33.) continued

For energy,

$$E_n = -\frac{ke^2}{2a_0}\frac{Z^2}{n^2}$$

$$= -\frac{ke^2 Z^2 (\mu k e^2)}{2\hbar^2 n^2}$$

$$\Rightarrow \boxed{E_1 = -18.9 \text{ MeV}}$$

Chapter 4

#4.) The "seeing" ability, or resolution, of radiation is determined by its wavelength. If the size of an atom is of the order of 0.1 nm, how fast must an electron travel to have a wavelength small enough to "see" an atom.

We require

$$\lambda = 0.1\,\text{nm} = \frac{h}{p}$$

$$\Rightarrow p = \frac{h}{0.1\,\text{nm}} = mv \qquad (v \ll c\text{, so this is ok})$$

$$\therefore \boxed{v = \frac{h}{m_e(0.1\,\text{nm})} = 7.28\times10^6\,\text{m/s}}$$

#6.) An electron and a proton each have kinetic energy equal to 50 keV. What are their de Broglie wavelengths?

Since $K = 50\text{ keV} \Rightarrow v \ll c$

$\therefore$ we can use

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\Rightarrow p = \sqrt{2mK} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{\sqrt{2mK}}$$

for e^-,

$$\boxed{\lambda_{e^-} = \frac{h}{\sqrt{2m_e K}} = 5.36\times10^{-3}\text{ nm}}$$

for proton,

$$\boxed{\lambda_p = \frac{h}{\sqrt{2m_p K}} = 1.28\times10^{-4}\text{ nm}}$$

#11.) For an electron to be confined to a nucleus, its de Broglie wavelength would have to be less than 10^{-14} m.

a.) What would be the kinetic energy of an electron confined to this region?

That is one small number, so to be safe we had better use relativity.

#11.) continued

We need

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{h}{1\times10^{-14}\,m} = 124\ MeV/c$$

Now,

$$E^2 = p^2c^2 + m^2c^4$$

$$\Rightarrow E = 0.124\ GeV \qquad \text{(Whoa momma!)}$$

$$\boxed{K = E - mc^2 = 124\ MeV}$$ That's a spicy meatball!

b.) Heck no. An e^- with that much kinetic energy would be able to bust out of a nucleus.

13.) Figure P4.13 shows the top three planes of a crystal with planar spacing d. If $2d\sin\theta = 1.01\lambda$ for the two waves shown, and high energy electrons of wavelength λ penetrate many planes deep into the crystal, which atomic plane produces a wave that cancels the surface reflection? Blah Blah Blah ...

To get destructive interference, we need

$$2d\sin\theta = \underbrace{(m+\tfrac{1}{2})\lambda}_{\text{path difference}} \quad \text{where } m = 0, 1, \ldots$$

In other words, we need to get the path difference to look like

somenumber.5

#13.) continued

The problem tells us that between two planes, the path difference is 1.01λ. If we multiply this by 50, we get

$$50.5\lambda$$

$$\Rightarrow \boxed{\text{the } 50^{th} \text{ plane produces the required wave}}$$

#16.) continued

$$v_g = v_p\big|_{k_0} + k \frac{dv_p}{dk}\bigg|_{k_0}$$

$$= \sqrt{\frac{k_0 S}{\rho}} + \frac{k_0}{2}\sqrt{\frac{S}{\rho k_0}}$$

$$v_g = \frac{3}{2}\sqrt{\frac{k_0 S}{\rho}} = \frac{3}{2} v_p\big|_{k_0}$$

$\Rightarrow$ $\boxed{v_g > v_p \text{, so the ripples appear to move inward}}$

#17.) The dispersion relation for free relativistic electron waves is

$$\omega(k) = \sqrt{c^2 k^2 + \left(\frac{m_e c^2}{\hbar}\right)^2}$$

Obtain expressions for the phase velocity v_p and group velocity v_g of these waves and show that their product is constant, independent of k. From your result, what can you conclude about v_g if $v_p > c$?

The phase velocity is just

$$v_p = \frac{\omega(k)}{k} = \frac{1}{k}\sqrt{c^2k^2 + \left(\frac{m_e c^2}{\hbar}\right)^2}$$

$$\boxed{v_p = \sqrt{c^2 + \left(\frac{m_e c^2}{\hbar k}\right)^2}}$$

#17) continued

The group velocity is

$$v_g = \frac{d\omega(k)}{dk} = \frac{d}{dk}\left(c^2k^2 + \left(\frac{m_e c^2}{\hbar}\right)^2\right)^{1/2}$$

$$= \frac{1}{2}\left(c^2k^2 + \left(\frac{m_e c^2}{\hbar}\right)^2\right)^{-1/2}(2c^2k)$$

$$\boxed{v_g = \frac{c^2k}{\sqrt{c^2k^2 + \left(\frac{m_e c^2}{\hbar}\right)^2}}}$$

Their product is

$$v_p \cdot v_g = \sqrt{c^2 + \left(\frac{m_e c^2}{\hbar k}\right)^2}\left(\frac{c^2 k}{\sqrt{c^2k^2 + \left(\frac{m_e c^2}{\hbar}\right)^2}}\right)$$

$$\boxed{\therefore\ v_p \cdot v_g = c^2 \neq f(k)}$$

$$\Rightarrow v_g = \frac{c^2}{v_p}$$

if $v_p = Ac$ where $A > 1$ (i.e., $v_p > c$)

$$\boxed{v_g = \frac{c^2}{Ac} = \frac{c}{A} < c}$$

#23.) A proton has a kinetic energy of 1.0 MeV. If its momentum is measured with an uncertainty of 5.0%, what is the minimum uncertainty in its position?

First of all,

$$\frac{K}{m_p c^2} = \frac{1.0\times10^6\,eV}{938.3\times10^6\,eV} = 0.11\,\% \Rightarrow \underline{\text{nonrelativistic}}$$

$$\Rightarrow K = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2mK} = 2.312\times10^{-20}\,\text{kg}\frac{\text{m}}{\text{s}}$$

$$\Delta p = 0.05\,p = 1.160\times10^{-21}\,\text{kg}\frac{\text{m}}{\text{s}}$$

$$\Delta p\, \Delta x_{min} = \frac{\hbar}{2}$$

$$\Rightarrow \boxed{\Delta x_{min} = \frac{\hbar}{2\Delta p} = 4.56\times10^{-14}\,\text{m}}$$

#24.) We wish to measure simultaneously the wavelength and position of a photon. Assume that the wavelength measurement gives $\lambda = 6000\,\text{Å}$ with an accuracy of one part in a million, i.e.

$$\frac{\Delta\lambda}{\lambda} = 1\times10^{-6}$$

What is the minimum uncertainty in the position of the photon?

We know that

$$p = \frac{h}{\lambda}$$

#24.) continued

$$\Rightarrow \Delta p = -\frac{h}{\lambda^2}\Delta\lambda$$

$$= -\frac{h}{\lambda}\left(\frac{\Delta\lambda}{\lambda}\right)$$

$$\Delta p \Delta x_{min} = \frac{\hbar}{2}$$

$$\Delta x_{min} = \frac{\hbar}{2\Delta p} = \frac{\hbar}{2}\left(-\frac{\lambda}{h}\left(\frac{\lambda}{\Delta\lambda}\right)\right)$$

$$= \frac{\lambda}{4\pi(\Delta\lambda/\lambda)}$$

$$\boxed{\Delta x_{min} = 4.78 \times 10^{-2}\ m}$$ (4 cm! That's quite a bit!)

#26.) A beam of electrons is incident on a slit of variable width. If it is possible to resolve a 1% difference in momentum, what slit width would be necessary to resolve the interference pattern of the electrons if their kinetic energy is

a.) 0.010 MeV

We're talking MeV here, so to be safe, let's use relativity.

2a.) continued

We know

$$K = mc^2(\gamma - 1)$$

$$\Rightarrow \gamma = \frac{K}{mc^2} + 1 = 1.02$$

$$= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

which gives $v = 5.91 \times 10^7$ m/s

Now,

$$p = \gamma m v = 5.49 \times 10^{-23} \frac{kg\,m}{s}$$

They tells us the uncertainty in momentum is

$$\Delta p = 0.01 p$$

And

$$\Delta x_{min} \Delta p = \frac{\hbar}{2}$$

$$\Delta x_{min} = \frac{\hbar}{2\Delta p} = \frac{\hbar}{(0.02 p)}$$

$$\boxed{\Delta x_{min} = 0.0961 \text{ nm}}$$

b.) 1.0 MeV

Using the same method, you get

26.) continued

$\gamma = 2.96$

$p = 7.60 \times 10^{-22}\ \frac{kg\,m}{s}$

$\Rightarrow \boxed{\Delta x_{min} = 0.00694\ nm}$

c.) 100 MeV

$\gamma = 197$

$p = 5.38 \times 10^{-20}\ kg\,m/s$

$\Rightarrow \boxed{\Delta x_{min} = 9.80 \times 10^{-14}\ m}$

#28.) An electron of momentum p is at a distance r from a stationary proton. The system has a kinetic energy $K = p^2/2m_e$ and potential energy $U = -ke^2/r$. Its total energy is $E = K + U$. If the electron is bound to the proton to form a hydrogen atom, its average position is at the proton but the uncertainty in its position is approximately equal to the radius r of the orbit. The electron's average momentum will be zero, but the uncertainty in its momentum will be given by the uncertainty principle. Treat the atom as a one dimensional system in the following:

#28.) continued

a.) Estimate the uncertainty in the electron's momentum in terms of r

We're told that

$$\Delta x = r$$

$$\Rightarrow \boxed{\Delta p \sim \frac{\hbar}{\Delta x} = \frac{\hbar}{r}}$$

(factors of $\frac{1}{2}$ or whatever don't really matter)

b.) Estimate the electron's kinetic, potential, and total energies in terms of r.

$$K = \frac{p^2}{2m}$$

$$p = p_{avg} + \Delta p = \Delta p$$

$$= \frac{(\Delta p)^2}{2m}$$

$$\boxed{K = \frac{\hbar^2}{2m_e r^2}}$$

$$\boxed{U = -\frac{ke^2}{r}}$$

$$\boxed{E = K + U = \frac{\hbar^2}{2m_e r^2} - \frac{ke^2}{r}}$$

#28.) continued

c.) The actual value of r is the one that minimizes the total energy, resulting in a stable atom. Find that value of r and the resulting total energy. Compare your answer with the predictions of the Bohr theory.

$$E = \frac{\hbar^2}{2m_e r^2} - \frac{ke^2}{r}$$

To minimize, take $\frac{dE}{dr}$ and set equal to zero

$$\frac{dE}{dr} = -\frac{\hbar^2}{m_e r^3} + \frac{ke^2}{r^2} = 0$$

$$\Rightarrow \boxed{r = \frac{\hbar^2}{m_e k e^2} = a_0 !}$$

$$\therefore \boxed{E_{min} = +\frac{\hbar^2}{2m_e a_0^2} - \frac{ke^2}{a_0} = -13.6 \text{ eV}}$$

Keen! That's the same as the Bohr theory. However, It's kind of just dumb luck (actually it's cuz I've seen this before) that we got the same numbers. When I said

$$\Delta p = \frac{\hbar}{r}$$

I could've thrown in any constants, like $\Delta p = \frac{\hbar}{2r}$ or somethin'!

#29.) An excited nucleus with a lifetime of 0.100 ns emits a γ ray of energy 2.00 MeV. Can the energy width (uncertainty in energy, ΔE) of this 2.00 MeV γ emission line be directly measured if the best gamma detectors can measure energies to ± 5 eV?

The lifetime of the particle is its time uncertainty, i.e.,

$$\Delta t = 0.100 \times 10^{-9}\ \text{s}$$

Using the energy-time form of the uncertainty principle,

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

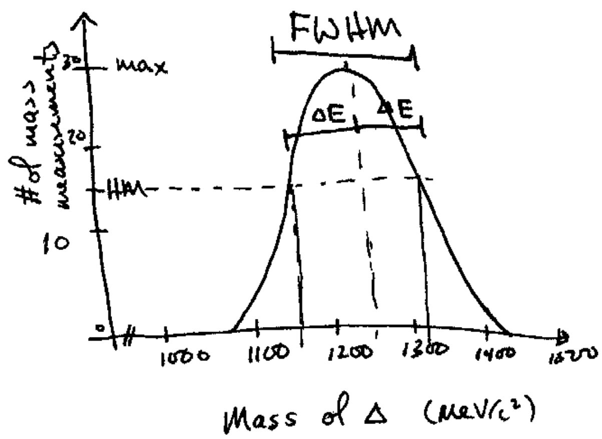
$$\Delta E \geq \frac{\hbar}{2\Delta t} = 3.29 \times 10^{-6}\ \text{eV}$$

So the width of the line is much less than ± 5 eV, and the detectors cannot measure it.

#30.) Typical measurements of the mass of a subatomic delta particle ($m \approx 1230$ MeV/ are shown in Fig. P4.30. Although the lifetime of the delta is much too short to measure directly, it can be calculated from the energy-time uncertainty principle. Estimate the lifetime from the full width at half maximum (FWHM) of the mass measurement distribution shown.

#30.) continued

Figure P 4.30 looks like



The max is 30 counts, which means half max is 15 counts

So FWHM = 110 MeV/c^2

$$\Rightarrow \Delta E = \frac{110 \text{ MeV}}{2} = 55 \text{ MeV}$$

And

$$\Delta E \, \Delta t_{min} = \frac{\hbar}{2}$$

$$\Rightarrow \boxed{\Delta t_{min} = \frac{\hbar}{2\Delta E} = 6.0 \times 10^{-24} \text{ s}}$$

#35.) An air rifle is used to shoot 1.0 g particles at 100 m/s through a hole of diameter 2.00 mm. How far from the rifle must an observer be to see the beam spread by 1.0 cm because of the uncertainty principle? Compare this answer with the diameter of the Universe (2×10^{26} m)

36

a) $E_{photon} = 1.8\,eV$, so $f = \frac{E}{h} = \boxed{4.35 \times 10^{14}\,Hz}$

b) $\lambda = \frac{c}{f} = 6.89 \times 10^{-7}\,m = \boxed{689\,nm}$

c) $\Delta E \Delta t \approx \frac{\hbar}{2}$, so since $\Delta t = 2\mu s$,

$$\Delta E \approx \frac{\hbar}{2(2\mu s)} = \boxed{1.6 \times 10^{-10}\,eV}$$ Tiny!

22] $p = mv = (0.05\,kg)(30\,\frac{m}{s}) = 1.5\,\frac{kg\,m}{s}$

So $\Delta p = (0.1\%)p = 1.5\times10^{-3}\,\frac{kg\cdot m}{s}$

$$\Rightarrow \Delta x\,\Delta p = \frac{\hbar}{2} \Rightarrow \Delta x = \frac{\hbar}{2\Delta p}$$

$$\boxed{\Delta x = 3.5\times10^{-32}\,m}$$

25] In the x-direction (horizontal) there's an uncertainty in velocity given by $\Delta x\, m\,\Delta v_x = \frac{\hbar}{2} \Rightarrow \Delta v_x = \frac{\hbar}{2m\Delta x}$

Since the average x-velocity is $\langle v_x \rangle = 0$, let's assume a worst-case scenario and say the pellet has $v_x = \Delta v_x = \frac{\hbar}{2m\Delta x}$. But we also know that in the time it takes to hit the ground, the pellet has moved a horizontal distance $\Delta x = \Delta v_x t$. + we can get,

since $\frac{1}{2}gt^2 = H \Rightarrow t = \sqrt{\frac{2H}{g}}$.

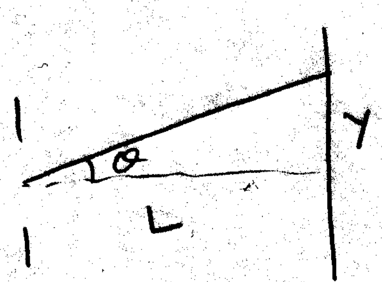
So $\Delta x = \Delta v_x \sqrt{\frac{2H}{g}} \Rightarrow (\Delta x)^2 = \left(\frac{\hbar}{2m}\right)\left(\frac{2H}{g}\right)^{1/2}$

$$\boxed{\Delta x = \left(\frac{\hbar}{2m}\right)^{1/2}\left(\frac{2H}{g}\right)^{1/4}}$$

31 For single slit diffraction, minima are given by

$$a \sin\theta = n\lambda, \quad n = 1, 2, 3, \ldots$$

For small θ, we can write θ in terms of the position on the screen and the dist. to the screen:



$$\theta \approx \tan\theta = \frac{Y}{L}$$

So $\frac{a Y_n}{L} = n\lambda$ where Y_n is the pos. of the n^{th} minimum.

So $Y_n = \frac{L}{a} n\lambda \Rightarrow Y_{n+1} - Y_n = \frac{L\lambda}{a} = 2.1\,\text{cm}$

$$\Rightarrow \lambda = \frac{(0.5\,\text{nm})(2.1\,\text{cm})}{20\,\text{cm}} = 5.25 \times 10^{-2}\,\text{nm}$$

So now $p = \frac{h}{\lambda} = 1.26 \times 10^{-23}\,\frac{\text{kg}\cdot\text{m}}{\text{s}}$

So $E = \frac{p^2}{2m} = 8.74 \times 10^{-17}\,\text{J} = \boxed{546\,\text{eV}}$

32

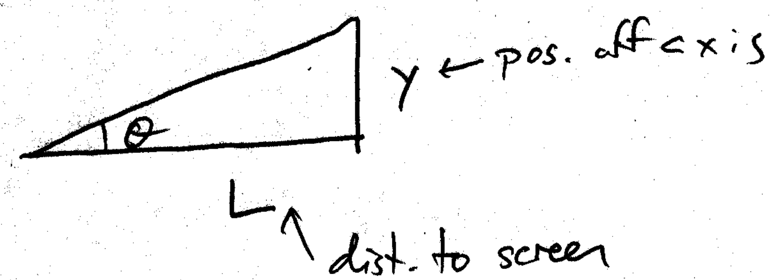
a) $\lambda = \frac{h}{p} = \frac{h}{mv} = \boxed{989\,nm}$

b) $d \sin\theta = \frac{\lambda}{2}$ for 1^{st} min.

$$(1mm)\sin\theta = \frac{989\,nm}{2}$$

$$\Rightarrow \sin\theta = 4.95\times10^{-4}$$

But $\theta = \frac{y}{L}$



$$\Rightarrow y = (10m)(4.95\times10^{-4})$$

$$\boxed{y = 4.94\,mm}$$

c) No! This diffraction pattern arises b/c of the wave-like properties of the neutron beam. If we knew which slit it went through, we'd never see a diffraction pattern.

33] With one slit open, we see ψ_1. With the other, we see ψ_2. The #/second is the probability (or related to it)

So $|\psi_1|^2 = 25|\psi_2|^2$.

Now, with both open, we see $\psi_1 + \psi_2$. The maximum Probability will be when these constructively interfere, and

$$P_{max} = (|\psi_1| + |\psi_2|)^2.$$

Similarly, min. probability is when we get destructive interference,

$$P_{min} = (|\psi_1| - |\psi_2|)^2$$

So $$\frac{P_{max}}{P_{min}} = \frac{|\psi_1|^2 + |\psi_2|^2 + 2|\psi_1||\psi_2|}{|\psi_1|^2 + |\psi_2|^2 - 2|\psi_1||\psi_2|}$$

$$= \frac{1 + 25 + 2 \cdot 5}{1 + 25 - 2 \cdot 5} = \boxed{\frac{36}{16} = 2.25}$$