.76\,eV.$

$KE_{max} = E_{photon} - \phi \Rightarrow \boxed{\phi = 1.36\,eV}$

4] a] $E_{incident} = \frac{hc}{\lambda} = 3.98 \times 10^{-15}\ J = 2.48 \times 10^{4}\ eV$

So $p = \frac{E}{c} = \frac{h}{\lambda} = \boxed{1.32 \times 10^{-23}\ \frac{kg \cdot m}{s} = 2.48 \times 10^{4}\ \frac{eV}{c}}$

b] $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \Rightarrow \lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\theta)$

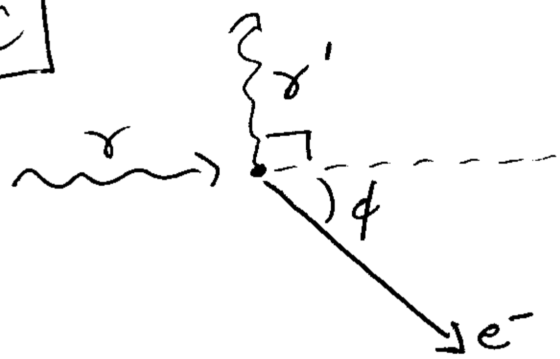
$\theta = 90^\circ$, $\cos 90^\circ = 0$, so $\lambda' = \lambda + \frac{h}{m_e c} \Rightarrow \lambda' = \lambda + 0.00243\,nm$

$\Rightarrow \lambda' = 5.243 \times 10^{-2}\ nm$

$\Rightarrow p' = \frac{h}{\lambda'} = \boxed{1.26 \times 10^{-23}\ \frac{kg \cdot m}{s} = 2.34 \times 10^{4}\ \frac{eV}{c}}$

AM

c]



By momentum conservation, $p_e \sin\phi = p_{\gamma'}$
$p_e \cos\phi = p_\gamma$

So $p_e^2 = p_{\gamma'}^2 + p_\gamma^2 \Rightarrow$

$$p_e^2 = 3.33\times10^{-46}\left(\frac{kg\cdot m}{s}\right)^2$$

$$p_e = \boxed{1.82\times10^{-23}\ \frac{kg\cdot m}{s} = 3.42\times10^{4}\ \frac{eV}{c}}$$

5] $E = \frac{p^2}{2m} + U$. Now, using $\Delta p = \frac{\hbar}{2\Delta x} \Rightarrow p = \frac{\hbar}{2x}$
(since $\langle p\rangle = \langle x\rangle = 0$)

$$\Rightarrow \boxed{E = \left(\frac{\hbar}{2x}\right)^2 \frac{1}{2m} + U(x)}$$ where $U(x) = \begin{cases} -\frac{U_0 x}{a} & x<0 \\ \frac{U_0 x}{a} & x>0 \end{cases}$

b] $$\frac{dE}{dx} = 2\left(\frac{\hbar}{2x}\right)\cdot\frac{-\hbar}{2x^2}\frac{1}{2m} \mp \frac{U_0}{a} = \frac{-\hbar^2}{4mx^3} \mp \frac{U_0}{a} = 0$$

where $\mp$ is for $x \lessgtr 0$.

Solve: $\mp \frac{U_0}{a} = \frac{\hbar^2}{4mx^3} \Rightarrow x^3 = \mp \frac{\hbar^2 a}{4mU_0}$

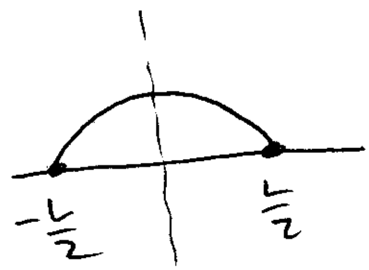
$$\Rightarrow x = \mp \left(\frac{\hbar^2 a}{4mU_0}\right)^{1/3}.$$

$$\text{So } E = \frac{\hbar^2}{8mx^2} \mp \frac{U_0 x}{a} = \frac{\hbar^2}{8m}\left(\frac{4mU_0}{\hbar^2 a}\right)^{2/3} + \frac{U_0}{a}\left(\frac{\hbar^2 a}{4mU_0}\right)^{1/3}$$

$$= \frac{1}{2}\left[\frac{\hbar^{2/3}U_0^{2/3}}{(4m)^{1/3}a^{2/3}}\right] + \frac{\hbar^{2/3}U_0^{2/3}}{a^{2/3}(4m)^{1/3}}\Bigg]$$

$$= \boxed{\frac{3}{2}\left(\frac{\hbar^2 U_0^2}{4ma^2}\right)^{1/3}}$$

6a] $\boxed{\psi(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)}$



b] $\langle x \rangle = 0$

c] $\langle p \rangle = 0$

d] $\langle p^2 \rangle = 2mE = 2m\frac{\hbar^2\pi^2}{2mL^2} = \frac{\hbar^2\pi^2}{L^2}$

e] $\langle x^2 \rangle = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{2}{L}x^2\cos^2\left(\frac{\pi x}{L}\right)dx \Rightarrow \begin{array}{l} u = \frac{\pi x}{L} \\ du = \frac{\pi}{L}dx \end{array}$

$$= \frac{2}{L}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{L}{\pi}\right)^3(\cos^2 u)u^2\,du = \frac{2L^2}{\pi^3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} u^2\cos^2 u\,du$$

can do from $\cos^2 u + \sin^2 u = 1$, using integral on eqn. sheet.

$$= \frac{2L^2}{\pi^3}\left[\frac{u^3}{6} + \frac{1}{4}u\cos 2u + \frac{1}{8}(-1+2u^2)\sin 2u\right|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{2L^2}{\pi^3}\left[\left(\frac{\pi}{2}\right)^3\frac{1}{6} + \frac{1}{4}\cdot\frac{\pi}{2}(-1) - \left[\left(-\frac{\pi}{2}\right)^3\frac{1}{6} + \frac{1}{4}\left(-\frac{\pi}{2}\right)(-1)\right]\right]$$

$$= \frac{2L^2}{\pi^3}\left[\frac{\pi^3}{24} - \frac{\pi}{4}\right] = \boxed{L^2\left[\frac{1}{12} - \frac{1}{2\pi^2}\right]}$$

f] $\Delta x = L\left(\frac{1}{12} - \frac{1}{2\pi^2}\right)^{1/2}$

$\Delta p = \frac{\hbar\pi}{L}$

$$\Rightarrow \Delta x\,\Delta p = \hbar\pi\left(\frac{1}{12} - \frac{1}{2\pi^2}\right)^{1/2}$$

$$= \boxed{0.56\hbar > \frac{\hbar}{2}}\ !$$

7] $$E_{n_x n_y n_z} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right)$$

$$= \frac{\hbar^2\pi^2}{2mL^2}\left(n_x^2 + n_y^2 + \frac{n_z^2}{9}\right)$$

So $$E_{111} = \frac{\hbar^2\pi^2}{2mL^2}\left(1+1+\frac{1}{9}\right) = \boxed{\frac{19\hbar^2\pi^2}{18mL^2}}$$

b] 1st excited state: $$E_{112} = \frac{\hbar^2\pi^2}{2mL^2}\left(1+1+\frac{4}{9}\right)$$

$$= \frac{\hbar^2\pi^2}{2mL^2}\left(\frac{22}{9}\right) = \boxed{\frac{11\hbar^2\pi^2}{9mL^2}}$$

This has degeneracy $\boxed{1}$ (i.e. not degenerate)

c] x & z integrals are 1, so just do y:

$$\frac{2}{L}\int_{.15L}^{.35L} \sin^2\left(\frac{\pi y}{L}\right)dy = \frac{2}{\pi}\int_{.15\pi}^{.35\pi} \sin^2 u\, du = \frac{2}{\pi}\left[\frac{u}{2} + \frac{\sin 2u}{4}\right|_{.15\pi}^{.35\pi}$$

$$= \boxed{0.2}$$

8 | Don't do angular integral, since they just give 1 !

$$\langle PE \rangle = \left\langle -\frac{ke^2}{r} \right\rangle = \int_0^\infty dr\, r^2 \left(-\frac{ke^2}{r}\right)\left(R_{32}(r)\right)^2$$

$$= \frac{1}{(3a_0)^3}\left(\frac{2\sqrt{2}}{27\sqrt{5}}\right)^2 \frac{-ke^2}{a_0^4} \int_0^\infty dr\, r^2 \cdot \frac{1}{r} \cdot r^4 e^{-2r/3a_0}$$

$$= \left[\text{Stuff}\right] \int_0^\infty r^5 e^{-2r/3a_0}\, dr$$

$$= \left[\text{Stuff}\right] \int_0^\infty \left(\frac{3a_0}{2}\right)^6 u^5 e^{-u}\, du$$

$$= \left[\text{stuff}\right] \left(\frac{3a_0}{2}\right)^6 \cdot 5!$$

$$= \frac{1}{3^3 a_0^3} \frac{4 \cdot 2}{3^6 \cdot 5} \frac{-ke^2}{a_0^4} \cdot \frac{3^6 a_0^6}{2^6} \cdot 5!$$

$$= \frac{-ke^2}{a_0}\left(\frac{4 \cdot 2 \cdot \cancel{3^6} \cdot 5!}{3^3 \cdot \cancel{3^6} \cdot 5 \cdot 2^6}\right) = \frac{-ke^2}{a_0}\left(\frac{4!}{3^3 2^3}\right)$$

$$= -\frac{ke^2}{a_0}\left(\frac{1}{9}\right) = \boxed{-\frac{ke^2}{9a_0}}$$

b]

$$\langle KE \rangle = \langle E \rangle - \langle PE \rangle$$

$$= \frac{-ke^2}{2a_0} \cdot \frac{1}{9} + \frac{ke^2}{9a_0} = \boxed{\frac{ke^2}{18a_0}}.$$