#26.) continued

Either way, you get

$$\boxed{\begin{array}{l} a.)\ p = 9.38\ \text{MeV}/c \\ b.)\ p = 540\ \text{MeV}/c \\ c.)\ p = 1930\ \text{MeV}/c \end{array}}$$

#28.) Consider the relativistic form of Newton's second law. Show that when $\vec{F}$ is parallel to $\vec{v}$

$$F = m\left(1 - \frac{v^2}{c^2}\right)^{-3/2} \frac{dv}{dt}$$

where m is the mass of an object and v is its speed

The relativistic form of Newton's second law is (eq. 1.36)

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left[\frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}\right]$$

Using the chain rule, (Assuming $\vec{F} \parallel \vec{v}$)

$$\frac{d}{dt} = \frac{dv}{dt}\frac{d}{dv}$$

$$\therefore F = \frac{d}{dv}\left[\frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}\right]\frac{dv}{dt}$$

$$= \left[\frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{mv}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}\left(-\frac{1}{2}\right)\left(-\frac{2v}{c^2}\right)\right]\frac{dv}{dt}$$

29

a) So $F = qE = \dfrac{dp}{dt}$.

Relativistically, though, $p = \dfrac{mv}{\sqrt{1-v^2/c^2}}$.

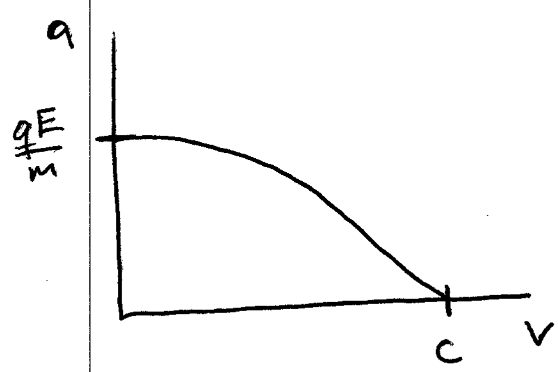
$$\text{So } qE = m\left[\frac{dv}{dt}\frac{1}{\sqrt{1-v^2/c^2}} + \frac{\cancel{m}v}{(1-v^2/c^2)^{3/2}}\cdot\frac{-1}{2}\overbrace{\left(-\frac{2v}{c^2}\right)\frac{dv}{dt}}^{\text{from } \frac{d}{dt}\left(\frac{v^2}{c^2}\right) = \frac{2v}{c^2}\frac{dv}{dt}}\right]$$

$$\Rightarrow qE = m\left[\frac{dv}{dt}\frac{1}{\sqrt{1-v^2/c^2}} + \frac{v^2/c^2}{(1-v^2/c^2)^{3/2}}\frac{dv}{dt}\right]$$

$$\Rightarrow qE = m\frac{dv}{dt}\left(1-v^2/c^2\right)^{-3/2}$$

$$\text{So } \boxed{a = \frac{dv}{dt} = \frac{qE}{m}\left(1-v^2/c^2\right)^{3/2}}$$

b) Let's graph this:



So as $v \to c$, $a \to 0$. Can't accelerate past $v = c$!

c] $$\frac{dv}{dt} = \frac{qE}{m}\left(1 - v^2/c^2\right)^{3/2}$$

So $$\frac{dv}{(1-v^2/c^2)^{3/2}} = \frac{qE}{m}\,dt$$

Integrate both sides: $$\int_0^v \frac{dv'}{(1-v'^2/c^2)^{3/2}} = \int_0^t \frac{qE}{m}\,dt'$$

$$\int \frac{dv}{(1-v^2/c^2)^{3/2}} = \frac{v}{(1-v^2/c^2)^{1/2}}$$ (look in table or do it)

So $$\frac{v}{(1-v^2/c^2)^{1/2}} = \frac{qE}{m}t$$

$$\Rightarrow \frac{v^2}{1-v^2/c^2} = \left(\frac{qE}{m}\right)^2 t^2 \Rightarrow \boxed{v = \frac{qE}{m}\left[\frac{t}{\left(1+\left(\frac{qEt}{mc}\right)^2\right)^{1/2}}\right]}$$

Note: As $t \to \infty$, $v \to c$, which is easy to see from rewriting $v = \frac{qEct}{\sqrt{(mc)^2 + (qEt)^2}}$. Nice!

Using $v = \frac{dx}{dt}$, integrate again:

$$\int_0^x dx' = \int_0^t \frac{qE}{m}\left(\frac{t'}{\sqrt{1+\left(\frac{qEt'}{mc}\right)^2}}\right)dt'$$

Writing (for the RHS) $u = 1 + \left(\frac{qEt'}{mc}\right)^2$, we see

$$du = 2t'\left(\frac{qE}{mc}\right)^2 dt'$$

So
$$\int \frac{qE}{m}\left(\frac{t'\,dt'}{\sqrt{1+\left(\frac{qEt'}{mc}\right)^2}}\right) = \int \frac{qE}{m}\, du \cdot \frac{1}{2}\left(\frac{mc}{qE}\right)^2 u^{-1/2}$$

$$= \frac{1}{2}c^2\left(\frac{m}{qE}\right)\cdot 2u^{1/2}\Big|_{u=1}^{u=1+\left(\frac{qEt}{mc}\right)^2}$$

$$= c^2\left(\frac{m}{qE}\right)\left[\sqrt{1+\left(\frac{qEt}{mc}\right)^2} - 1\right]$$

So $\boxed{x = \frac{c}{qE}\left[\sqrt{(mc)^2 + (qEt)^2} - mc\right]}$

As $t \to \infty$, $x \to ct$. As expected for something moving with speed c!

37] $KE = \gamma mc^2 - mc^2$

$$\gamma = \frac{1}{\sqrt{1-(.75)^2}} = 1.51$$

$m_{electron} = .511\ MeV/c^2$

So $KE = (0.51)(.511\ MeV) = 260\ keV$

Now, $KE_{proton} = 260\ keV$ and $m_{proton} = 938\ MeV/c^2$

So $(\gamma - 1)(938\ MeV) = 260\ keV$

$$\Rightarrow \gamma - 1 = \frac{.260}{938}$$

$$\Rightarrow \gamma = 1.000277$$

This is slow! Use $\gamma \approx 1 + \frac{1}{2}\frac{v^2}{c^2} \Rightarrow \frac{1}{2}\frac{v^2}{c^2} = 2.77 \times 10^{-4}$

$$\Rightarrow \boxed{v = 0.0235c}$$

that's 0.0235c

b] Now $p_{electron} = \gamma m v$

$$= (1.51)\left(\frac{.511 MeV}{c^2}\right)(.75c) = 0.579\ \frac{MeV}{c}$$

So $p_{proton} = 0.579 \frac{MeV}{c}$

$$\Rightarrow \gamma v \left(938 \frac{MeV}{c^2}\right) = 0.579 \frac{MeV}{c}$$

$$\Rightarrow \gamma v = (6.17 \times 10^{-4}) c$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \Rightarrow \frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} = (6.17 \times 10^{-4}) c$$

$$\Rightarrow \frac{v^2}{1 - \frac{v^2}{c^2}} = \left((6.17 \times 10^{-4}) c\right)^2$$

$$\Rightarrow \left(v^2/c^2\right)\left(1 + (6.17 \times 10^{-4})^2\right) = (6.17 \times 10^{-4})^2$$

$$\Rightarrow v = \frac{6.17 \times 10^{-4}}{\sqrt{1 + (6.17 \times 10^{-4})^2}} c$$

$$\boxed{v = 6.17 \times 10^{-4} c}$$

#38.) continued

b.) What is their kinetic energy in MeV?

$$K = E - mc^2$$
$$= 399\,mc^2$$
$$\boxed{K = 3.74 \times 10^5\ \text{MeV}}$$

#39.) How long will the Sun shine, Nellie? The Sun radiates about 4.0×10^{26} J of energy into space each second. (Who is Nellie? Am I Nellie?)

a.) How much mass is released as radiation each second?

Just use

$$E = mc^2$$
$$\Rightarrow \boxed{m = \frac{E}{c^2} = 4.4 \times 10^9\ \text{kg}}$$

b.) If the mass of the sun is 2.0×10^{30} kg, how long can the sun survive if the energy release continues at the present rate?

From part a.), the sun loses mass at the rate

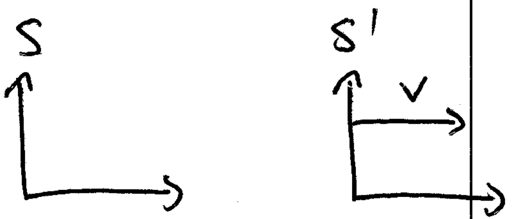
$$\frac{\Delta m}{\Delta t} = \frac{4.4 \times 10^9\ \text{kg}}{1\ \text{sec}}$$

$$\Rightarrow (2.0 \times 10^{30}\ \text{kg})\left(\frac{1\ \text{sec}}{4.4 \times 10^9\ \text{kg}}\right) = 454 \times 10^{18}\ \text{sec}$$

$$\therefore \boxed{\Delta t = 1.4 \times 10^{13}\ \text{yrs.}}$$

Smoke 'em if you got 'em

41]



So $p' = \dfrac{mu'}{\sqrt{1-u'^2/c^2}}$, $E' = \dfrac{mc^2}{\sqrt{1-u'^2/c^2}}$

$$u' = \frac{u-v}{1-\frac{uv}{c^2}}.$$

Thus $1 - u'^2/c^2 = 1 - \dfrac{1}{c^2}\left(\dfrac{u-v}{1-\frac{uv}{c^2}}\right)^2 = 1 - \dfrac{(u-v)^2}{c^2\left(1-\frac{uv}{c^2}\right)^2}$

$$\Rightarrow \boxed{p' = m\left[\frac{u-v}{1-\frac{uv}{c^2}}\right]\left[1 - \frac{(u-v)^2}{c^2\left(1-\frac{uv}{c^2}\right)^2}\right]^{-1/2}}$$ Nastay!

and $$\boxed{E' = mc^2\left[1 - \frac{(u-v)^2}{c^2\left(1-\frac{uv}{c^2}\right)^2}\right]^{-1/2}}$$ (So $p' \neq p$, $E' \neq E$).

b] Lots of algebra, but it works. I'm not gonna do it here.

45] $n \to p + e^- + \bar{\nu}$

Total initial energy $E_i = m_n c^2 = 939.6\ \text{MeV}$.

Total final energy $E_f = m_p c^2 + m_e c^2 + m_{\bar{\nu}} c^2 + K$

$m_p c^2 = 938.3\ \text{MeV}$

$m_e c^2 = 0.511\ \text{MeV}$

$m_{\bar{\nu}} c^2 = 0$

So $E_f = (938.3 + 0.511 + (0.781 \pm 0.005))\ \text{MeV}$

$= 939.592 \pm 0.005\ \text{MeV}$

939.6 MeV is in this range, so it's consistent.

DAMN!

#47.) The K° meson is an uncharged member of the particle "zoo" that decays into two charged pions according to

$$K^\circ \rightarrow \pi^+ + \pi^-$$

The pions have opposite charges as indicated and have the same mass, $m_\pi = 140\ MeV/c^2$. Suppose that a K° at rest decays into two pions in a bubble chamber in which a magnetic field of 2.0T is present. If the radius of curvature of the pions is 34.4cm find

a.) the momenta and speeds of the pions

We can use #31.) from the last homework set

$$p = 300\ BR$$

Plug in values to get

$$\boxed{p = 206\ MeV/c}$$

Since the K°'s momentum is zero, the pions' momenta must be equal and opposite in direction

$$p_{\pi^+} = -p_{\pi^-}$$

Note that

$$p = \gamma m u$$
$$E = \gamma m c^2$$

$$\Rightarrow \frac{p}{E} = \frac{u}{c^2}$$

$$\therefore \boxed{u = \frac{pc^2}{E} = \frac{(206 MeV/c)\ c^2}{pc/\sqrt{p^2c^2 + m^2c^4}} = 0.827c}$$

#47.) continued

b.) the mass of the K^0 meson

Conservation of mass-Energy tells us

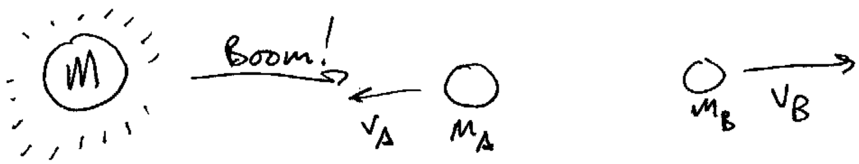
$$E_{K_0} = E_{\pi^+} + E_{\pi^-} = 2E$$

$$E_{K_0} = m_{K^0} c^2$$

$$2E = 2\sqrt{p_\pi^2 c^2 + m_\pi^2 c^4} = 498\,\text{MeV}$$

$$\therefore \quad \boxed{m_{K_0} = 498\ \text{MeV}/c^2}$$

48.) An unstable particle having a mass of 3.34×10^{-27} kg is initially at rest. The particle decays into two fragments that fly off with velocities of 0.987 c and −0.868 c. Find the rest masses of the fragments



Conserve mass energy

$$Mc^2 = \gamma_A m_A c^2 + \gamma_B m_B c^2 \qquad (3)$$

$$\gamma_A = 6.22$$
$$\gamma_B = 2.01$$

Conserve momentum

$$\gamma_A m_A u_A = \gamma_B m_B u_B$$

$$\Rightarrow m_A = \frac{\gamma_B}{\gamma_A} m_B u_B = 0.284\, m_B$$

#48.) continued

Substitute into (3) + plug in values

$$3.34\times10^{-27}\,kg = (6.22)(.284)\,m_A + 2.01\,m_A$$

$$\therefore m_A = 8.84\times10^{-28}\,kg$$

$$m_B = 0.284\,m_A = 2.51\times10^{-28}\,kg$$

#52.) If astronauts could travel at $v=0.95c$, we on Earth would say it takes $(4.2/0.95) = 4.4$ years to reach Alpha Centauri, 4.2 lightyears away. The astronauts disagree.

a.) How much time passes on the astronaut's clocks?

A time dilation problem. Recall

$\Delta t' = \gamma \Delta t$ (Δt proper time)

$$\Rightarrow \boxed{\Delta t = \frac{\Delta t'}{\gamma} = 1.37 \text{ years}}$$

b.) What distance to Alpha Centauri do the astronauts measure?

A Lorentz contraction problem. Recall

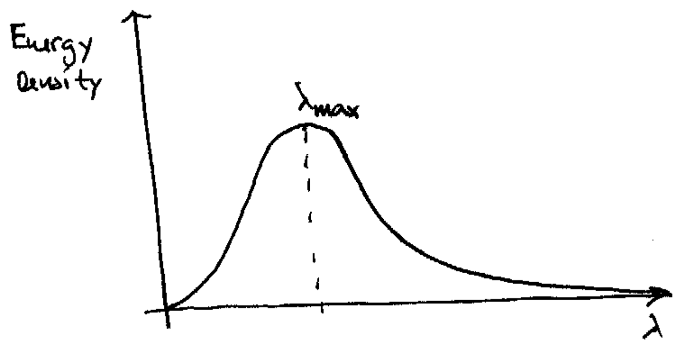
$\Delta x' = \gamma \Delta x$

$$\Rightarrow \boxed{\Delta x = \frac{\Delta x'}{\gamma} = 1.31 \text{ lightyears}}$$

Physics 2D Homework Ch.2

#2) The temperature of your skin is approximately 35°C. What is the wavelength at which the peak occurs in the radiation emitted from your skin?

Assume your skin emits like a black body (it doesn't, but we're going to anyway). So its radiation spectrum looks like



We're looking λ_{max}. Use Wien's displacement law (you kind of have to hunt for it)

$$\lambda_{max} T = 2.898 \times 10^{-3} \text{ m}\cdot\text{K} \qquad \text{eq. } 2.6$$

$$\Rightarrow \lambda_{max} = \frac{2.898 \times 10^{-3} \text{ m}\cdot\text{K}}{T}$$

make sure T is in Kelvin!

$$T = 35^\circ C = 308 \text{ K}$$

$$\therefore \boxed{\lambda_{max} = 9410 \text{ nm}}$$

4\] a\] $e_{TOTAL} = \sigma T^4$

$$= 5.67\times10^{-8} \frac{W}{m^2 K^4} (3000\,K)^4$$

$$= \boxed{4.6\times10^{6}\ W/m^2}$$

b\] So $75\,W = (4.6\times10^{6}\ W/m^2)\cdot \text{Area}$

$$\Rightarrow \text{Area} = \boxed{1.63\times10^{-5}\ m^2} = 16.3\ mm^2.$$