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Modern Physics (2D)  
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Quiz #1

**Problem 1: Cosmic Voyage! [12 pts]**

A certain star is 95.0 light years away. How long would it take a spacecraft traveling at  $0.960c$  to reach that star from the Earth, as measured by observers: (a) on Earth, (b) on the spacecraft? (c) what is the distance traveled according to observers on the spacecraft? (d) What will the spacecraft occupants compute their speed to be from the results of (b) and (c) ?

**Problem 2: Tell it to the Judge! [8 pts]**

(a) How fast must you be approaching a red traffic light ( $\lambda=675\text{nm}$ ) for it to appear yellow ( $\lambda=575\text{nm}$ )? Express your answer in terms of  $c$ , the speed of light. (b) If you used this as an excuse to not getting a ticket for running a red light, how much of a fine would you get for speeding? Assume that the Judge orders a fine of \$1.00 for each km/h that your speed exceeds the posted limit of 90km/h. [Note:  $1\text{nm} = 10^{-9}\text{ m}$ ]

## QUIZ 1

- (1)(a) the observer on earth measures the proper distance between the star & earth since there is no relative motion between the observer and the distance they wish to measure. The time measured is then:

(3 pts)

$$\Delta t = \frac{L}{v} = \frac{95(9.46 \cdot 10^{15} \text{ m})}{(0.96 c)} = 3.1 \cdot 10^9 \text{ s}$$

or in terms of years

$$\frac{95 c (1 \text{ year})}{0.96 c} = 98.96 \text{ years}$$

since 1 light year  $\approx$   $c \cdot 1 \text{ year}$

- (b) The spacecraft observer will measure the proper time since the departure from the earth and the arrival at the star take place at the same location. The measured is then:

$$\gamma = 3.57$$

(3 pts)

$$\Delta t' = \Delta t / \gamma = 0.87 \cdot 10^9 \text{ s}$$

or

$$27.7 \text{ years}$$

- (c) Observers on the spacecraft will see a length contraction since they are not at rest relative to the distance they wish to measure

(3 pts)

$$\Rightarrow L' = L / \gamma = \frac{95 \text{ light years}}{3.57}$$

$$= 26.6 \text{ light years}$$

or

$$2.5 \cdot 10^{17} \text{ m}$$

(d) to the observers on the spacecraft, it would appear that they have travelled a length  $L' = L/\gamma$  in a time

(3 pts)  $\Delta t' = \Delta t/\gamma$ , so their measured velocity would be

$$\frac{L'}{\Delta t'} = \frac{(L/\gamma)}{(\Delta t/\gamma)} = \frac{L}{\Delta t} = v = 0.96c$$

(2) (a)  $\lambda_s = 675 \text{ nm}$ ,  $\lambda_{\text{obs}} = 575 \text{ nm}$   
 $\uparrow$  red  $\uparrow$  green

you are driving towards the traffic light, so we will the frequency doppler shifted according to:

(4 pts)

$$f_{\text{obs}} = f_s \sqrt{\frac{(1 + \frac{v}{c})}{(1 - \frac{v}{c})}} \quad \left( \begin{array}{l} \text{observer} \\ \text{approaching source} \end{array} \right)$$

(7 pts)

also:  $f\lambda = c$

$$\Rightarrow \frac{c}{\lambda_{\text{obs}}} = \frac{c}{\lambda_s} \sqrt{\frac{(1 + \frac{v}{c})}{(1 - \frac{v}{c})}}$$

$$\frac{\lambda_s}{\lambda_{\text{obs}}} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$

$$\left( \frac{\lambda_s}{\lambda_{\text{obs}}} \right)^2 (1 - \frac{v}{c}) = 1 + \frac{v}{c}$$

solving for  $\frac{v}{c}$  yields

$$\frac{v}{c} = \frac{\left( \frac{\lambda_s}{\lambda_{\text{obs}}} \right)^2 - 1}{\left( \frac{\lambda_s}{\lambda_{\text{obs}}} \right)^2 + 1} = 0.16$$

$$\Rightarrow v = 0.16c$$

(b) in terms of m/s

$$v = 4.77 \cdot 10^7 \text{ m/s}$$

convert to km/h

$$v = 4.77 \cdot 10^7 \frac{\text{m}}{\text{s}} \cdot \left( \frac{1 \text{ km}}{1000 \text{ m}} \right) \cdot \left( \frac{60 \text{ s}}{1 \text{ min}} \right) \left( \frac{60 \text{ min}}{1 \text{ hr}} \right)$$

(1 pts)

$$= 171.7 \cdot 10^6 \text{ km/hr}$$

you are going

$$v - v_{\text{limit}} = 171.7 \cdot 10^6 \text{ km/hr} - 90 \text{ km/hr}$$
$$\approx 171.7 \cdot 10^6 \text{ km/hr}$$

over the limit

so you owe the government

\$ 171.7 million dollars

$$\left( \$ 171.7 \cdot 10^6 \right)$$