Physics 2D Week 4 HW Sols

Ch. 2: 2, 4, 9, 12, 13, 16, 19, 21, 24, 26, 30, 34, 36

2] Wien's law: $\lambda_{max} T = 2.898 \times 10^{-3}\, m \cdot K$

(it's on p. 60)

So using $T = 35°C = 308\,K$, we get

$$\lambda_{max} = 9.410 \times 10^{-6}\, m = \boxed{9410\, nm}$$

4] a] $e_{TOT} = \sigma T^4 = 5.67 \times 10^{-8} \frac{W}{m^2 K^4} (3000\,K)^4$

$$= \boxed{4.6 \times 10^6\, W/m^2}$$

b] $75\, W = e_{TOT} \cdot \text{Area}$

$$\Rightarrow \text{Area} = 1.63 \times 10^{-5}\, m^2 = \boxed{16.3\, mm^2}$$

9] 1 photon has $E = hf = (6.626 \times 10^{-34}\, J \cdot s)(94\, MHz)$

$$= 6.23 \times 10^{-26}\, J$$

$100\, kW = 10^5 \frac{J}{s}$, so this means $\frac{10^5}{6.23 \times 10^{-26}} = \boxed{1.6 \times 10^{30}\ \text{photons/sec}}$

12] So $K_{max} = 2.92 eV$.

Thus $2.92 eV = hf - \phi$

But $hf = \frac{hc}{\lambda} = \frac{1240 \, eV \cdot nm}{250 \, nm} = 4.96 \, eV$

So $\phi = (4.96 - 2.92 \, eV) = \boxed{2.04 eV}$

13] $K_{max} = hf - \phi = \frac{hc}{\lambda} - \phi = \frac{1240 eV \cdot nm}{350 nm} - 2.24 eV$

$= \boxed{1.30 eV}$

b] Cutoff wavelength:

$\phi = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\phi} = \frac{1240 \, eV \cdot nm}{2.24 eV}$

$= \boxed{554 \, nm}$

16] a] $E_{light} = \frac{hc}{\lambda} = 4.13 \, eV$

So $\boxed{\text{lithium \& beryllium}}$ will eject electrons, since

$\phi < E_{light}$

b] $K_{max} = hf - \phi = \begin{cases} 1.83 eV & \text{for lithium} \\ 0.23 eV & \text{for beryllium.} \end{cases}$

19] $K_{max} = \frac{hc}{\lambda} - \phi$, so for the first wavelength,

$1\,eV = \frac{hc}{\lambda} - \phi$. The second wavelength is $\frac{\lambda}{2}$, and so

$4\,eV = \frac{2hc}{\lambda} - \phi$. We want to solve for ϕ, so just divide the 2nd eqn by 2 and subtract:

$$
\begin{array}{r}
1\,eV = \frac{hc}{\lambda} - \phi \\
-\left(2\,eV = \frac{hc}{\lambda} - \frac{\phi}{2}\right) \\
\hline
-1\,eV = -\frac{\phi}{2}
\end{array}
\Rightarrow \boxed{\phi = 2\,eV}
$$

21] Need to find K_{max}, so need v for the electrons. For e^-'s in a magnetic field, we know $F = \boxed{qvB = \frac{mv^2}{r}}$

(it's safe to use non-relativistic mechanics here, since these are not very energetic electrons).

So $v = \frac{qrB}{m}$, and $K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{q^2r^2B^2}{m}$

again, nonrelativistic is OK.

Plugging in yields $K = \boxed{1.4\,eV}$

So $1.4\,eV = \frac{hc}{\lambda} - \phi \Rightarrow \phi = \frac{hc}{450\,nm} - 1.4\,eV$

$= \boxed{1.36\,eV}$

24] a] $\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$

$= \frac{h}{m_e c}(1 - \cos 30^\circ)$

$= 3.25 \times 10^{-13}\,m = \boxed{3.25 \times 10^{-4}\,nm}$

b] $\lambda' = \lambda_0 + \frac{h}{m_e c}(1 - \cos\theta)$.

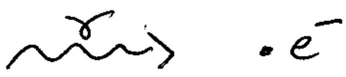
And $\lambda_0 = \frac{hc}{E} = \frac{1240\,eV \cdot nm}{300\,keV} = 4.13 \times 10^{-3}\,nm$.

So $\lambda' = 4.46 \times 10^{-3}\,nm \Rightarrow E' = \frac{hc}{\lambda} = \frac{1240\,eV\,nm}{4.46 \times 10^{-3}\,nm} = \boxed{278\,keV}$

c] $KE_{e^-} = E' - E = \boxed{22\,keV}$ (that's just the kinetic energy - you could include rest energy too if you wanted).

26 | The setup:

Before



After

Energy | $E_\gamma + m_e c^2 = E_e$

Momentum | $p_\gamma = p_e$

So since $E_\gamma = p_\gamma c$ and $E_e = \sqrt{p_e^2 c^2 + m^2 c^4}$

we can combine these to say

$$\boxed{pc + m_e c^2 = \sqrt{p^2 c^2 + m_e^2 c^4}} \quad (p = p_e = p_\gamma)$$

Square both sides: $p^2c^2 + 2pm_e c^3 + m_e^2 c^4 = p^2 c^2 + m_e^2 c^4$

$$\Rightarrow \boxed{2pm_e c^3 = 0}$$

This means $p = 0$, which says there's no photon to begin with. So the only solution is to have a free electron just hanging out.

30 We know $\lambda' = \lambda_o + \frac{h}{m_e c}(1-\cos\theta)$.

For max energy transfer, λ should increase (i.e. the energy of the photon should decrease) as much as possible.

So $\theta = 180° \Rightarrow \boxed{\lambda' = \lambda_o + \frac{2h}{m_e c}}$.

Now, $E = \frac{hc}{\lambda}$ for a photon.

Let's call $E' = \frac{hc}{\lambda'}$, $E_o = \frac{hc}{\lambda_o}$. So then

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\left(\lambda_o + \frac{2h}{m_e c}\right)}.$$

Let's assume $\frac{2h}{m_e c} \ll \lambda_o$ and check this later.

Then $\frac{1}{\lambda_o + \frac{2h}{m_e c}} = \frac{1}{\lambda_o\left(1 + \frac{2h}{m_e c \lambda_o}\right)} \approx \frac{1}{\lambda_o}\left(1 - \frac{2h}{m_e c \lambda_o}\right)$

So $\boxed{E' = E_o - \frac{2h^2c^2}{m_e c^2 \lambda_o^2}}$ (I multiplied top & bottom of the fraction by c)

This makes sense: $\Delta E < 0$ for the photon.

So $|\Delta E| = \frac{2h^2c^2}{m_e c^2 \lambda_o^2}$, and we know $|\Delta E| = 30\,keV$.

So $\lambda_0 = \left(\frac{2(hc)^2}{m_e c^2 |\Delta E|} \right)^{1/2}$

$$= \left(\frac{2(1240\,\text{eV}\cdot\text{nm})^2}{511\,\text{keV} \cdot 30\,\text{keV}} \right)^{1/2} = \boxed{1.42 \times 10^{-2}\,\text{nm}}$$

Is this consistent with $\frac{2h}{m_e c} \ll \lambda_0$?

Well, $\frac{2h}{m_e c} = \frac{2hc}{m_e c^2} = \frac{2 \cdot 1240}{511 \times 10^3}\,\text{nm} = 4.85 \times 10^{-3}\,\text{nm}$

so $\frac{\left(\frac{2h}{m_e c}\right)}{\lambda_0} = 0.34$ which is kind of small.

Not a great approximation, but a decent one.

34] From #30, we know $\lambda_0 = \left(\frac{2(hc)^2}{m_e c^2 |\Delta E|} \right)^{1/2}$

So $\lambda_0 = \left(\frac{2(1240\,\text{eV}\cdot\text{nm})^2}{511\,\text{keV} \cdot 50\,\text{keV}} \right)^{1/2} = 1.1 \times 10^{-2}\,\text{nm}$.

$$\Rightarrow E = \frac{hc}{\lambda_0} = \frac{1240\,\text{eV nm}}{1.1 \times 10^{-2}\,\text{nm}} = \boxed{113\,\text{keV}}$$

36] $E_{final, photon} = 80\,\text{keV}$

$E_e = 25\,\text{keV}$

So by energy conservation, $E_{i, photon} = 105\,\text{keV}$

$$\Rightarrow \lambda = \frac{hc}{E} = \boxed{1.18 \times 10^{-2}\,\text{nm}}$$

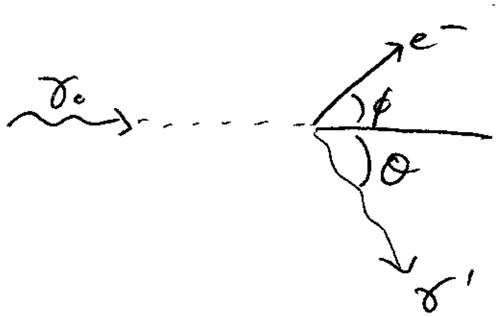
b] $\lambda' = \frac{hc}{E'} = 1.55 \times 10^{-2}$ nm

So since $\lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)$ and $\frac{h}{m_e c} = 2.43 \times 10^{-3}$ nm,

$$1 - \cos\theta = \frac{\lambda' - \lambda_0}{h/m_e c} \Rightarrow \cos\theta = -0.52$$

So $\boxed{\theta = 121^\circ}$

c]



Cons. of momentum: $p_e \sin\phi = p_{\gamma'} \sin\theta \Rightarrow \boxed{\sin\phi = \frac{p'}{p_e}\sin\theta}$

$p' = \frac{E'}{c}$, and $E' = 80$ keV $\Rightarrow p' = 80 \frac{\text{keV}}{c}$.

Since $E_e^2 = p_e^2 c^2 + m_e^2 c^4$, $p_e^2 = \frac{E_e^2 - m_e^2 c^4}{c^2}$

Careful — E_e = total energy of e^- = rest + kinetic = (511+25) keV

So $p_e = 162 \frac{\text{keV}}{c}$

$$\Rightarrow \sin\phi = \frac{80}{162}\sin\theta = 0.42 \Rightarrow \boxed{\phi = 25^\circ}$$